

**CIVL4120/7020 Advanced open channel hydraulics and design - Tutorial (1)**

**Unsteady open channel flows**

The course is a professional subject in which the students are expected to have a sound knowledge of the basic principles of continuity, energy and momentum, and understand the principles of fluid flow motion. The students should have completed successfully the core course Introduction to Fluid mechanics (CIVL3130) and have a solid knowledge in the core course Catchment Hydraulics (CIVL3140).

**Attendance to tutorials is very strongly advised.** Repeated absences by some individuals will be noted and these would demonstrate some disappointing responsible behaviour.

Past course results demonstrated a very strong correlation between the attendance of tutorials during the semester, the performances at the end-of-semester examination, and the overall course result.

**Unsteady Open Channel Flows. 1- Basic equations**

Rapid quizz

- a- Write the five basic assumptions used to develop the Saint-Venant equations.
- b- Were the Saint-Venant equations developed for movable boundary hydraulic situations?
- c- Are the Saint-Venant equations applicable to a steep slope?
- d- Express the differential form of the Saint-Venant equations in terms of the water depth and flow velocity. Compare the differential form of the momentum equation with the backwater equation.
- e- What is the dynamic wave equation? From which fundamental principle does it derive?
- f- What are the two basic differences between the dynamic wave equation and the backwater equation?
- g- Is the dynamic wave equation applicable to a hydraulic jump?
- h- Is the dynamic wave equation applicable to an undular hydraulic jump or an undular surge?
- i- Considering a channel bend, estimate the conditions for which the basic assumption of quasi-horizontal transverse free-surface is no longer valid. *Assume a rectangular channel of width  $W$  much smaller than the bend radius  $r$ .*
- j- Give the expression of the friction slope in terms of the flow rate, cross-section area, hydraulic diameter and Darcy friction factor only. Then, express the friction slope in terms of the flow rate and Chézy coefficient. Simplify both expressions for a wide rectangular channel.
- k- Considering the flood plain sketched in Figure 4.13 (Chapter 4, CHANSON 2004 p. 86), develop the expression of the friction slope in terms of the total flow rate and respective Darcy friction factors.

**Solution**

Textbook pages 290-313 & 313-315.

*Comments*

c- In open channel flow hydraulics, a "steep" slope is defined when the uniform equilibrium flow is supercritical (Textbook, Chap. 5). The notion of steep and mild slope is not only a function of the bed slope but is also a function of the flow resistance.

A basic assumption of the Saint-Venant equations is a bed slope that is small enough such that it is possible to assume  $\cos\theta \approx 1$  and  $\sin\theta \approx \tan\theta \approx \theta$ . This assumption is based solely upon the invert angle with the horizontal  $\theta$ . The following table summarises the error associated with the approximation with increasing angle  $\theta$ .

$\theta$ deg.	$\theta$ rad.	1-cos $\theta$	sin $\theta$ /tan $\theta$
0	0	0	1
0.5	0.008727	3.81E-05	0.999962
1	0.017453	0.000152	0.999848
2	0.034907	0.000609	0.999391
6	0.10472	0.005478	0.994522
10	0.174533	0.015192	0.984808
15	0.261799	0.034074	0.965926
25	0.436332	0.093692	0.906308

g- The dynamic wave equation is the differential form of the unsteady momentum equation. It might not be applicable to a discontinuity (e.g., a hydraulic jump), although the integral form of the Saint-Venant equations is (Textbook pp. 293-296).

i- In a channel bend, the flow is subjected to a centrifugal acceleration acting normal to the flow direction and equal to  $V^2/r$  where  $r$  is the radius of curvature. The centrifugal pressure force induces a greater water depth at the outer bank than in a straight channel.

In first approximation, the momentum equation applied in the transverse direction yields :

$$\frac{1}{2} * \rho * g * (d + \Delta d)^2 - \frac{1}{2} * \rho * g * (d - \Delta d)^2 = \rho * \frac{V^2}{r} * d * W$$

assuming  $W \ll r$  and a flat horizontal channel. The rise  $\Delta d$  in free-surface elevation is about :

$$\Delta d \approx \frac{V^2}{2 * r * g} * W$$

The change in water depth from the inner to outer bank is less than 1% if the channel width, curvature and water depth satisfy :

$$\frac{V^2}{r * g} * \frac{W}{d} < 0.01$$

(See textbook pp. 314)

Detailed application A

Considering a long channel, flow measurements at two gauging stations give at  $t = 0$  :

	<u>Station 1</u>	<u>Station 2</u>
Location $x$ (km) :	7.1	8.25
Water depth (m) :	2.2	2.45
Flow velocity (m/s) :	+0.35	+0.29

In the  $(x, t)$  plane, plot the characteristics issuing from each gauging station. (*Assume straight lines.*) Calculate the location, time and flow properties at the intersection of the characteristics issuing from the two gauging stations. Assume  $S_f = S_o = 0$ .

*Solution*

Answer:  $x = 7.7$  km,  $t = 120$  s,  $V = +0.06$  m/s,  $d = 2.34$  m

The flow conditions correspond to a reduction in flow rate. At  $x = 7.7$  km and  $t = 120$  s,  $q = 0.14$  m<sup>2</sup>/s, compared to  $q_1 = 0.77$  m<sup>2</sup>/s and  $q_2 = 0.71$  m<sup>2</sup>/s at  $t = 0$ .

Detailed application B

The analysis of flow measurements in a river reach gave:

	<u>Station 1</u>	<u>Station 2</u>
Location x (km) :	11.8	13.1
Water depth (m) :	0.65	0.55
Flow velocity (m/s) :	+0.5	+0.55

at  $t = 1$  hour. Predict the flood flow development. Assuming a kinematic wave (i.e.  $S_o = S_f$ ), plot the characteristics issuing from the measurement stations *assuming straight lines*. Calculate the flow properties at the intersection of the characteristics.

*Solution*

The solutions of the characteristic system of equations yields:  $x = 12.6$  km,  $t = 263$  s,  $V = 0.80$  m/s,  $C = 2.44$  m/s,  $d = 0.61$  m and  $q = 0.49$  m<sup>2</sup>/s.

As a comparison,  $q_1 = 0.325$  m<sup>2</sup>/s and  $q_2 = 0.30$  m<sup>2</sup>/s at  $t = 0$ . *The flow situation corresponds to an increase in flow rate.*

Textbook p. 316.

Detailed application C (2004 examination paper)

A long channel has a 12 m wide rectangular cross-section, is horizontal, and the bed roughness is equivalent to a Darcy-Weisbach friction factor of 0.015. Field measurements are conducted to validate a numerical model.

Flow measurements at three gauging stations give at  $t = 120$  s :

	<u>Station 1</u>	<u>Station 2</u>	<u>Station 3</u>
Location x (km) :	1.100	1.300	1.500
Water depth (m) :	1.95237	1.87321	1.74658
Flow velocity (m/s) :	0.10605	0.28492	0.57729

(a) Calculate the flow conditions at Station 2 at  $t = 140$  s. *Use the Hartree method assuming  $\Delta t = 20$  s.*

(b) Explain the type of hydraulic conditions (i.e. the flow situation) taking place in the channel. *Justify your answer in words and possibly with a sketch.*

*Solution*

Station 2,  $t = 140$  s :  $d = 1.825$  m,  $C = 4.23$  m/s,  $V = 0.395$  m/s,  $S_f = 1.6 \text{ E-}5$

Textbook pp. 308-309.

Quiz

a- Considering a supercritical flow (flow direction in the positive x-direction), how many boundary conditions are needed for  $t > 0$  and where ?

b- What is the difference between the diffusive wave equation, dynamic wave equation, and kinematic wave equation ? Which one(s) does(do) apply to unsteady flows ?

More exercises in textbook pp. 313-317 & 371-373.

"The Hydraulics of Open Channel Flow: An Introduction", *Butterworth-Heinemann Publ.*, 2nd edition, Oxford, UK, 2004.

## Unsteady Open Channel Flows. 2- Applications

\* A 0.2 m high small wave propagates downstream in a horizontal channel with initial flow conditions  $V = +0.1$  m/s and  $d = 2.2$  m. Calculate the propagation speed of the small wave.

*Solution*

$U = + 5.1$  m/s,  $C = 4.64$  m/s.

Textbook pp. 319-321.

\* Uniform equilibrium flow conditions are achieved in a long rectangular channel ( $W = 12.8$  m, concrete lined,  $S_0 = 0.0005$ ). The observed water depth is 1.75 m. Calculate the celerity of a small monoclinal wave propagating downstream. *Perform your calculations using the Darcy friction factor.*

*Solution*

$U = + 2.7$  m/s.

Textbook pp. 321-322.

\* The flow rate in a rectangular canal ( $W = 3.4$  m, concrete lined,  $S_0 = 0.0007$ ) is  $3.1$  m<sup>3</sup>/s and uniform equilibrium flow conditions are achieved. The discharge suddenly increases to  $5.9$  m<sup>3</sup>/s. Calculate the celerity of the monoclinal wave. How long will it take for the monoclinal wave to travel 20 km ? *Perform your calculations using the Darcy friction factor.*

*Solution*

$d_2 = 1.23$  m,  $U = + 1.81$  m/s,  $t = 11,100$  s (3 h 5 min.).

Textbook pp. 321-322.

\* Considering a long, horizontal rectangular channel ( $W = 4.2$  m), a gate operation, at one end of the canal, induces a sudden withdrawal of water resulting in a negative velocity. At the gate, the boundary conditions for  $t > 0$  are:  $V(x = 0, t) = - 0.2$  m/s. Calculate the extent of the gate operation influence in the canal at  $t = 1$  hour. The initial conditions in the canal are :  $V = 0$  and  $d = 1.4$  m.

\* Water flows in an irrigation canal at steady state ( $V = 0.9$  m/s,  $d = 1.65$  m). The flume is assumed smooth and horizontal. The flow is controlled by a downstream gate. At  $t = 0$ , the gate is very-slowly raised and the water depth upstream of the gate decreases at a rate of 5 cm per minute until the water depth becomes 0.85 m. (1) Plot the free-surface profile at  $t = 10$  minutes. (2) Calculate the discharge per unit width at the gate at  $t = 10$  minutes.

*Solution (Textbook pp. 322-327)*

The simple wave problem corresponds to a negative surge. In absence of further information, the flume is assumed wide rectangular. Let select a coordinate system with  $x = 0$  at the gate and  $x$  positive in the upstream direction. The initial flow conditions are :  $V_0 = -0.9$  and  $C_0 = 4.0$  m/s. In the  $(x, t)$  plane, the equation of the initial forward characteristics (issuing from  $x = 0$  and  $t = 0$ ) is given :

$$\frac{dt}{dx} = \frac{1}{V_0 + C_0} = 0.32 \text{ s/m}$$

At  $t = 10$  minutes, the maximum extent of the disturbance is  $x = 1,870$  m. That is, the zone of quiet is defined as  $x > 1.87$  km. At the gate ( $x = 0$ ), the boundary condition is :  $d(x=0, t_0 \leq 0) = 1.65$  m,  $d(x = 0, t_0) = 1.65 - 8.33E-4 * t_0$ , for  $0 < t_0 < 960$  s, and  $d(x=0, t_0 \geq 960 \text{ s}) = 0.85$  m. The second flow property is calculated using the backward characteristics issuing from the initial forward characteristics and intersecting the boundary at  $t = t_0$ :

$$V(x=0, t_0) = V_0 + 2 * (C(x=0, t_0) - C_0) \quad \text{Backward characteristics}$$

where  $C(x, t_0) = \sqrt{g * d(x=0, t_0)}$ .

At  $t = 10$  minutes, the flow property between  $x = 0$  and  $x = 1.87$  km are calculated from:

$$V(x, t = 600) + 2 * C(x, t = 600) = V(x=0, t_0) + 2 * C(x=0, t_0) \quad \text{forward characteristics}$$

$$V(x, t = 600) - 2 * C(x, t = 600) = V_0 - 2 * C_0$$

backward characteristics

where the equation of the forward characteristics is :

$$t = t_0 + \frac{x}{V(x=0, t_0) + C(x=0, t_0)}$$

forward characteristics

These three equations are three unknowns:  $V(x, t = 600)$ ,  $C(x, t = 600)$  and  $t_0 = t(x=0)$  for the C1 characteristics. The results of the calculation at  $t = 12$  minutes are presented in the below.

The flow rate at the gate is  $-2.56 \text{ m}^2/\text{s}$  at  $t = 600 \text{ s}$ . The negative sign shows that the flow direction is in the negative  $x$ -direction.

Table - Negative surge calculations at  $t = 10$  minutes

$t_0$ (x=0)	$d(x=0)$	$C(x=0)$	$V(x=0)$	Fr (x=0)	x	$V(x)$	$C(x)$	$d(x)$
C1			C2		t=10min	t=10min	t=10min	t=10min
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
0	1.65	4.02	-0.90	-0.22	1873	-0.90	4.02	1.65
60	1.6	3.96	-1.02	-0.26	1586	-1.02	3.96	1.60
120	1.55	3.90	-1.15	-0.29	1320	-1.15	3.90	1.55
180	1.5	3.83	-1.27	-0.33	1075	-1.27	3.83	1.50
240	1.45	3.77	-1.40	-0.37	852	-1.40	3.77	1.45
300	1.4	3.70	-1.53	-0.41	651	-1.53	3.70	1.40
360	1.35	3.64	-1.67	-0.46	473	-1.67	3.64	1.35
420	1.3	3.57	-1.80	-0.51	318	-1.80	3.57	1.30
480	1.25	3.50	-1.94	-0.55	187	-1.94	3.50	1.25
540	1.2	3.43	-2.08	-0.61	80.7	-2.08	3.43	1.20
600	1.15	3.36	-2.23	-0.66	0	-2.23	3.36	1.15

\* A 200 km long rectangular channel ( $W = 3.2 \text{ m}$ ) has a reservoir at the upstream end and a gate at the downstream end. Initially the flow conditions in the canal are uniform:  $V = 0.35 \text{ m/s}$ ,  $d = 1.05 \text{ m}$ . The water surface level in the reservoir begins to rise at a rate of  $0.2 \text{ m}$  per hour for 6 hours. Calculate the flow conditions in the canal at  $t = 2$  hours. Assume  $S_0 = S_f = 0$ .

Solution

x	$V(x)$	$C(x)$	$d(x)$
25616	0.35	3.21	1.05
23841	0.50	3.28	1.10
21630	0.65	3.36	1.15
19000	0.79	3.43	1.20
15964	0.93	3.50	1.25
13720	1.03	3.55	1.28
11306	1.12	3.59	1.32
8723	1.21	3.64	1.35
5976	1.30	3.68	1.38
3067	1.39	3.73	1.42
0	1.47	3.77	1.45

The flow situation corresponds to a positive surge formation. However the wave does not have time develop and steepen enough in 2 hours to form a discontinuity (i.e. surge front) ahead. In other terms, the forward characteristics issuing from the reservoir do not intersect for  $t \leq 2$  hours.

#### Tidal bore propagation (2004 examination paper)

Considering the propagation of a tidal bore in an estuary, the river flow conditions prior to the bore arrival are:  $Q = 52 \text{ m}^3/\text{s}$ ,  $d = 1.15 \text{ m}$ ,  $B = 95 \text{ m}$ . The river channel is assumed to be horizontal and rectangular. The tidal bore arrives and propagates upstream. Its celerity is measured by an observer standing on the right bank and recorded as  $3.35 \text{ m/s}$  (positive upstream).

- Calculate the new flow depth and flow velocity immediately shortly the passage of the bore. *Indicate clearly the direction of the flow after the passage of the bore.*
- What type of bore would the observer see ? *Justify your answer.*

Fig. - Photographs of tidal bores

(A) Undular bore of the Garonne River on 23 Sept. 2010 afternoon - Tidal bore between 18:13 and 18:21  
The bore was undular in the deep river channel and breaking in the shallower waters next to the left bank -  
Looking downstream at the incoming bore



(B) Breaking tidal bore of the Sée River on 1 October 2008 - Bore propagation from right to left



\* Waters flow in a horizontal, smooth rectangular channel. The initial flow conditions are  $d = 2.1$  m and  $V = +0.3$  m/s. The flow rate is stopped by some gate closure at the downstream end of the canal. The downstream gate is closed slowly at a rate corresponding to a linear decrease in flow rate from  $0.63$  m<sup>2</sup>/s down to zero in

15 minutes. (1) Predict the surge front development. (2) Calculate the free-surface profile at  $t = 1$  hour after the start of gate closure.

\* A 5 m wide forebay canal supplies a penstock feeding a Pelton turbine. The initial conditions in the channel are  $V = 0$  and  $d = 2.5$  m. (1) The turbine starts suddenly operating with  $6 \text{ m}^3/\text{s}$ . Predict the water depth at the downstream end of the forebay canal. (2) What is the maximum discharge that the forebay channel can supply? Use a simple wave theory.

*Solution (textbook pp. 335-339)*

(1)  $d(x=0) = 2.24$  m. (2)  $Q = -18.3 \text{ m}^3/\text{s}$ .

More exercises in textbook pp. 362-370 & 371-373.

"The Hydraulics of Open Channel Flow: An Introduction", *Butterworth-Heinemann Publ.*, 2nd edition, Oxford, UK, 2004.

### **Unsteady Open Channel Flows. 3- Applications to dam break wave**

\* A 15 m high dam fails suddenly. The dam reservoir had a 13.5 m depth of water and the downstream channel was dry. (1) Calculate the wave front celerity, and the water depth at the origin. (2) Calculate the free-surface profile 2 minutes after failure.

Assume an infinitely long reservoir and use a simple wave analysis ( $S_o = S_f = 0$ ).

*Solution*

$U = 23 \text{ m/s}$ ,  $d(x=0) = 6 \text{ m}$ .

\* A vertical sluice shut a trapezoidal channel (3 m bottom width, 1V:3H side slopes). The water depth was 4.2 m upstream of the gate and zero downstream (i.e. dry channel). The gate is suddenly removed. Calculate the negative celerity. Assuming an ideal dam break wave, compute the wave front celerity and the free-surface profile one minute after gate removal.

*Solution*

The assumption of hydrostatic pressure distribution is valid for  $t > 3 \cdot \sqrt{d_o/g} \sim 2 \text{ s}$ . That is, the Saint-Venant equations may be applied at  $t = 60 \text{ s}$ . For a trapezoidal channel, the celerity of a small disturbance is :

$$C = \sqrt{g \cdot \frac{A}{B}} = \sqrt{g \cdot \frac{d \cdot (W + d \cdot \cot \delta)}{W + 2 \cdot d \cdot \cot \delta}} \quad (\text{see Textbook, Chapter 3, paragraph 3.4.2})$$

where  $W$  is the bottom width and  $\delta$  is the sideslope angle with the horizontal (i.e.  $\cot \delta = 3$ ).

The celerity of the negative wave is:  $-C_o = -4.7 \text{ m/s}$ . The celerity of the wave front is  $U = +2 \cdot C_o = +9.6 \text{ m/s}$ . Considering a backward characteristics issuing from the dam break wave front, the inverse slope of the C2 characteristics is a constant :

$$\frac{dx}{dt} = V - C = 2 \cdot C_o - 3 \cdot C$$

The integration gives the free-surface profile equation at a given time  $t$  :

$$\frac{x}{t} = 2 \cdot \sqrt{g \cdot \frac{d_o \cdot (W + d_o \cdot \cot \delta)}{W + 2 \cdot d_o \cdot \cot \delta}} - 3 \cdot \sqrt{g \cdot \frac{d \cdot (W + d \cdot \cot \delta)}{W + 2 \cdot d \cdot \cot \delta}}$$

At  $t = 60 \text{ s}$ , the free-surface profile between the leading edge of the wave front and the negative wave most upstream location is :

$d \text{ (m)} :$	4.2	3	2	1.725	1	0.5	0
$x \text{ (m)} :$	-282	-160	-38.4	0	119	231	564

\* A 5 m high spillway gate fails suddenly. The water depth upstream of the gate was 4.5 m depth and the downstream concrete channel was dry and horizontal. (1) Calculate the wave front location and velocity at  $t = 30$  s. (2) Compute the discharge per unit width at the gate at  $t = 30$  s. (3) Calculate the wave front celerity at  $t = 2$  minutes.

Use the lecture note development for dam break wave with friction. Assume  $f = 0.03$  for a relatively new concrete lining.

*Solution*

(1)  $x_s = 207.6$  m,  $U = 4.8$  m/s. (2)  $q = 8.85$  m<sup>2</sup>/s. (3)  $U = 3.14$  m/s

\* A 65 m high concrete dam fails explosively (i.e. Malpasset dam type failure). The dam reservoir was nearly full and the depth of water upstream of the dam wall was 61.5 m. The downstream channel was dry and horizontal. (1) At  $t = 1$  minute, calculate the ideal wave front celerity and location. (2) For a real-fluid flow with flow resistance, calculate the ideal wave front celerity and location at  $t = 1$  minute. (3) Calculate the free-surface profile 5 minutes after failure for a real fluid.

Assume an infinitely long reservoir and assume  $f = 0.08$  for the downstream valley roughness.

Use the turbulent dam break wave's simple solution to solve dam break wave with friction.

*Solution*

(1)  $U = 49.1$  m/s,  $x_s = 2.95$  km.

(2)  $U = 16$  m/s,  $x_s = 1.46$  km,  $x_1 = -0.033$  km

(3)  $U = 9.4$  m/s,  $x_s = 5.8$  km,  $x_1 = -3.1$  km. The free-surface profile is an ideal-fluid flow "parabola" for  $-7.4$  km  $< x < -3.1$  km, and the wave tip region extends for  $-3.1$  km  $< x < 5.8$  km.

Fig. - Photographs of the Malpasset dam ruins (Courtesy of Didier TOULOUZE)

(A) General view from the top of the right bank





(B) Details of the left abutment



*References*

- CHANSON, H. (2009). "Application of the Method of Characteristics to the Dam Break Wave Problem." *Journal of Hydraulic Research, IAHR*, Vol. 47, No. 1, pp. 41-49 (DOI: 10.3826/jhr.2009.2865).  
{<http://espace.library.uq.edu.au/view/UQ:164021>}
- CHANSON, H. (2006). "Analytical Solutions of Laminar and Turbulent Dam Break Wave." *Proc. International Conference on Fluvial Hydraulics River Flow 2006, Lisbon, Portugal, 6-8 Sept., Topic A3*, R.M.L. FERREIRA, E.C.T.L. ALVES, J.G.A.B. LEAL, and A.H. CARDOSO Eds., Balkema Publ., Taylor & Francis Group, London, Vol. 1, pp. 465-474,  
{<http://espace.library.uq.edu.au/view.php?pid=UQ:7927>}
- CHANSON, H. (2005). "Applications of the Saint-Venant Equations and Method of Characteristics to the Dam Break Wave Problem." Report No. CH55/05, Dept. of Civil Engineering, The University of Queensland, Brisbane, Australia, May, 127 pages.  
{<http://espace.library.uq.edu.au/view.php?pid=UQ:9438>}

\* A horizontal, rectangular canal is shut by a vertical sluice. There is no flow motion on either side of the gate. The water depth is 3.2 m upstream of the gate and 1.2 m downstream. The gate is suddenly lifted. (1) Calculate the wave front celerity, and the surge front height. (2) Compute the water depth at the gate. Is it a function of time?

*Solution*

(1)  $d_1/d_0 = 0.375$ ,  $U = 5.25$  m/s,  $d(x=0) = 2.07$  m. (2)  $d_2 - d_1 = 0.87$  m.

\* A 35 m high dam fails suddenly. The initial reservoir height was 31 m above the downstream channel invert and the downstream channel was filled with 1.8 m of water initially at rest. (1) Calculate the wave front celerity, and the surge front height. (2) Calculate the wave front location 2 minutes after failure. (3) Predict the water depth 10 minutes after gate opening at two locations:  $x = 2$  km and  $x = 4$  km. Assume an infinitely long reservoir and use a simple wave analysis ( $S_o = S_f = 0$ ).

*Solution*

(1)  $d_1/d_0 = 0.06$ ,  $U = 18.1$  m/s,  $d_2 - d_1 = 8.34$  m. (2)  $x_s = 2.2$  km ( $t = 2$  minutes). (3)  $d(x=2$  km,  $t=10$  min.) = 11.3 m and  $d(x=4$  km,  $t=10$  min.) = 10.1 m

\* A senior coastal engineer wants to study sediment motion in the swash zone. For 0.5 m high breaking waves, the resulting swash is somehow similar to a dam break wave running over a dry bed. (1) Assuming an initial reservoir water depth of 0.5 m, calculate the wave front celerity and height at 3 seconds after wave/dam break. (2) Calculate the bed shear stress distribution in the wave front region. (3) Predict the occurrence of bed load motion and sediment suspension at 3 seconds after wave/dam break.

Assume  $S_0 = 0$ . The beach is made of fine sand ( $d_{50} = 0.25$  mm,  $d_{90} = 0.85$  mm). Assume  $f = 0.05$ . For sea water,  $\rho = 1024$  kg/m<sup>3</sup> and  $\mu = 1.22 \times 10^{-3}$  Pa.s.

Fig. - Breaking wave runoff on a beach face - Front propagation from left to right



### Solution

Let select a positive x-direction toward the shore. The dam break wave ( $d_0 = 0.5$  m) propagates in a dry channel. The x coordinate is zero ( $x = 0$ ) at wave breaking (i.e. pseudo-dam site) and the time origin is taken at the start of wave breaking.

(1)  $U = 1.34$  m/s,  $x_s = 7.9$  m.

(2) In the wave front region,  $V = U$ . Hence:  $\tau_0 = 11.5$  Pa,  $V_* = 0.105$  m/s.

(3)  $\tau_* = 2.8$ ,  $V_*/w_0 = 3.8$ .

$\tau_* > (\tau_*)_c \Rightarrow$  Bed load motion

$V_*/w_0 > 0.2$  to  $2 \Rightarrow$  Suspended load

### Remarks

+ The above development has a number of limitations. The reservoir is assumed infinite although a breaking wave has a finite volume, and the beach slope is assumed horizontal.

+ The Shields parameter  $\tau_*$  must be compared with the critical Shields parameter for bed load motion  $(\tau_*)_c \sim 0.035$  (Chapter 8, paragraph 8.3). For a 0.25 mm sand particle, the settling velocity is 0.028 m/s. The ratio  $V_*/w_0$  is used to assess sediment suspension (Chapter 9, paragraph 9.2).

More exercises in textbook pp. 362-370 &371-373.

"The Hydraulics of Open Channel Flow: An Introduction", *Butterworth-Heinemann Publ.*, 2nd edition, Oxford, UK, 2004.