Chapter 1-2 - Unsteady Open Channel Flows. 2- Applications

1- List the key assumptions of the Saint-Venant equations.

2- What is the celerity of a small disturbance in (1) a rectangular channel, (2) a 90-degree V-shaped channel and (3) a channel of irregular cross-section? (4) Application: a flow cross-section of a flood plain has the following properties: hydraulic diameter = 5.14 m, maximum water depth = 2.9 m, wetted perimeter = 35 m, free-surface width = 30 m. Calculate the celerity of a small disturbance.

3- A 0.2 m high small wave propagates downstream in a horizontal channel with initial flow conditions $V = +0.1$ m/s and $d = 2.2$ m. Calculate the propagation speed of the small wave.

4- Uniform equilibrium flow conditions are achieved in a long rectangular channel ($W = 12.8$ m, concrete lined, $S_o = 0.0005$). The observed water depth is 1.75 m. Calculate the celerity of a small monoclinal wave propagating downstream. Perform your calculations using the Darcy friction factor.

5- The flow rate in a rectangular canal ($W = 3.4$ m, concrete lined, $S_o = 0.0007$) is 3.1 $m^3/s$ and uniform equilibrium flow conditions are achieved. The discharge suddenly increases to 5.9 $m^3/s$. Calculate the celerity of the monoclinal wave. How long will it take for the monoclinal wave to travel 20 km?

6- What is the basic definition of a simple wave? May the simple wave theory be applied to (1) a sloping, frictionless channel, (2) a horizontal, rough canal, (3) a positive surge in a horizontal, smooth channel with constant water depth, (4) a smooth, horizontal canal with an initially accelerating flow?

7- What is the "zone of quiet"?

8- Considering a long, horizontal rectangular channel ($W = 4.2$ m), a gate operation, at one end of the canal, induces a sudden withdrawal of water resulting in a negative velocity. At the gate, the boundary conditions for $t > 0$ are: $V(x = 0, t) = -0.2$ m/s. Calculate the extent of the gate operation influence in the canal at $t = 1$ hour. The initial conditions in the canal are: $V = 0$ and $d = 1.4$ m.

9- Water flows in an irrigation canal at steady state ($V = 0.9$ m/s, $d = 1.65$ m). The flume is assumed smooth and horizontal. The flow is controlled by a downstream gate. At $t = 0$, the gate is very-slowly raised and the water depth upstream of the gate decreases at a rate of 5 cm per minute until the water depth becomes 0.85 m. (1) Plot the free-surface profile at $t = 10$ minutes. (2) Calculate the discharge per unit width at the gate at $t = 10$ minutes.

10- A 200 km long rectangular channel ($W = 3.2$ m) has a reservoir at the upstream end and a gate at the downstream end. Initially the flow conditions in the canal are uniform: $V = 0.35$ m/s, $d = 1.05$ m. The water surface level in the reservoir begins to rise at a rate of 0.2 m per hour for 6 hours. Calculate the flow conditions in the canal at $t = 2$ hours. Assume $S_o = S_f = 0$.

12- Waters flow in a horizontal, smooth rectangular channel. The initial flow conditions are $d = 2.1$ m and $V = +0.3$ m/s. The flow rate is stopped by sudden gate closure at the downstream end of the canal. Using the quasi-steady flow analogy, calculate the new water depth and the speed of the fully-developed surge front.

13- Let consider the same problem as above (i.e. a horizontal, smooth rectangular channel, $d = 2.1$ m and $V = 0.3$ m/s) but the downstream gate is closed slowly at a rate corresponding to a linear decrease in flow rate from 0.63 m$^2$/s down to zero in 15 minutes. (1) Predict the surge front development. (2) Calculate the free-surface profile at $t = 1$ hour after the start of gate closure.

14- A 5 m wide forebay canal supplies a penstock feeding a Pelton turbine. The initial conditions in the channel are $V = 0$ and $d = 2.5$ m. (1) The turbine starts suddenly operating with 6 m$^3$/s. Predict the water depth at the downstream end of the forebay canal. (2) What is the maximum discharge that the forebay channel can supply? Use a simple wave theory.
15- Write the kinematic wave equation for a wide rectangular channel, in terms of the flow rate, bed slope and water depth. What is the speed of a kinematic wave? Does the kinematic wave routing predict subsidence?

16- A wide channel has a bed slope $S_0 = 0.0003$ and the channel bed has an equivalent roughness height of 25 mm. The initial flow depth is 2.3 m and uniform equilibrium flow conditions are achieved. The water depth is abruptly increased to 2.4 m at the upstream end of the channel. Calculate the speed of the diffusion wave and the diffusion coefficient.

17- A 8 m wide rectangular canal (concrete lining) operates at uniform equilibrium flow conditions for a flow rate of 18 m$^3$/s resulting in a 1.8 m water depth. At the upstream end, the discharge is suddenly increased to 18.8 m$^3$/s. Calculate the flow rate in the canal 1 hour later at a location $x = 15$ km. Use diffusion routing.

18- A 15 m high dam fails suddenly. The dam reservoir had a 13.5 m depth of water and the downstream channel was dry. (1) Calculate the wave front celerity, and the water depth at the origin. (2) Calculate the free-surface profile 2 minutes after failure. Assume an infinitely long reservoir and use a simple wave analysis ($S_0 = S_f = 0$).

19- A vertical sluice shut a trapezoidal channel (3 m bottom width, 1V:3H side slopes). The water depth was 4.2 m upstream of the gate and zero downstream (i.e. dry channel). The gate is suddenly removed. Calculate the negative celerity. Assuming an ideal dam break wave, compute the wave front celerity and the free-surface profile one minute after gate removal.

20- A 5 m high spillway gate fails suddenly. The water depth upstream of the gate was 4.5 m depth and the downstream concrete channel was dry and horizontal. (1) Calculate the wave front location and velocity at $t = 3$ minutes. (2) Compute the discharge per unit width at the gate at $t = 3$ minutes. Use DRESSLER's theory assuming $f = 0.01$ for new concrete lining. (3) Calculate the wave front celerity at $t = 3$ minutes using WHITHAM's theory.

21- A horizontal, rectangular canal is shut by a vertical sluice. There is no flow motion on either side of the gate. The water depth is 3.2 m upstream of the gate and 1.2 m downstream. The gate is suddenly lifted. (1) Calculate the wave front celerity, and the surge front height. (2) Compute the water depth at the gate. Is it a function of time?

22- A 35 m high dam fails suddenly. The initial reservoir height was 31 m above the downstream channel invert and the downstream channel was filled with 1.8 m of water initially at rest. (1) Calculate the wave front location and the surge front height. (2) Calculate the wave front location 2 minutes after failure. (3) Predict the water depth 10 minutes after gate opening at two locations: $x = 2$ km and $x = 4$ km. Assume an infinitely long reservoir and use a simple wave analysis ($S_0 = S_f = 0$).

23- Dr NIELSEN, senior coastal engineer, wants to study sediment motion in the swash zone. For 0.5 m high breaking waves, the resulting swash is somehow similar to a dam break wave running over retreating waters. (1) Assuming an initial reservoir water depth of 0.5 m, an initial water depth $d_1 = 0.07$ m and an initial flow velocity $V_1 = -0.4$ m/s, calculate the surge front celerity and height. Assume a simple wave ($S_0 = S_f = 0$). (2) Calculate the bed shear stress immediately behind the surge front. The beach is made of fine sand ($d_{50} = 0.3$ mm, $d_{90} = 0.8$ mm). Assume $k_s = 2*d_{90}$ (Chapter 12, paragraph 12.4.2, Table 12.2). For sea water, $\rho = 1024$ kg/m$^3$ and $\mu = 1.22$ E-3 Pa.s. (3) Predict the occurrence of bed load motion and sediment suspension. (4) During a storm event, breaking waves near the shore may be 3 to 5 m high. For a 2 m high breaking wave, calculate the surge front height and bed shear stress behind the surge front assuming an initial reservoir water depth of 2 m, an initial water depth $d_1 = 0.15$ m and an initial flow velocity $V_1 = -1$ m/s.


Tutorial - Solutions

Chapter 1-2

2- (3) $C = \sqrt{g*d/2}$. (4) $C = 3.8$ m/s.

3- $U = +5.1$ m/s, $C = 4.64$ m/s.
4- \( U = + 2.7 \text{ m/s.} \)

5- \( d_2 = 1.23 \text{ m, } U = + 1.81 \text{ m/s, } t = 11,100 \text{ s (3 h 5 min.).} \)

6- (1) No. (2) No. (3) Yes. (4) No.

8- The problem may be analysed as a simple wave. The initial flow conditions are : \( V_0 = 0 \) and \( C_0 = 3.7 \text{ m/s.} \) Let select a coordinate system with \( x = 0 \) at the gate and \( x \) positive in the upstream direction. In the \((x, t)\) plane, the equation of the initial forward characteristics (issuing from \( x = 0 \) and \( t = 0 \)) is given:

\[
\frac{dt}{dx} = \frac{1}{V_0 + C_0} = 0.27 \text{ s/m}
\]

At \( t = 1 \text{ hour,} \) the extent of the influence of the gate operation is 13.3 km.

9- The simple wave problem corresponds to a negative surge. In absence of further information, the flume is assumed wide rectangular.

Let select a coordinate system with \( x = 0 \) at the gate and \( x \) positive in the upstream direction. The initial flow conditions are : \( V_0 = -0.9 \) and \( C_0 = 4.0 \text{ m/s.} \) In the \((x, t)\) plane, the equation of the initial forward characteristics (issuing from \( x = 0 \) and \( t = 0 \)) is given:

\[
\frac{dt}{dx} = \frac{1}{V_0 + C_0} = 0.32 \text{ s/m}
\]

At \( t = 10 \text{ minutes,} \) the maximum extent of the disturbance is \( x = 1.870 \text{ m.} \) That is, the zone of quiet is defined as \( x > 1.87 \text{ km.} \)

At the gate \((x = 0), \) the boundary condition is : \( d(x=0, t_0 \leq 0) = 1.65 \text{ m, } d(x = 0, t_0) = 1.65 - 8.33E-4 \times t_0 , \) for \( 0 < t_0 < 960 \text{ s,} \) and \( d(x=0, t_0 \geq 960) = 0.85 \text{ m.} \) The second flow property is calculated using the backward characteristics issuing from the initial forward characteristics and intersecting the boundary at \( t = t_0 : \)

\[
V(x=0, t_0) = V_0 + 2 \times (C(x=0, t_0) - C_0)
\]

where \( C(x, t_0) = \sqrt{g \times d(x=0, t_0)}. \)

At \( t = 10 \text{ minutes,} \) the flow property between \( x = 0 \) and \( x = 1.87 \text{ km} \) are calculated from:

\[
V(x, t = 600) + 2 \times C(x, t = 600) = V(x=0, t_0) + 2 \times C(x=0, t_0) \quad \text{forward characteristics}
\]

\[
V(x, t = 600) - 2 \times C(x, t = 600) = V_0 - 2 \times C_0 \quad \text{backward characteristics}
\]

where the equation of the forward characteristics is:

\[
t = t_0 + \frac{x}{V(x=0, t_0) + C(x=0, t_0)}
\]

These three equations are three unknowns : \( V(x, t = 600), C(x, t = 600) \) and \( t_0 = t(x=0) \) for the C1 characteristics. The results of the calculation at \( t = 12 \text{ minutes} \) are presented in the below.

The flow rate at the gate is \( -2.56 \text{ m}^2/\text{s} \) at \( t = 600 \text{ s.} \) The negative sign shows that the flow direction is in the negative \( x \)-direction.

**Table - Negative surge calculations at \( t = 10 \text{ minutes} \)**

<table>
<thead>
<tr>
<th>( t_0 ) (x=0)</th>
<th>( d(x=0) )</th>
<th>( C(x=0) )</th>
<th>( V(x=0) )</th>
<th>( Fr ) (x=0)</th>
<th>( x )</th>
<th>( V(x) )</th>
<th>( C(x) )</th>
<th>( d(x) )</th>
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<tr>
<td>C1 (1)</td>
<td>C2 (4)</td>
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<td>t=10min (7)</td>
<td>t=10min (8)</td>
<td>t=10min (9)</td>
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<td>0</td>
<td>-2.23</td>
<td>3.36</td>
<td>1.15</td>
</tr>
</tbody>
</table>

10- Answer
Note that the flow situation corresponds to a positive surge formation. However the wave does not have time develop and steepen enough in 2 hours to form a discontinuity (i.e. surge front) ahead. In other terms, the forward characteristics issuing from the reservoir do not intersect for \( t \leq 2 \) hours.

12- \( U = 4.46 \, \text{m/s}, d_2 = 2.24 \, \text{m} \) (gentle undular surge).

14- (1) \( d(x=0) = 2.24 \, \text{m} \). (2) \( Q = -18.3 \, \text{m}^3/\text{s} \).

16- Initial uniform equilibrium flow conditions are : \( d = 2.3 \, \text{m}, V = +1.42 \, \text{m/s}, f = 0.026 \). Diffusion wave : \( U = 2.1 \, \text{m/s}, D_t = 5.4 \times 10^3 \, \text{m}^2/\text{s} \) (for a wide rectangular channel).

17- Initially, uniform equilibrium flow conditions are : \( V = +1.25 \, \text{m/s}, S_o = S_f = 0.00028, f = 0.017 \). The celerity of the diffusion wave is :

\[
U = \frac{Q}{B \sqrt{8 * g * f * A^2 * \frac{DH}{4}}} = \frac{\sqrt{S_f}}{B} \frac{3}{2} B \sqrt{8 * g * f * \frac{A}{P_w} + \left(1 - \frac{2}{3} \frac{A}{B} \frac{P_w}{P} \right)} = \frac{3}{2} V \left(1 - \frac{2}{3} \frac{A}{B} \frac{P_w}{P} \right) = 1.68 \, \text{m/s}
\]

The diffusion coefficient is :

\[
D_t = \frac{8 * g * A^2 * \frac{DH}{4}}{2 * B * |Q|} = 4.02 \times 10^3 \, \text{m}^2/\text{s}
\]

The analytical solution of the diffusion wave equation yields :

\[
Q(x, t) = Q_o + \frac{\delta Q}{2} \left(1 - \text{erf} \left(\frac{x - U*t}{\sqrt{4*dt*t}}\right) + \exp \left(\frac{U*x}{D_t}\right) \left(1 - \text{erf} \left(\frac{x + U*t}{\sqrt{4*D_t*t}}\right)\right)\right)
\]

assuming constant diffusion wave celerity and diffusion coefficient, where \( Q_o = 18 \, \text{m}^3/\text{s} \) and \( \delta Q = 0.8 \, \text{m}^3/\text{s} \). For \( t = 1 \) hour and \( x = 15 \, \text{km}, Q = 18.04 \, \text{m}^3/\text{s} \).

18- \( U = 23 \, \text{m/s}, d(x=0) = 6 \, \text{m} \).

19- The assumption of hydrostatic pressure distribution is valid for \( t > 3*\sqrt{\frac{d_o}{g}} \approx 2 \, \text{s} \). Hence the Saint-Venant equations may be applied for \( t = 60 \, \text{s} \). Note the non-rectangular channel cross-section. For a trapezoidal channel, the celerity of a small disturbance is :

\[
C = \sqrt{g * \frac{A}{B}} = \sqrt{g * \frac{d * (W + d * \cot \delta)}{W + 2 * d * \cot \delta}}
\]

(see Chapter 3, paragraph 3.4.2)

where \( W \) is the bottom width and \( \delta \) is the sideslope angle with the horizontal (i.e. \( \cot \delta = 3 \)).

The method of characteristics predicts that the celerity of the negative wave is: - \( C_o = -4.7 \, \text{m/s} \). The celerity of the wave front is \( U = +2*C_o = +9.6 \, \text{m/s} \). Considering a backward characteristics issuing from the dam break wave front, the inverse slope of the C2 characteristics is a constant :

\[
\frac{d x}{dt} = V - C = 2 * C_o - 3 * C
\]
The integration gives the free-surface profile equation at a given time $t$:

$$\frac{x}{t} = 2 \sqrt{\frac{g}{W + 2d_0 \cot \delta} \left( W + d_0 \cot \delta \right)} - 3 \sqrt{\frac{g}{W + 2d \cot \delta} \left( W + d \cot \delta \right)}$$

At $t = 60$ s, the free-surface profile between the leading edge of the wave front and the negative wave most upstream location is:

<table>
<thead>
<tr>
<th>$d$ (m)</th>
<th>$x$ (m)</th>
</tr>
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<tbody>
<tr>
<td>4.2</td>
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<td>231</td>
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</tr>
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</table>

20- $x_s = 330$ m, $U = V(x=x_s) = 4.05$ m/s, $q(x=0) = 0.21$ m$^2$/s (DRESSLER's theory). $U = 4.7$ m/s (WHITHAM's theory).

21- (1) $d_1/d_0 = 0.375$, $U = 5.25$ m/s, $d(x=0) = 2.07$ m. (2) $d_2 - d_1 = 0.87$ m.

22- (1) $d_1/d_0 = 0.06$, $U = 18.1$ m/s, $d_2 - d_1 = 8.34$ m. (2) $x_s = 2.2$ km ($t = 2$ minutes). (3) $d(x=2$ km, $t=10$ min.) = 11.3 m and $d(x=4$ km, $t=10$ min.) = 10.1 m

23- Let select a positive $x$-direction toward the shore. The dam break wave ($d_0 = 0.5$ m) propagates in a channel initially filled with water ($d_1 = 0.045$ m) with an opposing flow velocity ($V_1 = -0.4$ m/s). The $x$ coordinate is zero ($x = 0$) at wave breaking (i.e. pseudo-dam site) and the time origin is taken at the start of wave breaking.

The characteristic system of equation, and the continuity and momentum principles at the wave front must be solved theoretically. The free-surface profile is horizontal between the leading edge of the positive surge (point E3) and the intersection with the $C_1$ forward characteristics issuing from the initial negative characteristics. The flow depth $d_2$ and velocity $V_2$ behind the surge front satisfy the continuity and momentum equations as well as the condition along the $C_1$ forward characteristics:

$$d_1 * (U - V_1) = d_2 * (U - V_2) \quad \text{Continuity equation (III-5-39)}$$

$$d_2 * (U - V_2)^2 - d_1 * (U - V_1)^2 = \frac{1}{2} \frac{g}{d_1} d_1^2 - \frac{1}{2} \frac{g}{d_2} d_2^2 \quad \text{Momentum equation (III-5-40)}$$

$$V_2 + 2 \sqrt{\frac{g}{d} d_2} = 2 \sqrt{\frac{g}{d_0} d_0} \quad \text{Forward characteristics (III-5-33)}$$

Equations (III-5-33), (III-5-39) and (III-5-40) form a system of three non-linear equations with three unknowns $V_2$, $d_2$ and $U$.

An iterative calculation shows that the surge front celerity is: $U \sqrt{g/d_1} = 3.2$ and $U = 2.12$ m/s.

At the wave front, the continuity and momentum equations yield:

$$\frac{d_2}{d_1} = \frac{1}{2} \left( \sqrt{1 + 8 \frac{(U - V_1)^2}{g d_1} - 1} \right) = 4.6 \quad \text{hence } d_2 = 0.21 \text{ m}$$

Equation (III-5-33) may be rewritten:

$$\frac{V_2}{\sqrt{g / d_2}} = 2 \sqrt{\frac{d_0}{d_2} - 1} = 1.1 \quad \text{hence } V_2 = 1.58 \text{ m/s}$$

Behind the surge front, the boundary shear stress equals:

$$\tau_0 = \frac{f}{8} \rho V_2^2 = 7.7 \text{ Pa}$$

The Shields parameter $\tau_*$ equals 1.55 which is almost one order of magnitude greater than the critical Shields parameter for bed load motion ($\tau_c^* = 0.035$ (Chapter 8, paragraph 8.3). For a 0.3 mm sand particle, the settling velocity is 0.034 m/s. The ratio $V* \omega_0$ equals 2.5 implying sediment suspension (Chapter 9, paragraph 9.2).

For a 2 m high breaking wave during a storm event, the surge front height equals: $d_2 - d_1 = 0.69$ m. The boundary shear stress behind the surge front equal: $\tau_0 = 21$ Pa.

**Remarks**

+ The above development has a number of limitations. The reservoir is assumed infinite although a breaking wave has a finite volume, the beach slope is assuming frictionless and horizontal.
+ Note that the calculations of $U$, $V_2$ and $d_2$ are independent of time.