IDEAL-FLUID FLOW TUTORIALS

Attendance to tutorials is very strongly advised.

Repeated absences by some individuals will be noted and these would demonstrate some disappointing responsible behaviour.

1. Introduction : Pre-Requisite Knowledge - Tutorials

The first tutorial consists of basic pre-requisite knowledge.

1.1 Give the following fluid and physical properties(at 20 Celsius and standard pressure) with a 4-digit accuracy.

	Value	Units
Air density :		
Water density :		
Air dynamic viscosity :		
Water dynamic viscosity :		
Gravity constant (in Brisbane) :		
Surface tension (air and water) :		

1.2 What is the definition of an ideal fluid ?

What is the dynamic viscosity of an ideal fluid ?

1.3 From what fundamental equation does the Navier-Stokes equation derive : (a) continuity, (b) momentum equation, (c) energy equation, (d) other ?

From what fundamental principle derives the Bernoulli equation ?

1.4 Sketch the streamlines of the following two-dimensional flow situations :

A- A laminar flow past a circular cylinder,

B- A turbulent flow past a circular cylinder,

In each case, show the possible extent of the wake (if any). Indicate clearly in which regions the ideal fluid flow assumptions are valid, and in which areas they are not.

Remember the CIVL3130 Fluid Mechanics experiment "Flow past a cylinder".

2. Ideal Fluid Flow - Irrotational Flows - Tutorials

2.1 Quizz

- What is the definition of the velocity potential ?
- Is the velocity potential a scalar or a vector ?
- Units of the velocity potential ?
- What is definition of the stream function ? Is it a scalar or a vector ? Units of the stream function ?

For an ideal fluid with irrotational flow motion :

- Write the condition of irrotationality as a function of the velocity potential.
- Does the velocity potential exist for 1- an irrotational flow and 2- for a real fluid ?
- Write the continuity equation as a function of the velocity potential.

Further, answer the following questions :

- What is a stagnation point ?
- For a two-dimensional flow, write the stream function conditions.
- How are the streamlines at the stagnation point ?

Reference

CHANSON, H. (2008). "Applied Hydrodynamics: an Introduction to Ideal and Real Fluid Flows." *Lecture Notes*, Subject CIVL4160, Div. of Civil Engrg., Univ. of Queensland, Australia, 355 pages.

VALLENTINE, H.R. (1969). "Applied Hydrodynamics." *Butterworths*, London, UK, SI edition.

2.2 Basic applications

(1) Considering the following velocity field :

$$V_x = y \times z \times t$$

$$V_y = z \times x \times t$$

$$V_z = x \times y \times t$$

- Is the flow a possible flow of an incompressible fluid ?
- Is the motion irrotational ? If yes : what is the velocity potential ?

(2) Considering the following velocity field :

$$V_x = 2 \times x$$

$$V_y = -2 \times y$$

Is the motion irrotational ? In the affirmative, what is the velocity potential ?

(3) Draw the streamline pattern of the following stream functions :

(3.1) $\psi = 50 \times x$

(3.2) $\psi = - 20 \times y$

(3.3) $\psi = - 40 \times x - 30 \times y$

(3.4) $\psi = -10 \times x^2$ (from x = 0 to x = 5)

2.3 Two-dimensional flow

Considering a two-dimensional flow, find the velocity potential and the stream function for a two-dimensional flow having the following velocity components :

$$V_x = -\frac{2 \times x \times y}{(x^2 + y^2)^2}$$

$$V_y = \frac{x^2 - y^2}{(x^2 + y^2)^2}$$

2.4 Applications

(a) Using the software 2DFlowPlus, investigate the flow field of a vortex (at origin, strength 2) superposed to a sink (at origin, strength 1). Visualise the streamlines, the contour of equal velocity ad the contour of constant pressure.

Repeat the same process for a vortex (at origin, strength 2) superposed to a sink (at x=-5, y=0, strength 1). How would you describe the flow region surrounding the vortex.

(b) Investigate the superposition of a source (at origin, strength 1) and an uniform velocity field (horizontal direction, V = 1). How many stagnation point do you observe ? What is the pressure at the stagnation point ? What is the "half-Rankine" body thickness at x = +1 ? (You may do the calculations directly or use 2DFlowPlus to solve the flow field.)

(c) Using 2DFlowPlus, investigate the flow past a circular building (for an ideal fluid with irrotational flow motion). How many stagnation points is there ? Compare the resulting flow pattern with real-fluid flow pattern behind a circular bluff body (search Reference text in the library).

(d) Investigate the seepage flow to a sink (well) located close to a lake. What flow pattern would you use ?

Note : the software 2DFlowPlus is described in the lecture notes, Appendix D.

Reference

CHANSON, H. (2008). "Applied Hydrodynamics: an Introduction to Ideal and Real Fluid Flows." *Lecture Notes*, Subject CIVL4160, Div. of Civil Engrg., Univ. of Queensland, Australia, 355 pages.

2.5 Basic equations (2)

Considering an two-dimensional irrotational flow of ideal fluid, which basic principle(s) is(are) used to determine the pressure field ?

Exercise Solutions

Exercise 2.2

Solution (1)

The equations satisfy the equation of continuity for :

$$\frac{\partial V_x}{\partial x} = \frac{\partial V_y}{\partial y} = \frac{\partial V_z}{\partial z} = 0$$

so that :

$$\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} = 0$$

The components of the vorticity are :

$$\left(\frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z}\right) = (x \times t - x \times t) = 0$$

$$\left(\frac{\partial V_x}{dz} - \frac{\partial V_z}{\partial x}\right) = (y \times t - y \times t) = 0$$

$$\left(\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}\right) = (z \times t - z \times t) = 0$$

hence vorticity is zero and the field could represent irrotational flow. The velocity potential would then be the solution of :

$$\frac{\partial \phi}{\partial x} = -V_x = -y \times z \times t \qquad \phi = -x \times y \times z \times t + f_1(y,z,t)$$

$$\frac{\partial \phi}{\partial y} = -V_y = -x \times z \times t \qquad \phi = -x \times y \times z \times t + f_2(x,z,t)$$

$$\frac{\partial \phi}{\partial z} = -V_z = -x \times y \times t \qquad \phi = -x \times y \times z \times t + f_3(x,y,t)$$

and hence :

$$\phi = -x \times y \times z \times t + f(t)$$

is a possible velocity potential.

Solution (3)

Irrotational flow motion.

Solution (3)

(1) Vertical uniform flow : V_o = +50 m/s

(2) Horizontal uniform flow : V_o = + 20 m/s

(3) Uniform flow : V_o = 50 m/s and α = 127 degrees

(4) Non uniform vertical flow

Exercise 2.3

Solution

In polar coordinate the velocity components are :

$$V_x = -\frac{2 \times \cos\theta \times \sin\theta}{r^2} \qquad V_y = \frac{\cos^2\theta - \sin^2\theta}{r^2}$$

Using :

$$V_r = V_x \times \cos\theta + V_y \times \sin\theta V_\theta = V_y \times \cos\theta - V_x \times \sin\theta$$

we deduce :

$$V_r = -\frac{\sin\theta}{r^2} \qquad V_\theta = \frac{\cos\theta}{r^2}$$

In polar coordinates the velocity potential and stream function are :

$$V_r = -\frac{\partial \phi}{\partial r} \qquad V_\theta = -\frac{1}{r} \times \frac{\partial \phi}{\partial \theta}$$

$$V_r = -\frac{1}{r} \times \frac{\partial \psi}{\partial \theta} \qquad V_\theta = \frac{\partial \psi}{\partial r}$$

Hence :

$$\phi = -\frac{\sin\theta}{r} + \text{constant}$$

Discussion

The resulting flow pattern is a doublet at the origin aligned along the vertical axis. See CHANSON (2008), Chapter I-4.

Exercise 2.4

The software 2DFlow+ is installed on the engineering network.

Exercise 2.5

Solution

Bernoulli principle. See CHANSON (2008), Chapter I-2.

Flow net analysis & Graphical solutions - Discussion

A **streamline** is the line drawn so that the velocity vector is always tangential to it (i.e. no flow across a streamline). Some important characteristics of streamlines are :

(1) There can be no flow across a streamline.

(2) Streamlines converging in the direction of the flow indicate a fluid acceleration.

(3) Streamlines do not cross.

(4) In steady flow the pattern of streamlines does not change with time.

(5) Solid stationary boundaries are streamlines provided that separation of the flow from the boundary does not occur.

The following theorems are proved as a consequence of kinetic energy considerations and are limited to irrotational flows of ideal fluid :

(1) Irrotational motion is impossible if all of the boundaries are fixed.

(2) Irrotational motion of a fluid will cease when the boundaries come at rest.

(3) The pattern of irrotational flow which satisfies the Laplace equation and prescribed boundary conditions is unique and is determined by the motion of the boundaries.

(4) Irrotational motion of a fluid at rest at infinity is impossible if the interior boundaries are at rest.

(5) Irrotational motion of a fluid at rest at infinity is unique and determined by the motion of the interior solid boundaries.

Two-dimensional flow patterns can be represented visually by drawing a family of streamlines and the corresponding family of equipotential lines with the constants varying in arithmetical progression. The resulting network of lines is called a **flow net**. It is usual to select the ϕ-lines and ψ-lines so that : $\delta\psi = \delta\phi = \delta c$. The flow net consists of an orthogonal grid that reduces to perfect squares, when the grid size approaches zero. In uniform flow region the squares are of equal size. In diverging flow their size increases, and in converging flow they decrease in size, in the direction of flow.

Basic characteristics of flow nets are :

(1) Flow nets are based upon the assumption of irrotational flow of ideal fluid, but it is not necessary that the flow is steady.

(2) For given boundary conditions there is only one possible flow pattern. This is a property of the Laplace equation.

(3) Streamlines are everywhere tangent to the velocity vector. Fixed boundaries are streamlines. The volumetric flow rate between two streamlines is : $\delta q = \delta\psi$ where $\delta\psi$ is the difference in stream function value between adjacent streamlines.

(4) There is no velocity component tangent to an equipotential line and hence the velocity vector must be everywhere normal to an equipotential line. Equipotential lines intersect fixed boundaries normally.

(5) Streamlines and equipotential lines are orthogonal (i.e. intersect at right angles).

The problem of finding the flow net to satisfy given fixed boundaries is purely a graphical exercise: i.e., the construction of an orthogonal system of lines that compose boundaries and reduce to perfect squares in the limit as the number of lines increases. Typically the construction of a flow net includes :

(1) Construction of the streamlines in the regions where the velocity distributions are evident : e.g., parallel or radial flow, fixed boundaries.

(2) Sketch the remaining portions of streamlines with smooth curves.

(3) Construction of the equipotential lines : (3a) normal to all streamlines and fixed boundaries, and (3b) forming squares with the streamlines. This is a process of trial-and-error. The diagonal of the 'squares' should form smooth curves. At stagnation points the occurrence of five-sided 'square' results from the impossibility to obtain infinite spacing of streamlines in the region.

(4) Location of free surfaces by trial.

(5) Tendency to separate is indicated where the velocity at a boundary reaches a maximum and thereafter decreases in the direction of flow.

3. Two-Dimensional Flows (1) Basic equations and flow analogies - Tutorials

3.1 Flow net (2006 examination paper)

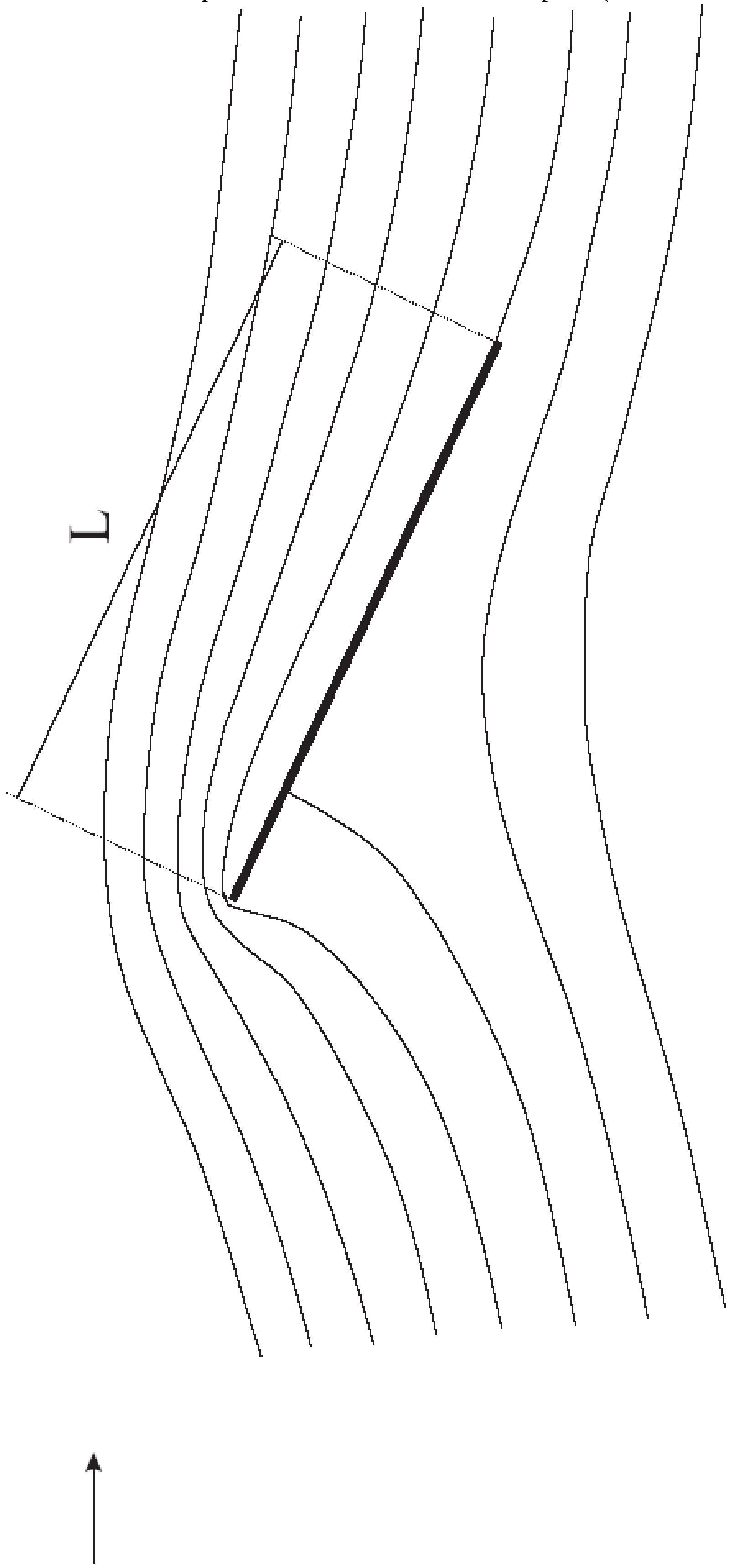
A hydrodynamic study of the flow past a flat plate is conducted in water. The plate length is L = 0.2 m and the plate width equals the test section width (0.5 m). For a particular angle of incidence, dye injection in a water tunnel gives the flow pattern shown in Figure E3-1 on the next page. The mean flow is horizontal.

(a) Complete the flow net by drawing the suitable equipotentials. (Draw the complete flow net on the examination paper.)

(b) The upstream velocity is 6.5 m/s. Estimate the drag force and the lift force acting on the plate. *Indicate clearly the sign convention. You may have to draw more streamline and equipotentials to obtain a good accuracy.*

Assume water at 20 Celsius (ρ = 998.2 kg/m3, μ = 1.005 E-5 Pa.s, σ = 0.0736 N/m).

Fig. E3-1 - Flow visualisation and streamline patterns around an inclined flat plate (undistorted scale)



3.2 Flow net

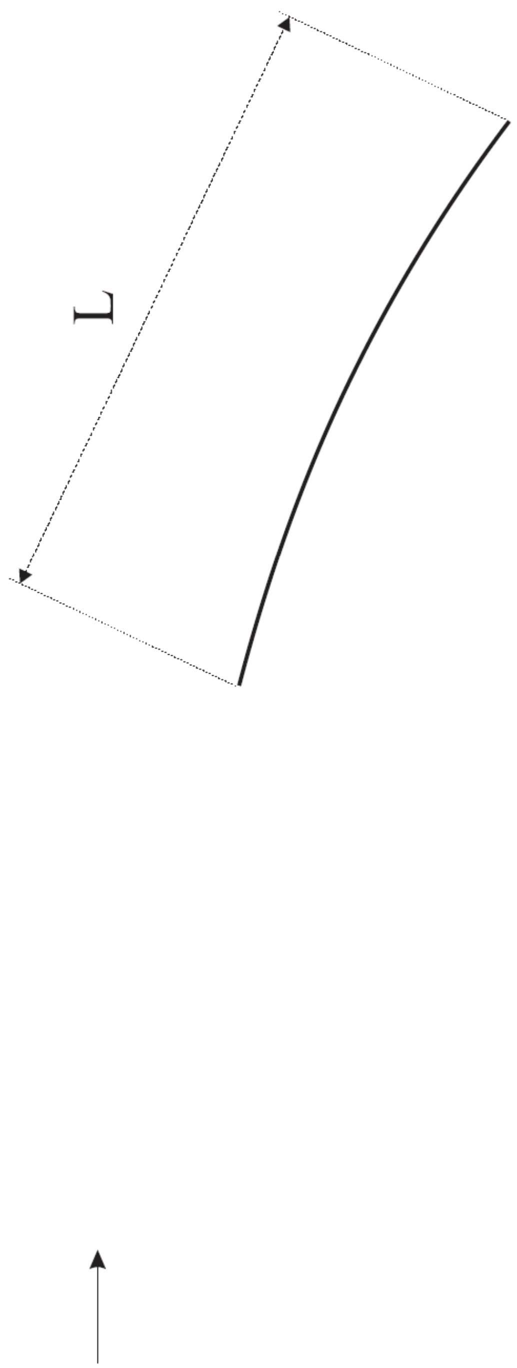
Let us consider thewater flow past a circular arc sketched in Figure E3-2. The chord is L = 0.35 m and the plate width equals 1 m. For a particular angle of incidence shown in Figure E3-2, conduct a graphical analysis of the flow field. The mean flow is horizontal.

(a) Draw the flow net by drawing the suitable streamlines and equipotentials. (Draw the complete flow net with sufficient details in the vicinity of the circular arc.)

(b) The upstream velocity is 15.2 m/s. Estimate the drag force and the lift force acting on the cambered plate. *Indicate clearly the sign convention..*

Assume water at 20 Celsius (ρ = 998.2 kg/m3, μ = 1.005 E-5 Pa.s, σ = 0.0736 N/m).

Fig. E3-2 - Flow past a circular arc (undistorted scale)



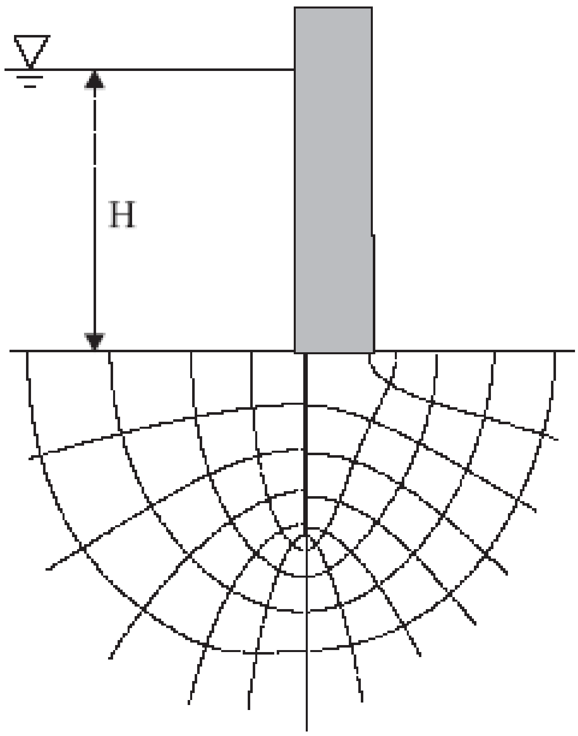
3.3 Flow net beneath a cutoff wall

For a two-dimensional seepage under a impervious structure with a cutoff wall (Fig. E3-3), the boundary conditions are : H = 6 m, K = 2.0 m/day.

(a) What is the hydraulic conductivity in m/s ? What type of soil is it ?

(b) Using the flow net, estimate the seepage flow par meter width of dam.

Fig. E3-3 - Flow net beneath an impervious dam



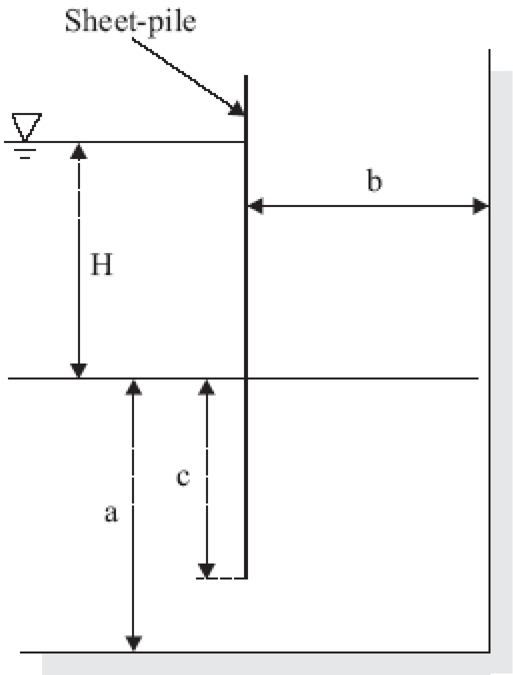
3.4 Flow net under a sheetpiling

For a two-dimensional seepage under sheetpiling with a permeable foundation (Fig. E3-4), the boundary conditions are : a = 9.4 m, b = 4.7 m, c = 4 m, H = 2.5 m, K = 2.0 10^{-3} cm/s.

Using a dimensioned flow net (with 5 stream tubes or more), estimate the seepage flow par meter width of dam.

Use graph paper.

Fig. E3-4 - Flow net beneath a sheet pile



3.5 Flow net under an impervious dam

For a two-dimensional seepage under an impervious dam with apron (Lecture Notes, CHANSON 2008, Fig. 3-1B), the boundary conditions are : H = 80 m, K = 5 10^{-5} m/s.

(A) Calculate the seepage flow rate in presence of an apron and a cutoff wall (Fig. 3-1B).

(B) In absence of the cutoff wall, sketch the flow net and determine the seepage flow. Determine the pressure distribution along the base of the dam and beneath the apron. Calculate the uplift forces on the dam foundation and on the apron.

3.6 Flow net under an impervious dam with cutoff wall

For a two-dimensional seepage under an impervious dam with a cutoff wall (Fig. E3-5), the boundary conditions are : H_1 = 60 m, H_2 = 5 m, a = 60 m, b = 100 m, L = 70 m, K = 1 10^{-5} m/s.

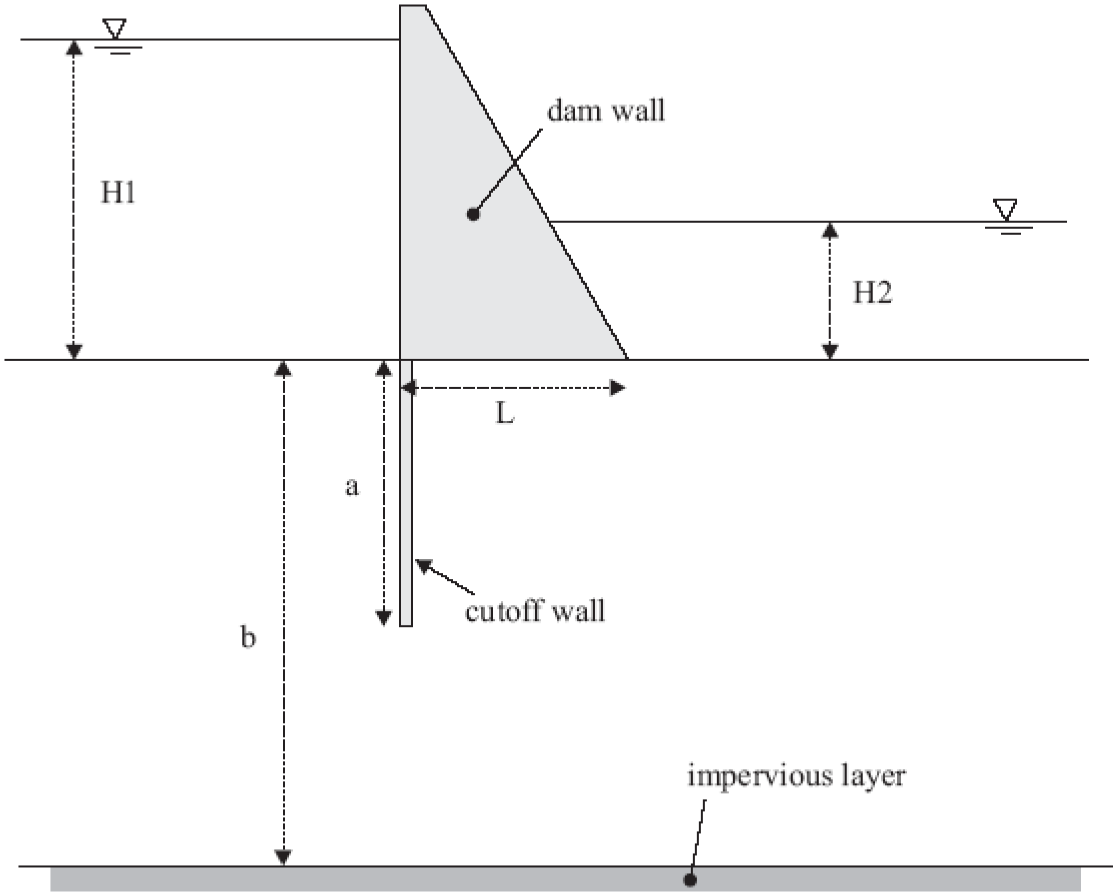
(A) In absence of cutoff wall (i.e. a = 0 m), sketch the flow net; determine the seepage flow; determine the pressure distribution along the base of the dam; calculate the uplift force.

(B) With the cutoff wall (i.e. a = 60 m): same questions : sketch the flow net; determine the seepage flow; determine the pressure distribution along the base of the dam; calculate the uplift force

(C) Comparison and discuss the results.

Use graph paper.

Fig. E3-5 - Flow net beneath a dam with cutoff wall

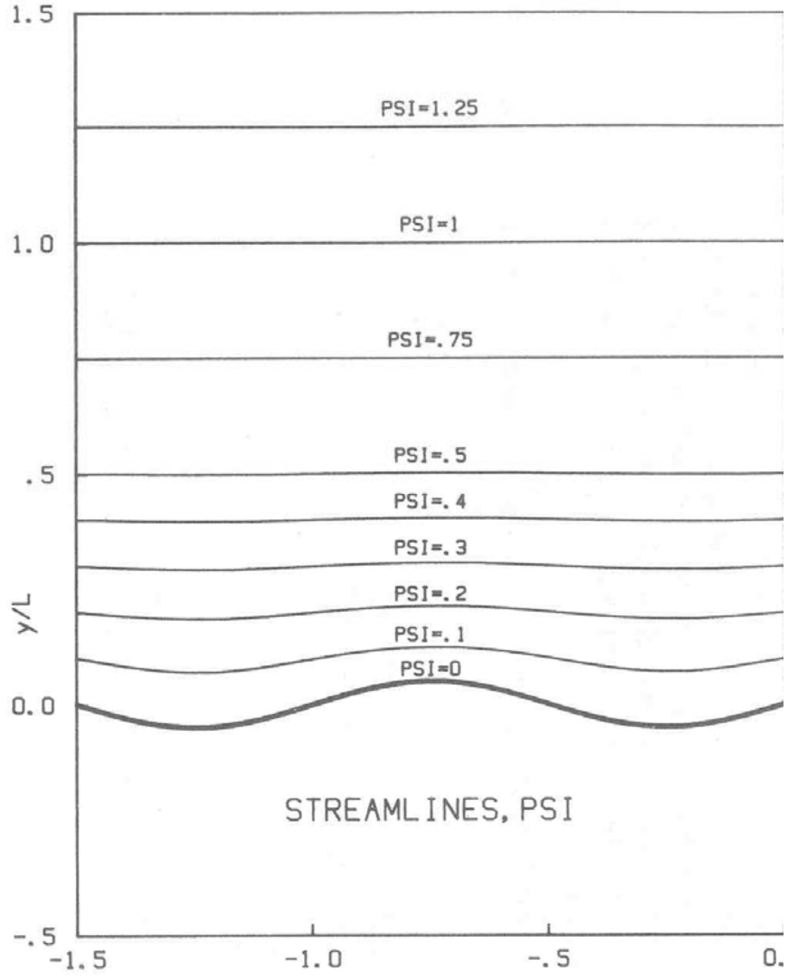


3.7 Pressure distribution above some bed forms

Figure E3-6 illustrates an open channel flow above some quasi-sinusoidal bed forms. The wave length is 2.2 m and the average water depth is 3.3 m. Neglecting sediment motion, calculate the pressure distributions at the bed form crest and trough.

On a flat bed, the pressure distribution is hydrostatic. Determine the deviations from the hydrostatic pressure distribution.

Fig. E3-6 - Flow net beneath a dam with cutoff wall



Exercise Solutions

Exercise 3.1

The problem is solved by drawing the equipotentials and completing the flow net. Close to the foil, additional streamlines and equipotentials may be drawn to improve the estimates of the velocities next to the extrados and intrados of the foil.

The pressure field is derived from the Bernoulli equation and the integration of the pressure distributions yields the lift and drag forces.

Remarks

- The flow net method and technique are presented in the Lecture Notes (CHANSON 2008).
- The complete theory of lift and drag on airfoils, wings and hydrofoils is developed in the Lecture Notes (CHANSON 2008, chapter I-6).

Exercise 3.2

The problem may be solved analytically using the Joukowski transformation and theorem of Kutta-Joukowski. The complete theory of lift and drag on airfoils, wings and hydrofoils is developed in the Lecture Notes (CHANSON 2008, chapter I-6).

The flow net solution can be compared with the theory of lift and drag.

Exercise 3.3

Solution

$q = 4.6$ m^2/day

Remark

See Lecture Notes (CHANSON 2008), pp. I3-3 to I-3-14.

Exercise 3.4

Solution

$q = 2.7$ m^2/day

Discussion

The flow pattern may be analysed analytically using a finite line source (for the sheet pile) and the theory of images. The resulting streamlines are the equipotentials of the sheet-pile flow.

Remember : A velocity potential can be found for each stream function. If the stream function satisfies the Laplace equation the velocity potential also satisfies it. Hence the velocity potential may be considered as stream function for another flow case. The velocity potential ϕ and the stream function ψ are called "*conjugate functions*" (Chapter I-2).

Remark

See Lecture Notes (CHANSON 2008), pp. I-3-3 to I-3-14.

Exercise 3.5

Solution

(A) The problem is similar to the flow net sketched in Figure 3-1B (Lecture Notes, Chapter I-3, paragraph 2.2).

(B) In absence of cutoff wall, the streamlines are shorter and the seepage flow rate is greater.

The pressure distribution beneath the dam foundation and aprong may be deduced from the equipotential lines, since $\phi = K \times H$ where K is the hydraulic conductivity and H is the piezometri head (Chapter I-3, paragraph 3.3).

The uplift pressure on the dam foundation and apron are very significant. The apron structure would be subjected to high risks of uplift and damage.

Remark

See Lecture Notes (CHANSON 2008), pp. I-3-3 to I-3-14.

Exercise 3.7

Solve graphically the problem : (a) Complete the equipotential lines; (b) Calculate the velocity magnitude at all vertical elevation; (c) Apply the Bernoulli principle.

Remember that the pressure gradient is hydrostatic far away upstream.

Application

This flow pattern is typical of the flow above standing wave bed forms, although these tend to be significantly larger (KENNEDY 1963, CHANSON 2000). The height of the wall undulations is closer to large ripples and small dunes (CHANSON 2204, pp. 151-155 & 223-231).

KENNEDY, J.F. (1963). "The Mechanics of Dunes and Antidunes in Erodible-Bed Channels." *Jl of Fluid Mech.*, Vol. 16, No. 4, pp. 521-544 (& 2 plates).

CHANSON, H. (2000). "Boundary Shear Stress Measurements in Undular Flows : Application to Standing Wave Bed Forms." *Water Res. Res.*, Vol. 36, No. 10, pp. 3063-3076 (ISSN 0043-1397). {http://espace.library.uq.edu.au/view.php?pid=UQ:11117}

CHANSON, H. (2004). "The Hydraulics of Open Channel Flow : An Introduction." *Butterworth-Heinemann*, Oxford, UK, 2nd edition, 630 pages (ISBN 978 0 7506 5978 9).

4. Two-Dimensional Flows (2) Basic flow patterns - Tutorials

4.1 Doublet in uniform flow (1)

Select the strength of doublet needed to portray an uniform flow of ideal fluid with a 20 m/s velocity around a cylinder of radius 2 m.

4.2 Source and sink

A source discharging 0.72 m^2/s is located at (-1, 0) and a sink of twice the strength is located at (+2, 0). For a remote pressure (far away) of 7.2 kPa, ρ = 1,240 kg/m^3, find the velocity and pressure at (0, 1) and (1, 1).

Note : When some measurements are conducted with a Prandtl-Pitot tube, the pressure tapping at the leading edge of the tube gives the dynamic pressure, while the pressure tappings on the side give the piezometric pressure. Remember that, at the leading edge of the tube, stagnation occurs.

Remarks

The Pitot tube is named after the Frenchman Henri PITOT. The first presentation of the concept of the Pitot tube was made in 1732 at the French Academy of Sciences by Henri PITOT. The original Pitot tube included basically a total head reading. Ludwig PRANDTL improved the device by introducing a pressure (or piezometric head) reading. The modified Pitot tube is sometimes called a Pitot-Prandtl tube.
For many years, aeroplanes used Prandtl-Pitot tubes to estimate their relative velocity.

4.3 Flow pattern (2)

In two-dimensional flow we now consider a source, a sink and an uniform stream. For the pattern resulting from the combinations of a source (located at (-L, 0)) and sink (located at (+L, 0)) of equal strength Q in uniform flow (velocity $+V_o$ parallel to the x-axis) :

(a) Sketch streamlines and equipotential lines;

(b) Give the velocity potential and the stream function.

This flow pattern is called the flow past a Rankine body. W.J.M. RANKINE (1820-1872) was a Scottish engineer and physicist who developed the theory of sources and sinks. The shape of the body may be altered by varying the distance between source and sink (i.e. 2×L) or by varying the strength of the source and sink. Other shapes may be obtained by the introduction of additional sources and sinks and RANKINE developed ship contours in this way.

(c) What is the profile of the Rankine body (i.e. find the streamline that defines the shape of the body)?

(d) What is the length and height of the body ?

(e) Explain how the flow past a cylinder can be regarded as a Rankine body. Give the radius of the cylinder as a function of the Rankine body parameter.

4.4 Flow pattern (3)

In two-dimensional flow we consider again a source, a sink and an uniform stream. But. the source is located at (+L, 0) and the sink is located at (-L, 0) (i.e. opposite to a Rankine body flow pattern). They are of equal strength q in an uniform flow (velocity $+V_o$ parallel to the x-axis).

Derive the relationship between the discharge q, the length L and the flow velocity such that no flow injected at the source becomes trapped into the sink.

4.5 Doublet in uniform flow (2)

We consider the air flow (V_o = 9 m/s, standard conditions) past a suspension bridge cable (Ø = 20 mm),

(a) Select the strength of doublet needed to portray the uniform flow of ideal fluid around the cylindrical cable.

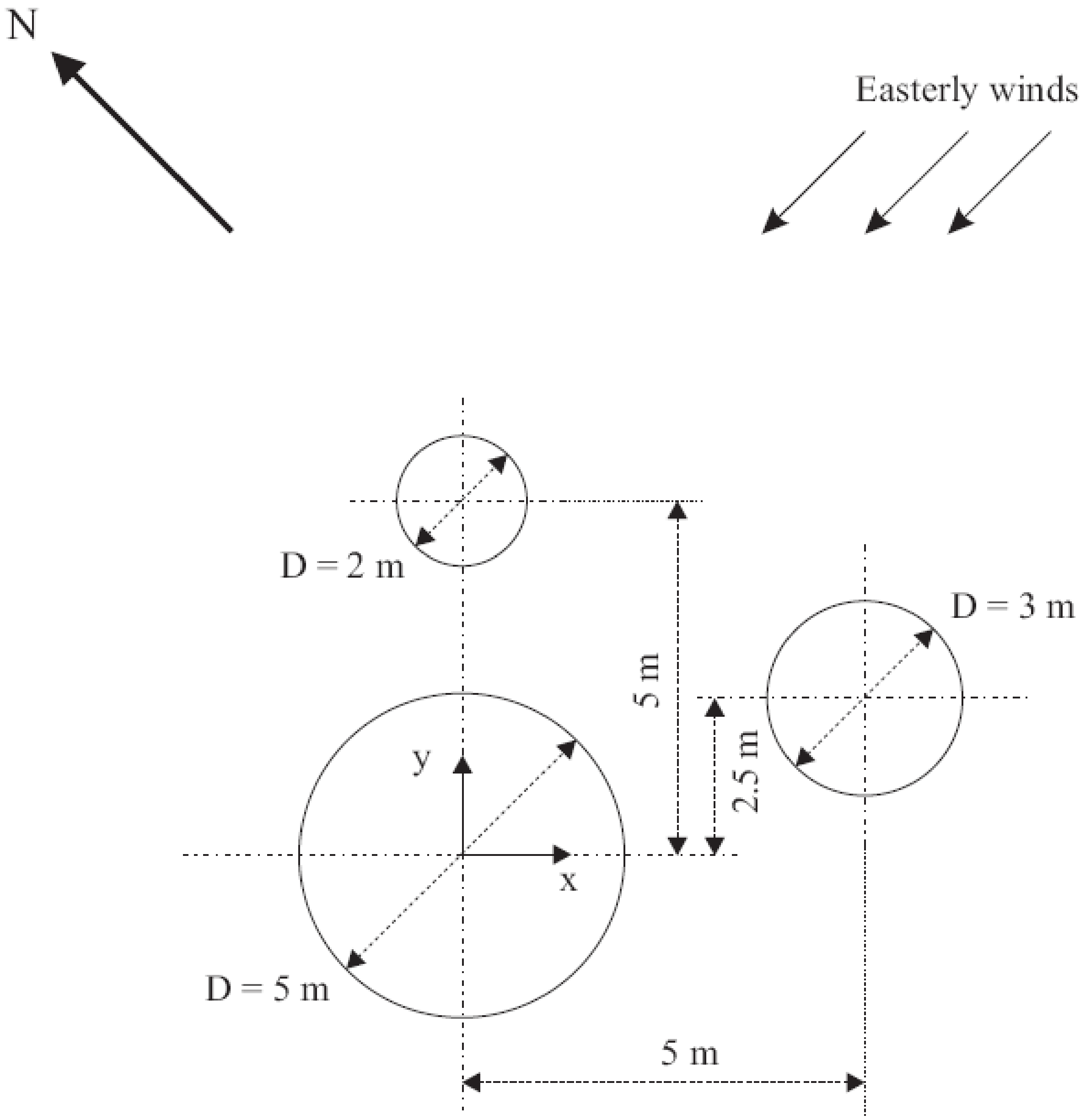
(b) In real fluid flow, calculate the hydrodynamic frequency of the vortex shedding.

4.6 Flow past buildings - 2003 exam paper

Let us consider a new architectural landmark to be built at the Mt Cootha Lookout. The structure consists of three circular cylinders (Height : 25 m - Diameter : 2, 3 and 5 m). The landmark will be facing North-East, while the dominant winds are Easterlies (Fig. 1).

You will assume that the wind flow around the structure is a two-dimensional irrotational flow of ideal fluid. The atmospheric conditions are : $P = P_{atm} = 10^5$ Pa ; T = 25 Celsius.

Fig. 1 - Sketch of the Mt Cootha landmark - View in elevation



(1) On graph paper, sketch the flow net with the landmark for a 25 m/s Easterly wind. Indicate clearly on the graph the discharge between two streamlines, the x-axis and y-axis, their direction, and use the centre of the 5-m diameter cylinder as the origin of your system of coordinates (with x in the South-East direction and y in the North-East direction).

(2) The Brisbane City Council is concerned about wind velocities between the buildings that may blow down tourists and damage cars.

(a) From your flow net, compute the wind velocity and the pressure at :

x = 1.5 m, y = 6 m

x = 5 m, y =4.5 m

x = 2.1 m, y = 2.1 m

These locations would be typical of tourists standing in front of the vertical cylinders.

(b) Where is located the point of maximum velocity and minimum pressure ?

(c) What is the maximum velocity and minimum pressure between the cylinders ? Indicate that location on your flow net.

(d) Discuss your results. Do you think that this result is realistic ? Why ?

(3) Explain what standard flow patterns you would use to describe the flow around these three buildings.

(4) Write the stream function and the velocity potential as a function of the wind speed V_o (25 m/s) and the two cylinder diameters D_1 (5 m), D_2 (3 m) and D_3 (2 m).

Do not use numbers. Express the results as functions of the above symbols.

(5) For a real fluid flow, what is (are) the drag force(s) on each cylinder (Height: 25 m) ?

4.7 Magnus effect (1)

Two 15-m high rotors 3 m in diameter are used to propel a ship. estimate the total longitudinal force exerted upon the rotors when the relative wind velocity is 25 knots, the angular velocity of the rotors is 220 revolutions per minute and the wind direction is at 60º from the bow of the ship.

Perform the calculations for (a) an ideal fluid with irrotational motion and (b) a real fluid.

(c) What orientation of the vector of relative wind velocity would yield the greatest propulsion force upon the rotorship ? Calculate the magnitude of this force.

Assume a real fluid flow. The result is trivial for ideal fluid with irrotational flow motion.

(d) Determine how nearly into the wind the rotorship could sail. That is, at wind angle would the resultant propulsion force be zero ignoring the wind effect on the ship itself.

4.8 Magnus effect (2)

A infinite rotating cylinder (R = 1.1 m) is placed in a free-stream flow (V_o = 15 knots) of water. The cylinder is rotating at 45 rpm and the ambient pressure far away is 1.1 E+5 Pa.

(a) Calculate and plot the pressure distribution on the cylinder surface.

(b) Find the maximum and minimum pressures on the cylinder surface.

(c) Find the location θ where the pressure on the cylinder surface is the fluid pressure far away from the cylinder.

4.9 Whirlpools

Whirlpools may be approximated by a series of vortices of same signs advected into an uniform flow.

(a) Consider two vortices of equal strength K = +1 located at (-2,) and (+2, 0). Estimate how far away the effect of the vortices is perceived to be that of an unique vortex. What would be the strength of that vortex ?

(b) Consider two vortices of equal strength K = +1 located at (-2,) and (+2, 0) in a horizontal uniform flow V = +0.03. What are the stream function and velocity potential of the resulting flow motion.

4.10 Magnus effect aircraft

Two rotating cylinders are used instead of conventional wings to provide the lift to an aircraft. Calculate the length of each cylinder wing for the following design conditions :

Cruise speed : 320 km/h

Cruise altitude : 2,000 m

Aircraft mass : 8 E+6 kg

Cylinder radius : 1.8 m

Cylinder rotation speed : 500 rpm

Exercise Solutions

Exercise 4.1

Solution

(a) A doublet and uniform flow is analog to the flow past a cylinder of radius :

$$R = \sqrt{\frac{-\mu}{V_o}}$$

where μ is the strength of the doublet. Hence :

$$\mu = -V_o \times R^2 = 80 \text{ m}^3/\text{s}$$

Remark

See Lecture Notes (CHANSON 2008), Chapter I-4.

Exercise 4.3

Solution

The flow past a Rankine body is the pattern resulting from the combinations of a source and sink of equal strength in uniform flow (velocity $+V_o$ parallel to the x-axis) :

$$\phi = -V_o \times r \times \cos\theta - \left(+ \frac{q}{2 \times \pi} \times \text{Ln}\left(\frac{r_1}{r_2}\right)\right)$$

$$\psi = -V_o \times r \times \sin\theta - \left(+ \frac{q}{2 \times \pi} \times (\theta_1 - \theta_2)\right)$$

where the subscript 1 refers to the source, the subscript 2 to the sink and q is positive for the source located at (-L, 0) and the sink located at (+L, 0).

The profile of the Rankine body is the streamline $\psi = 0$:

$$\psi = -V_o \times r \times \sin\theta + \frac{q}{2 \times \pi} \times (\theta_1 - \theta_2) = 0$$

$$r = \frac{q \times (\theta_1 - \theta_2)}{2 \times \pi \times V_o \times \sin\theta}$$

The length of the body equals the distance between the stagnation points where :

$$V = V_o + \frac{q}{2 \times \pi \times r_1} - \frac{q}{2 \times \pi \times r_2} = V_o + \frac{q}{2 \times \pi} \times \left(\frac{1}{r_s - L} - \frac{1}{r_s + L} \right) = 0$$

and hence :

$$L_{body} = 2 \times r_s = 2 \times L \times \sqrt{1 + \frac{q}{\pi \times L \times V_o}}$$

The half-width of the body h is deduced from the profile equation at the point (h, $\pi/2$) :

$$h = \frac{q \times (\theta_1 - \theta_2)}{2 \times \pi \times V_o}$$

where : $\theta_1 = \alpha$ and $\theta_2 = \pi - \alpha$ and hence :

$$\alpha = \frac{\pi}{2} - \frac{\pi \times h \times V_o}{q}$$

But also :

$$\tan\alpha = \frac{h}{L}$$

So the half-width of the body is the solution of the equation :

$$h = L \times \cot\left(\frac{\pi \times V_o}{q} \times h \right)$$

Remark

See Lecture Notes (CHANSON 2008), Chapter I-4.

Exercise 4.4

See Lecture Notes (CHANSON 2008), Chapter I-4.

Exercise 4.5

Solution

(a) A doublet and uniform flow is analog to the flow past a cylinder of radius :

$$R = \sqrt{\frac{- \mu}{V_o}}$$

where μ is the strength of the doublet. Hence :

$$\mu = - V_o \times R^2 = - 9 \text{ E-4 m}^3\text{/s}$$

(b) The Reynolds number of the flow is 1.1 E-4. For that range of Reynolds number, the vortex shedding behind the cable is characterised by a well-defined von Karman street of vortex. The hydrodynamic frequency satisfies :

$$St = \frac{\omega \times 2 \times R}{V_o} \sim 0.2$$

It yields : ω = 2.8 Hz. If the hydrodynamic frequency happens to coincide with the natural frequency of the structure, the effects may be devastating : e.g., Tacoma Narrows bridge failure on 7 November 1940.

4.6 2003 exam paper

(2)

(a1) x = 1.5 m, y = 6 m	V ~ 23.2 m/s	$P - P_{atm}$ = +52 Pa
(a2) x = 5 m, y =4.5 m	V ~ 28.5 m/s	$P - P_{atm}$ = -112 Pa
(a3) x = 2.1 m, y = 2.1 m	V ~ 10.5 m/s	$P - P_{atm}$ = +309 Pa

(b) between the 5-m and 2-m diameter cylinders

(c) V ~ 60 m/s, $P - P_{atm}$ = -1785 Pa

Such a large maximum wind velocity may cause a potential hazard for pedestrians and cyclists.

(d) The flow is turbulent : $\rho \times V \times D/\mu$ ~ 3.2 E+6 (D = 2 m). Separation is likely to occur behind the cylinder. However, since the location of maximum velocity is likely to be outside of a wake region, the above results are very likely representative.

(3) Use 3 doublet patterns (μ = 100, 225, 625 m^3/s)

(5)

D (m)=	2	3	5
Re =	3.2 E+6	4.8 E+6	8 E+6
C_D =	0.7	0.75	0.75
Drag (N) =	1.9 E+4	2.1 E+4	3.5 E+4

Exercise 4.7

Solution

The tangential velocity of the rotors is :

$$R \times \omega = 2\times\pi \times 220 / 60 \times 1.5 = 34.56 \text{ m/s}$$

The relative velocity of the wind is :

$$V_o = 25 \times 1852 / 3600 = 12.86 \text{ m/s}$$

Note : if the wind comes from starboard, the rotation of the rotor masts must be in the trigonometric positive direction to propel the ship forward.

(a) For an ideal fluid with irrotational motion :

$$\text{Total Lift} = 2 \times 15 \times 1.2 \times 12.86 \times 13.57 = 150.8 \text{ kN}$$

In the direction of flow motion, the total force is :

$$\text{Total force} = \text{Total Lift} \times \cos 30^\circ = 131 \text{ kN}$$

(b) The total lift and drag forces are :

Total Lift ~ 62.5 kN

Total Drag ~ 17.9 kN

In the direction of flow motion, the total force is :

Total force = 45.2 kN

Exercise 4.8

Solution

(a) The flow pattern has two stagnation points.

(b) The minimum and maximum pressures at the cylinder surface are respectively -7.24 E+4 and +1.40 E+5 Pa for θ = 270º and 20º(and 160°) respectively.

Notes : (1) The minimum pressure is sub-atmospheric and may lead to some cavitation. (2) There are two locations the pressure is maximum which correspond both the location of a stagnation point.

Exercise 4.9

Solution

Use 2D Flow Plus to asses the flow pattern.

A whirlpool is a vortex of vertical axis, with a downward velocity component near its centre. A good example is the bathtub vortex. VAN DYKE (1982, p. 59) presented a superb illustration. See also the Queensland Science Museum. A related example is the vortex dropshaft design. In coastal zones, whirlpools are produced by the interaction of rising and falling tides. They are often observed at the edges of straits with large tidal currents. (At Naruto, currents of up to 9 knots were observed.) The vortex (whirlpool) is a coherent structure typical of shear flows where there is a velocity difference across the shear layer. It affect the surrounding flow and water can be seen going back and forth across the shear layer between vortices. Notable oceanic whirlpools include those of Garofalo along the coast of Calabria in southern Italy, and of Messina in the strait between Sicily and peninsular Italy, the Maelstrøm (from Dutch for "whirling stream") located near the Lofoten Islands off the coast of Norway. Whirlpools near the Hebrides and Orkney islands, and in the Naruto strait between Awaji and Shikoku islands, are also well known.

See Lecture Notes (CHANSON 2008), App. I-E.

Also : {http://www.uq.edu.au/~e2hchans/whirlpl.html}.

Exercise 4.10

Solution

The air density at 2,000 m altitude is about 1.1 kg/m^3.

The ideal fluid flow pattern is the superposition of an uniform flow (V_o = 88.9 m/s), a doublet (strength μ) and a vortex strength K.

The cylinder radius and doublet strength are linked as :

$$R = \sqrt{\frac{(-\mu)}{V_o}}$$

while the vortex strength and rotation speed (ω = 52.36 rad/s) satisfy :

$$\omega = \frac{K}{2 \times \pi \times R^2} = \frac{K \times V_o}{2 \times \pi \times \mu}$$

The lift force per unit length of wing is

$$\text{Lift} = -\rho \times V_o \times K$$

The right wing must rotate in the anti-clockwise direction as seen by the pilot and ω must be negative to provide a positive lift.

The calculations for cruise conditions imply that each wing is 376 m long (!).

Note : The Magnus effect lift force decreases with decreasing speed and it becomes small at take-off and landing conditions.