$$C_{max} = \frac{M}{A * \sqrt{4 * \pi * K * \frac{x}{V}}} * \exp(- k * t)$$

The result leads to Equation (6-7) for k = 0 (no reaction).
2- The standard deviation of the contaminant cloud equals : $\sigma = \sqrt{2*K*t}$.

Sudden increase in mass concentration at the origin

The concentration is initially zero everywhere. At the initial time t = 0, the concentration is suddenly raised to C_o at the origin x = 0 and held constant : $C_m(0, t \geq 0) = C_o$. The dispersion equation becomes :

$$V * \frac{\partial C_m}{\partial x} = K * \frac{\partial^2 C_m}{\partial x^2} - k * C_m \qquad (7\text{-}13)$$

with the boundary condition : $C_m = 0$ at $x = + \infty$. A solution of the dispersion equation is :

$$C_m = C_o * \exp\left(- \frac{k * x}{V} * \left(\frac{2}{\alpha} * (\sqrt{\alpha + 1} - 1) \right) \right) \qquad (7\text{-}14)$$

where :

$$\alpha = \frac{4 * K * k}{V^2}$$

and C_o is an integration constant. Neglecting the existence of the initial zone, the initial concentration C_o is equivalent to the steady release of $\dot{M}$ units of mass per unit time into a river (discharge Q) :

$$C_o = \frac{\dot{M}}{Q} * \left(\frac{2}{\alpha} * (\sqrt{\alpha + 1} - 1) \right) \qquad (7\text{-}15)$$

Remarks

1- $\alpha = 4*K*k/V^2$ is a dimensionless coefficient of reaction-dispersion. For biochemical oxygen demand, α is typically of the order : $\alpha < 0.4$.
2- In natural rivers and using Equations (5-6) and (6-8), it yields :

$$\alpha = \frac{4 * K * k}{V^2} = 0.0264 * \frac{W^2 * k}{\varepsilon_t}$$

where W is the channel width and ε_t is the transverse mixing coefficient.

Discussion

The one-dimensional dispersion analysis (Eq. (7-10)) assumes that the contaminant is fully-mixed across the channel : i.e., it does not apply to the initial zone (Fig. 5-1). For a steady flow in rivers, the distance required for cross-section mixing is about (section 5.3) :

$L \sim 0.4*V*W^2/\varepsilon_t$ complete mixing of side discharge

while the characteristic distance of contaminant decay is V/k ([1]). The latter exceeds the cross-section mixing distance (i.e. x' = 0.4) only if the dimensionless coefficient of reaction-dispersion α is less than about 0.066.

---

[1]Remember that the time required for decay of the contaminant to a factor e^{-1} is 1/k, during which the fluid is advected to a distance V/k.