In a waterway, the sources of dissolved oxygen are the re-oxygenation at the free-surface and the photosynthesis. The sinks of oxygen include the biochemical oxygenation of organic matter, the decomposition of sludge deposits and the respiration of aquatic plants. In a first approximation, the budget of the dissolved oxygen (DO) is primarily a function of the re-oxygenation rate and the biochemical oxygenation demand (BOD) (Fig. 7-6). At steady state, the DO balance may be expressed as:

$$V \frac{\partial C_m}{\partial x} = k_a *(C_{sat} - C_m) - k_r * BOD$$  \hspace{1cm} (7-18)$$

where x is the distance in the flow direction, $C_m$ is the dissolved oxygen concentration, BOD is the biochemical oxygen demand, $C_{sat}$ is the concentration of dissolved gas in water at equilibrium (Appendix 7B), $k_a$ is the re-oxygenation rate constant and $k_r$ is the BOD decay rate.

In Equation (7-18), the right hand-side term includes the re-oxygenation term ($+k_a*(C_{sat} - C_m)$) plus the biochemical oxygenation demand ($-k_r*C_m$). Equation (7-18) may be integrated by a method of superposition. It yields:

$$C_m = C_{sat} - (C_{sat} - C_o) * \exp\left(-\frac{k_r * x}{V}\right) - \frac{k_r}{k_a - k_r} * BOD_o * \left(\exp\left(-\frac{k_r * x}{V}\right) - \exp\left(-\frac{k_a * x}{V}\right)\right)$$ \hspace{1cm} (7-19)$$

where $C_o$ is initial oxygen mass concentration (at x = 0), $BOD_o$ is the initial biochemical demand (at x = 0+), and assuming that the effluent (wastewater) is released at the origin (x = 0).

**Remarks**

1- The biochemical oxygen demand (BOD) is the amount of oxygen used by micro-organisms in the process of breaking down organic matter in water.

2- The above analysis of waterway self-cleaning capacity is sometimes called sag analysis or dissolved oxygen sag analysis. It was first proposed by STREETER and PHELPS in 1925 (2). GRAF and ALTINAKAR (1998, pp. 567-572), and METCALF & EDDY (1991, pp. 1216-1220) discussed the analysis in more details.

3- The oxidation of BOD consumes oxygen and it is an oxygen sink for the ambient waters. The most widely used parameter in surface waters is biochemical oxygen demand of consumed oxygen during a period of 5 days at a temperature of 20 Celsius in obscurity, denoted BOD$_5$.

METCALF & EDDY (1991, pp. 71-82) argued that BOD$_5$ is only a representative index of oxygen consumption. It is not accurate nor precise.

4- The BOD consumption decays as Equation (7-17):

$$BOD = BOD_o * \exp\left(-\frac{k_r * x}{V}\right)$$

It yields the following relationship between the initial biochemical oxygen demand $BOD_o$ and the initial BOD measured at 5 days:

$$BOD_o = \frac{(BOD_5)_o}{1 - \exp(-k_r * 5 * 86400)}$$

where $k_r$ is expressed in s$^{-1}$ and 5 days equal 86400 seconds.

5- Assuming that the effluent volume flow rate is negligible compared to the river discharge, and that the BOD of the stream is very small, the initial biochemical demand $BOD_o$ equals:

$$BOD_o = \frac{M}{V * A}$$

where $M$ is the effluent BOD mass discharge, $V$ is the stream velocity and $A$ is the stream cross-section area.

6- In the general case, the initial biochemical oxygen demand equals:

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