CIVL4120/7020 Advanced open channel hydraulics and design - TUTORIALS (1)

Revision - Open Channel Flows

1. Considering a broad-crested weir, draw a sketch of the weir in a rectangular horizontal channel. What is the main purpose of a broad-crest weir ?

A broad-crested weir is installed in a horizontal and smooth channel of rectangular cross-section. The channel width is 10 m. The bottom of the weir is 1.5 m above the channel bed. The water discharge is 11 m^3/s and the upstream water depth is 2.235 m. (1) Compute the depth of flow downstream of the weir (in absence of downstream control), assuming that critical flow conditions take place at the weir crest. (2) Calculated the horizontal, sliding force acting on the weir. Give the direction of the force exerted by the flow onto the sill.

Solution (Broad-crested weir)

d_2 = 0.173 m. $F_{sliding}$ = 178 kN. The force exerted by the fluid onto the weir sill acts in the downstream direction.

2. A rectangular (5.5 m width) concrete channel carries a steady discharge of 6 m^3/s. The longitudinal bed slope is 1.2 m per km. (a) What is the normal depth at uniform equilibrium ? (b) At uniform equilibrium what is the average boundary shear stress ? (c) At normal flow conditions, is the flow supercritical, super-critical or critical ? Would you characterise the channel as mild, critical or steep ?

For man-made channels, perform flow resistance calculations based upon the Darcy-Weisbach friction factor.

Solution (Uniform equilibrium flow)

(a) d = 0.64 m (c) Fr = 0.68 : near-critical flow, although sub-critical (hence mild slope) (See discussion in textbook page 95-96)

CIVL4120/7020 Advanced open channel hydraulics and design - TUTORIALS (2)

Unsteady Open Channel Flows. 1- Basic equations

Rapid quizz

a- Write the five basic assumptions used to develop the Saint-Venant equations.

b- Were the Saint-Venant equations developed for movable boundary hydraulic situations ?

c- Are the Saint-Venant equations applicable to a steep slope ?

d- Express the differential form of the Saint-Venant equations in terms of the water depth and flow velocity. Compare the differential form of the momentum equation with the backwater equation.

e- What is the dynamic wave equation ? From which fundamental principle does it derive ?

f- What are the two basic differences between the dynamic wave equation and the backwater equation ?

g- Is the dynamic wave equation applicable to a hydraulic jump ?

h- Is the dynamic wave equation applicable to an undular hydraulic jump or an undular surge ?

i- Considering a channel bend, estimate the conditions for which the basic assumption of quasi-horizontal transverse free-surface is no longer valid. *Assume a rectangular channel of width W much smaller than the bend radius r.*

j- Give the expression of the friction slope in terms of the flow rate, cross-section area, hydraulic diameter and Darcy friction factor only. Then, express the friction slope in terms of the flow rate and Chézy coefficient. Simplify both expressions for a wide rectangular channel.

k- Considering the flood plain sketched in Figure 4.13 (Chapter 4, CHANSON 2004 p. 86), develop the expression of the friction slope in terms of the total flow rate and respective Darcy friction factors.

Solution

Textbook pages 290-313 & 313-315.

Comments

c- In open channel flow hydraulics, a "steep" slope is defined when the uniform equilibrium flow is supercritical (Textbook, Chap. 5). The notion of steep and mild slope is not only a function of the bed slope but is also a function of the flow resistance.

A basic assumption of the Saint-Venant equations is a bed slope that is small enough such that it is possible to assume $\cos\theta \approx 1$ and $\sin\theta \approx \tan\theta \approx \theta$. This assumption is based solely upon the invert angle with the horizontal θ.

The following table summarises the error associated with the approximation with increasing angle θ.

θ deg.	θ rad.	1-cosθ	sinθ/tanθ
0	0	0	1
0.5	0.008727	3.81E-05	0.999962
1	0.017453	0.000152	0.999848
2	0.034907	0.000609	0.999391
6	0.10472	0.005478	0.994522
10	0.174533	0.015192	0.984808
15	0.261799	0.034074	0.965926
25	0.436332	0.093692	0.906308

g- The dynamic wave equation is the differential form of the unsteady momentum equation. It might not be applicable to a discontinuity (e.g., a hydraulic jump), although the integral form of the Saint-Venant equations is (Textbook pp. 293-296).

i- In a channel bend, the flow is subjected to a centrifugal acceleration acting normal to the flow direction and equal to V^2/r where r is the radius of curvature. The centrifugal pressure force induces a greater water depth at the outer bank than in a straight channel.
In first approximation, the momentum equation applied in the transverse direction yields :

$$\frac{1}{2} * \rho * g * (d + \Delta d)^2 - \frac{1}{2} * \rho * g * (d - \Delta d)^2 = \rho * \frac{V^2}{r} * d * W$$

assuming W << r and a flat horizontal channel. The rise Δd in free-surface elevation is about :

$$\Delta d \approx \frac{V^2}{2 * r * g} * W$$

The change in water depth from the inner to outer bank is less than 1% if the channel width, curvature and water depth satisfy :

$$\frac{V^2}{r * g} * \frac{W}{d} < 0.01$$

(See textbook pp. 314)

<u>Detailed applications</u>

A- Considering a long channel, flow measurements at two gauging stations give at t = 0 :

	Station 1	Station 2
Location x (km) :	7.1	8.25
Water depth (m) :	2.2	2.45
Flow velocity (m/s) :	+0.35	+0.29

In the (x, t) plane, plot the characteristics issuing from each gauging station. (*Assume straight lines*.) Calculate the location, time and flow properties at the intersection of the characteristics issuing from the two gauging stations. *Assume* $S_f = S_o = 0$.

Solution

Answer : x = 7.7 km, t = 120 s, V = +0.06 m/s, d = 2.34 m

The flow conditions correspond to a reduction in flow rate. At x = 7.7 km and t = 120 s, q = 0.14 m^2/s, compared to $q_1 = 0.77$ m^2/s and $q_2 = 0.71$ m^2/s at t = 0.

B- The analysis of flow measurements in a river reach gave :

	Station 1	Station 2
Location x (km) :	11.8	13.1
Water depth (m) :	0.65	0.55
Flow velocity (m/s) :	+0.5	+0.55

at t = 1 hour. Predict the flood flow development. Assuming a kinematic wave (i.e. $S_o = S_f$), plot the characteristics issuing from the measurement stations *assuming straight lines*. Calculate the flow properties at the intersection of the characteristics.

Solution

The solutions of the characteristic system of equations yields : x =12.6 km, t = 3,600 + 263 s, V = 0.80 m/s, C = 2.44 m/s, d = 0.61 m and q = 0.49 m^2/s.

As a comparison, q_1 = 0.325 m^2/s and q_2 = 0.30 m^2/s at t = 1 hour. *The flow situation corresponds to an increase in flow rate.*

Textbook p. 316.

C- **2004 Examination paper**

A long channel has a 12 m wide rectangular cross-section, is horizontal, and the bed roughness is equivalent to a Darcy-Weisbach friction factor of 0.015. Field measurements are conducted to validate a numerical model.

Flow measurements at three gauging stations give at t = 120 s :

	Station 1	Station 2	Station 3
Location x (km) :	1.100	1.300	1.500
Water depth (m) :	1.95237	1.87321	1.74658
Flow velocity (m/s) :	0.10605	0.28492	0.57729

(a) Calculate the flow conditions at Station 2 at t =140 s. *Use the Hartree method assuming Δt = 20 s.*

(b) Explain the type of hydraulic conditions (i.e. the flow situation) taking place in the channel. *Justify your answer in words and possibly with a sketch.*

+ **Solution**

Station 2, t = 140 s : d = 1.825 m, C = 4.23 m/s, V = 0.395 m/s, S_f = 1.6 E-5

(V_L = 0.204 m/s, C_L = 4.325 m/s / V_R = 0.397 m/s, C_R = 4.23m/s)

This flow situation corresponds to an increase in discharge at Station 2 associated with some flow accelaration and slight reduction in water depth.

Textbook pp. 308-309.

+ **Remark**

Addition informations on the Hartree method are available in [1] pp. 491-495 and [2] pp. 208-211.

[1] MONTES, J.S. (1998). "Hydraulics of Open Channel Flow." *ASCE Press*, New-York, USA, 697 pages.

[2] CHANSON, H. (2004). "Environmental Hydraulics of Open Channel Flows." *Elsevier Butterworth-Heinemann*, Oxford, UK, 483 pages (ISBN 0 7506 6165 8).

Quizz

a- Considering a supercritical flow (flow direction in the positive x-direction), how many boundary conditions are needed for t > 0 and where ?

b- What is the difference between the diffusive wave equation, dynamic wave equation, and kinematic wave equation ? Which one(s) does(do) apply to unsteady flows ?

More exercises in textbook pp. 313-317 &371-373.

"The Hydraulics of Open Channel Flow: An Introduction", *Butterworth-Heinemann Publ.*, 2nd edition, Oxford, UK, 2004.