In a waterway, the sources of dissolved oxygen are the re-oxygenation at the free-surface and the photo-synthesis. The sinks of oxygen include the biochemical oxygenation of organic matter, the decomposition of sludge deposits and the respiration of aquatic plants. In a first approximation, the budget of the dissolved oxygen (DO) is primarily a function of the re-oxygenation rate and the biochemical oxygenation demand (BOD) (Fig. 7-6). At steady state, the DO balance may be expressed as :

$$V * \frac{\partial C_m}{\partial x} = k_a * (C_{sat} - C_m) - k_r * BOD \qquad (7\text{-}18)$$

where x is the distance in the flow direction, C_m is the dissolved oxygen concentration, BOD is the biochemical oxygen demand, C_{sat} is the concentration of dissolved gas in water at equilibrium (Appendix 7B), k_a is the re-oxygenation rate constant and k_r is the BOD decay rate.

In Equation (7-18), the right handside term includes the re-oxygenation term $(+k_a*(C_{sat}-C_m))$ plus the biochemical oxygenation demand $(-k_r*C_m)$. Equation (7-18) may be integrated by a method of superposition. It yields :

$$C_m = C_{sat} - (C_{sat} - C_o) * \exp\left(-\frac{k_a * x}{V}\right) - \frac{k_r}{k_a - k_r} * BOD_o * \left(\exp\left(-\frac{k_r * x}{V}\right) - \exp\left(-\frac{k_a * x}{V}\right)\right) \qquad (7\text{-}19)$$

where C_o is initial oxygen mass concentration (at x = 0), BOD_o is the initial biochemical demand (at x = 0+), and assuming that the effluent (wastewater) is released at the origin (x = 0).

Remarks

1- The biochemical oxygen demand (BOD) is the amount of oxygen used by micro-organisms in the process of breaking down organic matter in water.

2- The above analysis of waterway self-cleaning capacity is sometimes called sag analysis or dissolved oxygen sag analysis. It was first proposed by STREETER and PHELPS in 1925 ([2]). GRAF and ALTINAKAR (1998, pp. 567-572). and METCALF & EDDY (1991, pp. 1216-1220) discussed the analysis in more details.

3- The oxidation of BOD consumes oxygen and it is an oxygen sink for the ambient waters. The most widely used parameter in surface waters is biochemical oxygen demand of consumed oxygen during a period of 5 days at a temperature of 20 Celsius in obscurity, denoted BOD_5.

METCALF & EDDY (1991, pp. 71-82) argued that BOD_5 is only a representative index of oxygen consumption. It is <u>not</u> accurate <u>nor</u> precise.

4- The BOD consumption decays as Equation (7-17):

$$BOD = BOD_o * \exp\left(-\frac{k_r * x}{V}\right)$$

It yields the following relationship between the initial biochemical oxygen demand BOD_o and the initial BOD measured at 5 days :

$$BOD_o = \frac{(BOD_5)_o}{1 - \exp(-k_r * 5 * 86400)}$$

where k_r is expressed in s^{-1} and 5 days equal 86400 seconds.

5- Assuming that the effluent volume flow rate is negligible compared to the river discharge, and that the BOD of the stream is very small, the initial biochemical demand BOD_o equals :

$$BOD_o = \frac{\dot{M}}{V * A}$$

where $\dot{M}$ is the effluent BOD mass discharge, V is the stream velocity and A is the stream cross-section area.

6- In the general case, the initial biochemical oxygen demand equals :

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[2] STREETER, H.W., and PHELPS, E.B. (1925), *US Publ. Heath Bulletin*, Vol. 146.