

IDEAL-FLUID FLOW TUTORIALS**TUTORIAL 1**

Attendance to tutorials is very strongly advised. Repeated absences by some individuals will be noted and these would demonstrate some disappointing responsible behaviour.

Past course results demonstrated a **very strong correlation** between the performances at the end-of-semester examination, the attendance of tutorials during the semester and the overall course result.

1. Pre-Requisite Knowledge - Tutorials

The first tutorial consists of basic pre-requisite knowledge.

1.1 Give the following fluid and physical properties(at 20 Celsius and standard pressure) with a 4-digit accuracy.

	<u>Value</u>	<u>Units</u>
Air density:		
Water density:		
Air dynamic viscosity:		
Water dynamic viscosity:		
Gravity constant in Brisbane:		
Surface tension (air and water) :		

1.2 What is the definition of an ideal fluid ?

What is the dynamic viscosity of an ideal fluid ?

1.3 From what fundamental equation does the Navier-Stokes equation derive : (a) continuity, (b) momentum equation, (c) energy equation, (d) other ?

From what fundamental principle derives the Bernoulli equation ?

1.4 Sketch the streamlines of the following two-dimensional flow situations :

A- A laminar flow past a circular cylinder,

B- A turbulent flow past a circular cylinder,

In each case, show the possible extent of the wake (if any). Indicate clearly in which regions the ideal fluid flow assumptions are valid, and in which areas they are not.

Remember the CIVL3130 Fluid Mechanics experiment "Flow past a cylinder".

2. Ideal Fluid Flow - Irrotational Flows

2.1 Quizz

- What is the definition of the velocity potential ?
- Is the velocity potential a scalar or a vector ?
- Units of the velocity potential ?
- What is definition of the stream function ? Is it a scalar or a vector ? Units of the stream function ?

For an ideal fluid with irrotational flow motion :

- Write the condition of irrotationality as a function of the velocity potential.
- Does the velocity potential exist for 1- an irrotational flow and 2- for a real fluid ?
- Write the continuity equation as a function of the velocity potential.

Further, answer the following questions :

- What is a stagnation point ?
- For a two-dimensional flow, write the stream function conditions.
- How are the streamlines at the stagnation point ?

Reference

CHANSON, H. (2009). "Applied Hydrodynamics: An Introduction to Ideal and Real Fluid Flows." CRC Press, Taylor & Francis Group, Leiden, The Netherlands, 478 pages.

VALLENTINE, H.R. (1969). "Applied Hydrodynamics." *Butterworths*, London, UK, SI edition.

2.2 Basic applications

(1) Considering the following velocity field :

$$V_x = y \times z \times t$$

$$V_y = z \times x \times t$$

$$V_z = x \times y \times t$$

- Is the flow a possible flow of an incompressible fluid ?
- Is the motion irrotational ? If yes : what is the velocity potential ?

(2) Considering the following velocity field :

$$V_x = 2 \times x$$

$$V_y = -2 \times y$$

Is the motion irrotational ? In the affirmative, what is the velocity potential ?

(3) Draw the streamline pattern of the following stream functions :

$$(3.1) \quad \psi = 2 \times x$$

$$(3.2) \quad \psi = 3 \times y$$

$$(3.3) \quad \psi = 3 \times x - 4 \times y$$

$$(3.4) \quad \psi = -1.5 \times x^2$$

$$(3.5) \quad \psi = 4 \times \ln(x) - 2/y$$

Remember: a streamline is curve along which ψ is constant

2.3 Two-dimensional flow

Considering a two-dimensional flow, find the velocity potential and the stream function for a two-dimensional flow having the following velocity components :

$$V_x = -\frac{2 \times x \times y}{(x^2 + y^2)^2}$$

$$V_y = \frac{x^2 - y^2}{(x^2 + y^2)^2}$$

2.4 Applications

(a) Using the software 2DFlowPlus, investigate the flow field of a vortex (at origin, strength 2) superposed to a sink (at origin, strength 1). Visualise the streamlines, the contour of equal velocity and the contour of constant pressure.

Repeat the same process for a vortex (at origin, strength 2) superposed to a sink (at $x=-5$, $y=0$, strength 1). How would you describe the flow region surrounding the vortex.

(b) Investigate the superposition of a source (at origin, strength 1) and an uniform velocity field (horizontal direction, $V = 1$). How many stagnation point do you observe ? What is the pressure at the stagnation point ? What is the "half-Rankine" body thickness at $x = +1$? (You may do the calculations directly or use 2DFlowPlus to solve the flow field.)

(c) Using 2DFlowPlus, investigate the flow past a circular building (for an ideal fluid with irrotational flow motion). How many stagnation points is there ? Compare the resulting flow pattern with real-fluid flow pattern behind a circular bluff body (search Reference text in the library).

(d) Investigate the seepage flow to a sink (well) located close to a lake. What flow pattern would you use ?

Note : the software 2DFlowPlus is described in the textbook, Appendix E. A demonstration copy can be downloaded from the course website {www.uq.edu.au/~2ehchans/civ4160.html}. It is installed in the undergraduate laboratory network.

Reference

CHANSON, H. (2009). "Applied Hydrodynamics: An Introduction to Ideal and Real Fluid Flows." CRC Press, Taylor & Francis Group, Leiden, The Netherlands, 478 pages.

2.5 Basic equations (2)

Considering an two-dimensional irrotational flow of ideal fluid, which basic principle(s) is(are) used to determine the pressure field ?

2.6 Laplace equation

- What is the Laplacian of a function ? Write the Laplacian of the scalar function ϕ in Cartesian and polar coordinates.
- Rewrite the definition of the Laplacian of a scalar function as a function of vector operators (e.g. grad, div, curl).

Solution

$$\Delta \phi(x,y,z) = \nabla \times \nabla \phi(x,y,z) = \text{div } \vec{\text{grad}} \phi(x,y,z) = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2}$$

Laplacian of scalar

$$\Delta \vec{F}(x,y,z) = \nabla \times \nabla \vec{F}(x,y,z) = \vec{i} \Delta F_x + \vec{j} \Delta F_y + \vec{k} \Delta F_z \quad \text{Laplacian of vector}$$

$$\Delta \phi(r,\theta,z) = \frac{1}{r} \times \frac{\partial}{\partial r} \left(r \times \frac{\partial \phi}{\partial r} \right) + \frac{1}{r^2} \times \frac{\partial^2 \phi}{\partial \theta^2} + \frac{\partial^2 \phi}{\partial z^2} \quad \text{Polar coordinates}$$

It yields:

$$\Delta f = \text{div } \vec{\text{grad}} f$$

$$\Delta \vec{F} = \vec{\text{grad}} \text{div } \vec{F} - \text{curl} \left(\vec{\text{curl}} \vec{F} \right)$$

where f is a scalar.

Note the following operations:

$$\Delta (f + g) = \Delta f + \Delta g$$

$$\Delta \left(\vec{F} + \vec{G} \right) = \Delta \vec{F} + \Delta \vec{G}$$

$$\Delta (f \times g) = g \times \Delta f + f \times \Delta g + 2 \times \vec{\text{grad}} f \times \vec{\text{grad}} g$$

where f and g are scalars.

2.7 Basic equations (2)

For a two-dimensional ideal fluid flow, write:

- (a) the continuity equation,
- (b) the streamline equation,
- (c) the velocity potential and stream function,
- (d) the condition of irrotationality and

(e) the Laplace equation
in polar coordinates.

Solution

Continuity equation

$$\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} = 0 \qquad \frac{1}{r} \times \frac{\partial (r \times V_r)}{\partial r} + \frac{1}{r} \times \frac{\partial V_\theta}{\partial \theta} = 0$$

Momentum equation

$$\frac{\partial V_x}{\partial t} + V_x \times \frac{\partial V_x}{\partial x} + V_y \times \frac{\partial V_x}{\partial y} = -\frac{\partial}{\partial x} \left(\frac{P}{\rho} + g \times z \right)$$

$$\frac{\partial V_y}{\partial t} + V_x \times \frac{\partial V_y}{\partial x} + V_y \times \frac{\partial V_y}{\partial y} = -\frac{\partial}{\partial y} \left(\frac{P}{\rho} + g \times z \right)$$

$$\frac{\partial V_r}{\partial t} + V_r \times \frac{\partial V_r}{\partial r} + \frac{V_\theta}{r} \times \frac{\partial V_r}{\partial \theta} - \frac{V_\theta^2}{r} = -\frac{\partial}{\partial r} \left(\frac{P}{\rho} + g \times z \right)$$

$$\frac{\partial V_\theta}{\partial t} + V_r \times \frac{\partial V_\theta}{\partial r} + \frac{V_\theta}{r} \times \frac{\partial V_\theta}{\partial \theta} + \frac{V_r \times V_\theta}{r} = -\frac{1}{r} \times \frac{\partial}{\partial \theta} \left(\frac{P}{\rho} + g \times z \right)$$

Streamline equation

$$V_x \times dy - V_y \times dx = V_r \times r \times d\theta - V_\theta \times dr = 0$$

Velocity potential and stream function

$$V_x = -\frac{\partial \phi}{\partial x} = -\frac{\partial \psi}{\partial y} V_r = -\frac{\partial \phi}{\partial r} = -\frac{1}{r} * \frac{\partial \psi}{\partial \theta}$$

$$V_y = -\frac{\partial \phi}{\partial y} = +\frac{\partial \psi}{\partial x} V_\theta = -\frac{1}{r} \times \frac{\partial \phi}{\partial \theta} = +\frac{\partial \psi}{\partial r}$$

$$\delta Q = \delta \psi \delta Q = \delta \psi$$

Condition of irrotationality

$$\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y} = 0 \frac{\partial V_\theta}{\partial r} - \frac{1}{r} \times \frac{\partial V_r}{\partial \theta} = 0$$

Laplace equation

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 \frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r^2} \times \frac{\partial^2 \phi}{\partial \theta^2} = 0$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0 \frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r^2} * \frac{\partial^2 \psi}{\partial \theta^2} = 0$$

2.8 Basic equations (3)

Considering an two-dimensional irrotational flow of ideal fluid:

- write the Navier-Stokes equation (assuming gravity forces),
- substitute the irrotational flow condition and the velocity potential,

- integrate each equation with respect to x and y,
- what is the final integrated form of the three equations of motion ?

This equation is called the Bernoulli equation for unsteady flow.

- For a steady flow write the Bernoulli equation. When the velocity is known, how do you determine the pressure ?

Solution

$$1- \quad \frac{\partial V_x}{\partial t} + V_x \times \frac{\partial V_x}{\partial x} + V_y \times \frac{\partial V_x}{\partial y} = - \frac{\partial}{\partial x} \left(\frac{P}{\rho} + g \times z \right)$$

$$\frac{\partial V_y}{\partial t} + V_x \times \frac{\partial V_y}{\partial x} + V_y \times \frac{\partial V_y}{\partial y} = - \frac{\partial}{\partial y} \left(\frac{P}{\rho} + g \times z \right)$$

- 2- Substituting the irrotational flow conditions:

$$\frac{\partial V_x}{\partial y} = \frac{\partial V_y}{\partial x}$$

and the velocity potential:

$$V_x = - \frac{\partial \phi}{\partial x} \qquad V_y = - \frac{\partial \phi}{\partial y}$$

the equations may be expressed as (STREETER 1948, p. 24):

$$\frac{\partial}{\partial x} \left(- \frac{\partial \phi}{\partial t} + \frac{V_x^2}{2} + \frac{V_y^2}{2} + \frac{P}{\rho} + g \times z \right) = 0$$

$$\frac{\partial}{\partial y} \left(- \frac{\partial \phi}{\partial t} + \frac{V_x^2}{2} + \frac{V_y^2}{2} + \frac{P}{\rho} + g \times z \right) = 0$$

- 3- Integrating with respect to x and y:

$$- \frac{\partial \phi}{\partial t} + \frac{V^2}{2} + \frac{P}{\rho} + g \times z = F_x(y, t)$$

$$- \frac{\partial \phi}{\partial t} + \frac{V^2}{2} + \frac{P}{\rho} + g \times z = F_y(x, t)$$

where V is defined as the magnitude of the velocity: $V^2 = V_x^2 + V_y^2$. The integration of the three motion equations are identical and the left-hand sides of the equations are the same:

$$F_x(y, t) = F_y(x, t)$$

- 4- The final integrated form of the three equations of motion is the Bernoulli equation for unsteady flow:

$$- \frac{\partial \phi}{\partial t} + \frac{V^2}{2} + \frac{P}{\rho} + g \times z = F(t)$$

In the general case of a volume force potential U (i.e. $\vec{F}_v = - \text{grad } U$):

$$- \frac{\partial \phi}{\partial t} + \frac{V^2}{2} + \frac{P}{\rho} + U = F(t)$$

More exercises in textbook pp. 24-25, 45-53.

CHANSON, H. (2009). "Applied Hydrodynamics: An Introduction to Ideal and Real Fluid Flows." CRC Press, Taylor & Francis Group, Leiden, The Netherlands, 478 pages (ISBN: 978-0-415-49271-3 (Hardback); 978-0-203-87626-8 (eBook)) (PSE library: TC171 .C54 2009).

IDEAL-FLUID FLOW TUTORIALS**TUTORIAL 2**

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3. Two-Dimensional Flows (1) Basic equations and flow analogies - Tutorials**3.1 Flow net (2006 examination paper)**

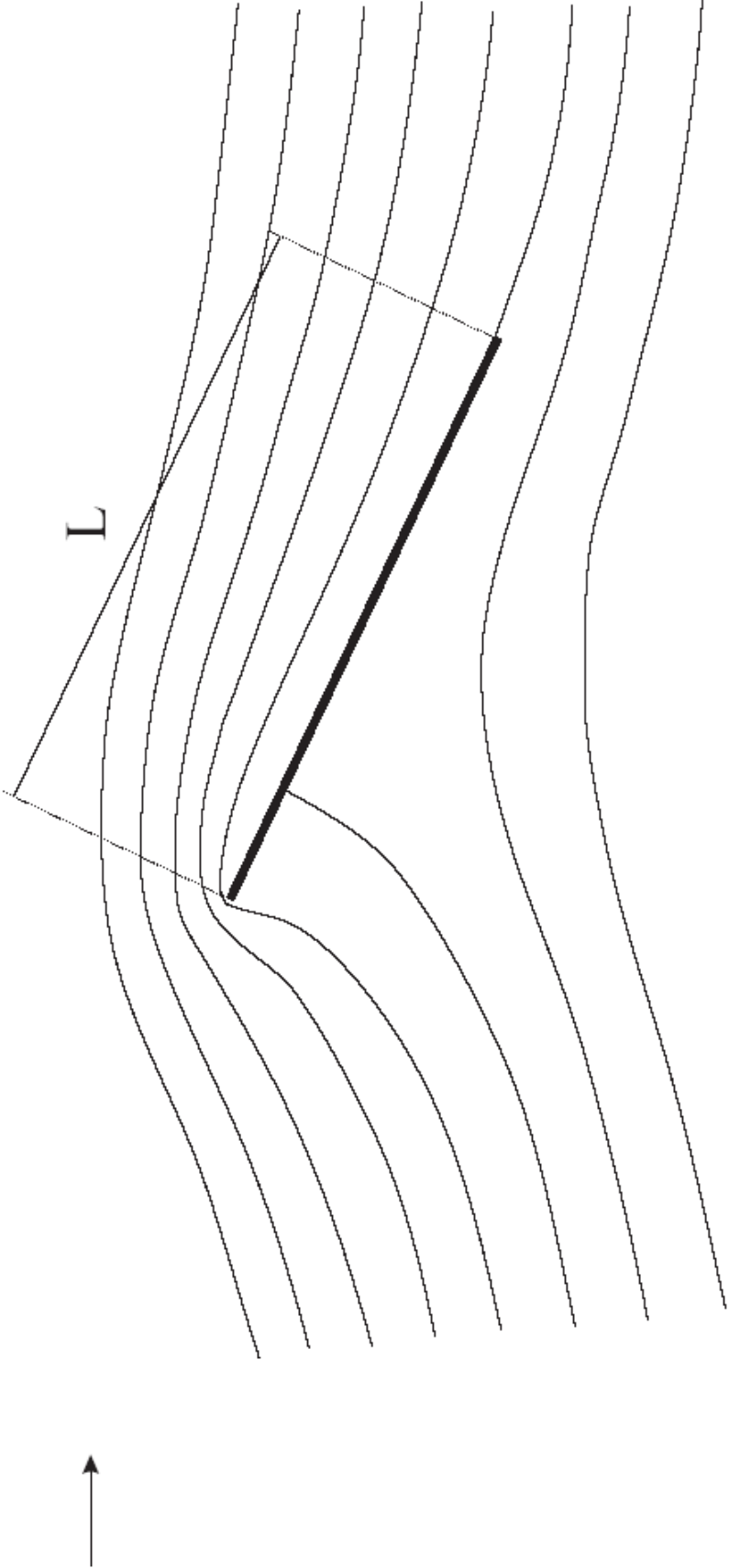
A hydrodynamic study of the flow past a flat plate is conducted in water. The plate length is $L = 0.2$ m and the plate width equals the test section width (0.5 m). For a particular angle of incidence, dye injection in a water tunnel gives the flow pattern shown in Figure E3-1 on the next page. The mean flow is horizontal.

(a) Complete the flow net by drawing the suitable equipotentials. (Draw the complete flow net on the examination paper.)

(b) The upstream velocity is 6.5 m/s. Estimate the drag force and the lift force acting on the plate. *Indicate clearly the sign convention. You may have to draw more streamline and equipotentials to obtain a good accuracy.*

Assume water at 20 Celsius ($\rho = 998.2$ kg/m³, $\mu = 1.005 \times 10^{-5}$ Pa.s, $\sigma = 0.0736$ N/m).

Fig. E3-1 - Flow visualisation and streamline patterns around an inclined flat plate (undistorted scale)



3.2 Flow net

Let us consider the water flow past a circular arc sketched in Figure E3-2. The chord is $L = 0.35$ m and the plate width equals 1 m. For a particular angle of incidence shown in Figure E3-2, conduct a graphical analysis of the flow field. The mean flow is horizontal.

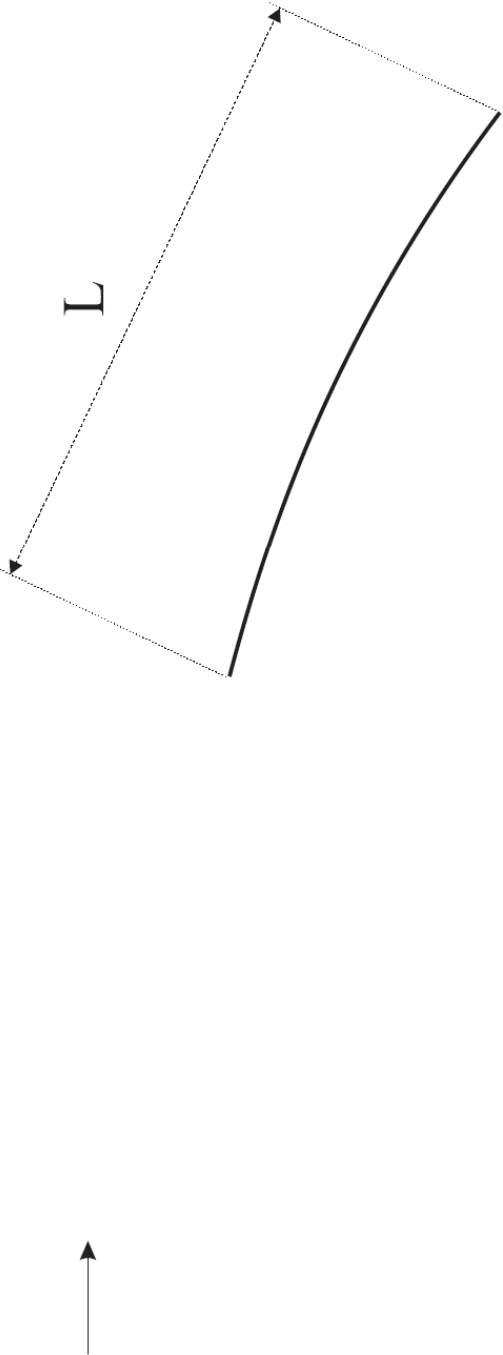
(a) Draw the flow net by drawing the suitable streamlines and equipotentials. (Draw the complete flow net with sufficient details in the vicinity of the circular arc.)

(b) The upstream velocity is 15.2 m/s. Estimate the drag force and the lift force acting on the cambered plate.

Indicate clearly the sign convention..

Assume water at 20 Celsius ($\rho = 998.2$ kg/m³, $\mu = 1.005 \times 10^{-5}$ Pa.s, $\sigma = 0.0736$ N/m).

Fig. E3-2 - Flow past a circular arc (undistorted scale)

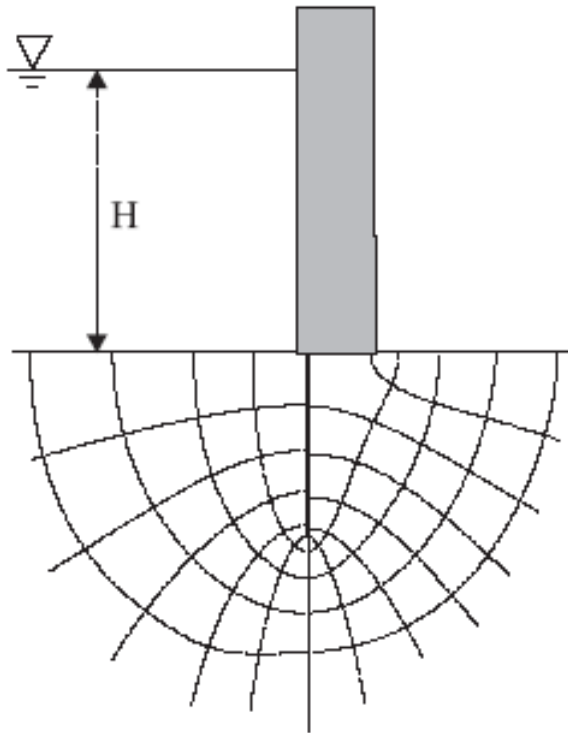


3.3 Flow net beneath a cutoff wall

For a two-dimensional seepage under a impervious structure with a cutoff wall (Fig. E3-3), the boundary conditions are : $H = 6$ m, $K = 2.0$ m/day.

- What is the hydraulic conductivity in m/s ? What type of soil is it ?
- Using the flow net, estimate the seepage flow par meter width of dam.

Fig. E3-3 - Flow net beneath an impervious dam



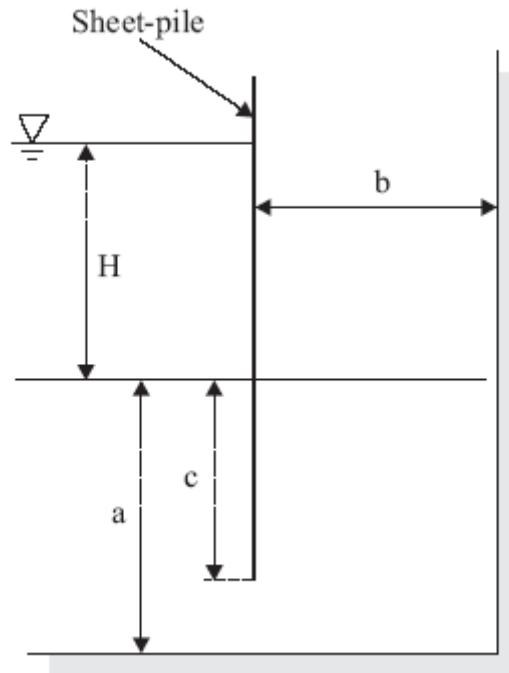
3.4 Flow net under a sheetpiling

For a two-dimensional seepage under sheetpiling with a permeable foundation (Fig. E3-4), the boundary conditions are : $a = 9.4$ m, $b = 4.7$ m, $c = 4$ m, $H = 2.5$ m, $K = 2.0 \cdot 10^{-3}$ cm/s.

Using a dimensioned flow net (with 5 stream tubes or more), estimate the seepage flow par meter width of dam.

Use graph paper.

Fig. E3-4 - Flow net beneath a sheet pile



3.5 Flow net under an impervious dam

For a two-dimensional seepage under an impervious dam with apron (textbook, CHANSON 2009, Fig. 3-1B), the boundary conditions are : $H = 80$ m, $K = 5 \cdot 10^{-5}$ m/s.

(A) Calculate the seepage flow rate in presence of an apron and a cutoff wall (Fig. 3-1B).

(B) In absence of the cutoff wall, sketch the flow net and determine the seepage flow. Determine the pressure distribution along the base of the dam and beneath the apron. Calculate the uplift forces on the dam foundation and on the apron.

3.6 Flow net under an impervious dam with cutoff wall

For a two-dimensional seepage under an impervious dam with a cutoff wall (Fig. E3-5), the boundary conditions are : $H_1 = 60$ m, $H_2 = 5$ m, $a = 60$ m, $b = 100$ m, $L = 70$ m, $K = 1 \cdot 10^{-5}$ m/s.

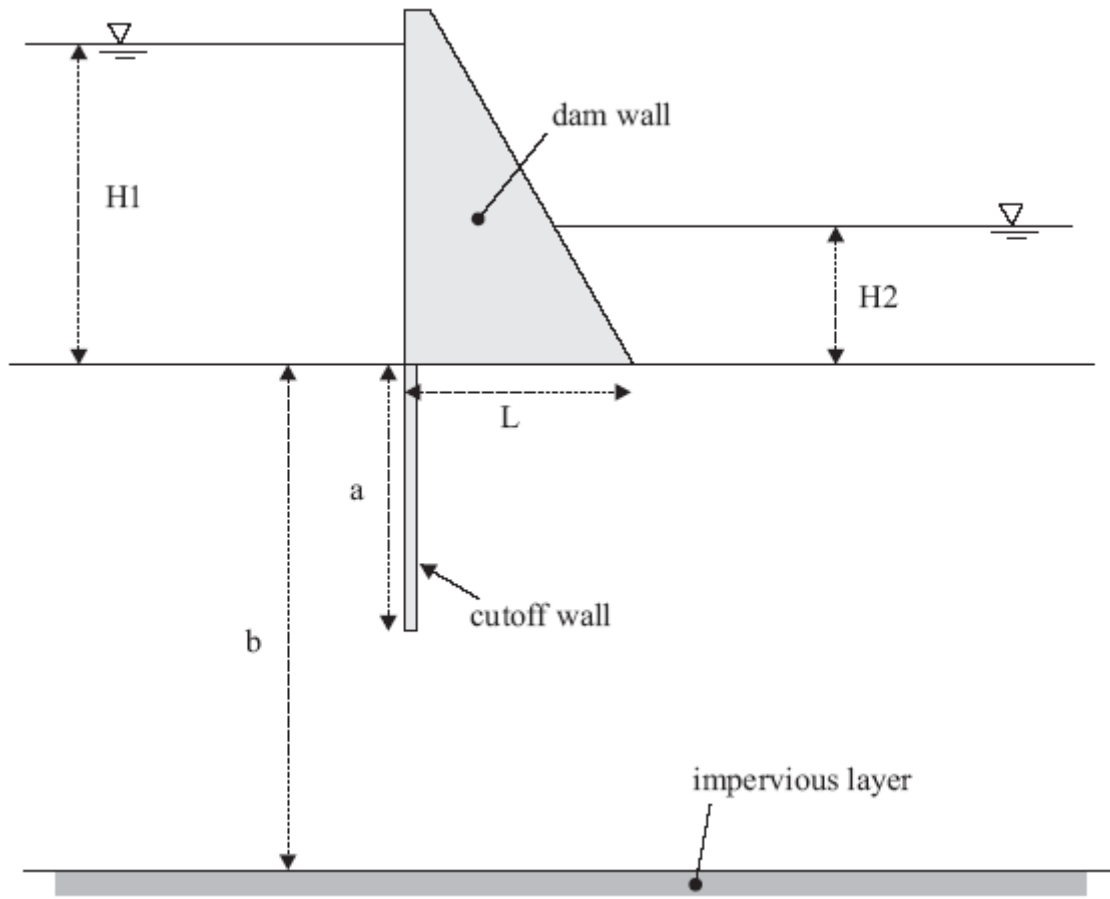
(A) In absence of cutoff wall (i.e. $a = 0$ m), sketch the flow net; determine the seepage flow; determine the pressure distribution along the base of the dam; calculate the uplift force.

(B) With the cutoff wall (i.e. $a = 60$ m): same questions : sketch the flow net; determine the seepage flow; determine the pressure distribution along the base of the dam; calculate the uplift force

(C) Comparison and discuss the results.

Use graph paper.

Fig. E3-5 - Flow net beneath a dam with cutoff wall

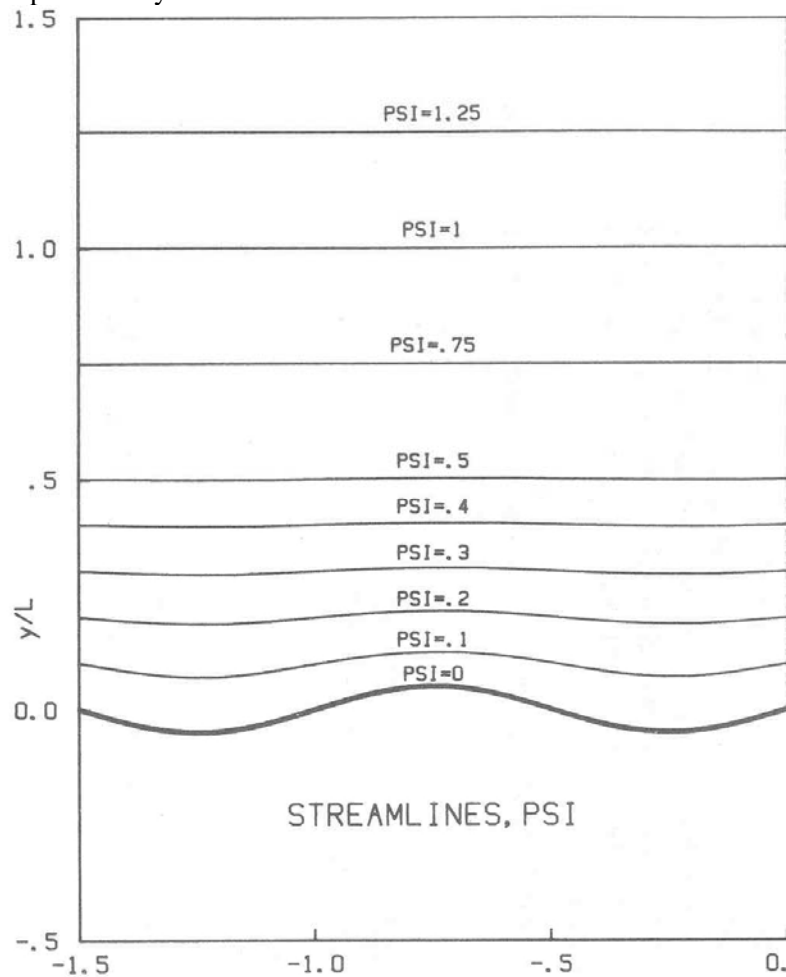


3.7 Pressure distribution above some bed forms

Figure E3-6 illustrates an open channel flow above some quasi-sinusoidal bed forms. The wave length is 2.2 m and the average water depth is 3.3 m. Neglecting sediment motion, calculate the pressure distributions at the bed form crest and trough.

On a flat bed, the pressure distribution is hydrostatic. Determine the deviations from the hydrostatic pressure distribution.

Fig. E3-6 - Streamlines past a wavy bed



3.8 Cauchy-Riemann equations

- Determine the stream function for parallel flow with a velocity V inclined at an angle α to the x -axis.
- Find the velocity potential using the Cauchy-Riemann equations.

3.9 Flow analogy

A Hele-Shaw cell apparatus is built with a gap between plates $\delta = 1.5$ mm filled with Glycerol ($\rho = 1260$ kg/m³, $\mu = 1.4$ Pa.s). Estimate the hydraulic conductivity of the apparatus.

More exercises in textbook pp. 74-80.

CHANSON, H. (2009). "Applied Hydrodynamics: An Introduction to Ideal and Real Fluid Flows." CRC Press, Taylor & Francis Group, Leiden, The Netherlands, 478 pages (ISBN: 978-0-415-49271-3 (Hardback); 978-0-203-87626-8 (eBook)) (PSE library: TC171 .C54 2009).

Exercise SolutionsExercise 3.1

The problem is solved by drawing the equipotentials and completing the flow net. Close to the foil, additional streamlines and equipotentials may be drawn to improve the estimates of the velocities next to the extrados and intrados of the foil.

The pressure field is derived from the Bernoulli equation and the integration of the pressure distributions yields the lift and drag forces.

Remarks

- The flow net method and technique are presented in the textbook (CHANSON 2009).
- The complete theory of lift and drag on airfoils, wings and hydrofoils is developed in the textbook (CHANSON 2009, chapter I-6).

Exercise 3.2

The problem may be solved analytically using the Joukowski transformation and theorem of Kutta-Joukowski. The complete theory of lift and drag on airfoils, wings and hydrofoils is developed in the textbook (CHANSON 2009, chapter I-6).

The flow net solution can be compared with the theory of lift and drag.

Exercise 3.3*Solution*

$$q = 4.6 \text{ m}^2/\text{day}$$

Remark

See Textbook (CHANSON 2009), pp. 75-76.

Exercise 3.4*Solution*

$$q = 2.7 \text{ m}^2/\text{day}$$

Discussion

The flow pattern may be analysed analytically using a finite line source (for the sheet pile) and the theory of images. The resulting streamlines are the equipotentials of the sheet-pile flow.

Remember : A velocity potential can be found for each stream function. If the stream function satisfies the Laplace equation the velocity potential also satisfies it. Hence the velocity potential may be considered as stream function for another flow case. The velocity potential ϕ and the stream function ψ are called "*conjugate functions*" (Chapter I-2).

Exercise 3.5

Solution

(A) The problem is similar to the flow net sketched in Figure 3-1B (Textbook, pp. 58-60).

(B) In absence of cutoff wall, the streamlines are shorter and the seepage flow rate is greater.

The pressure distribution beneath the dam foundation and apron may be deduced from the equipotential lines, since $\phi = K \times H$ where K is the hydraulic conductivity and H is the piezometric head (Chapter I-3, paragraph 3.3).

The uplift pressure on the dam foundation and apron are very significant. The apron structure would be subjected to high risks of uplift and damage.

Remark

See Textbook (CHANSON 2009), pp. 60.

Exercise 3.6

See Textbook (CHANSON 2009), pp. 77-78.

Exercise 3.7

Solve graphically the problem : (a) Complete the equipotential lines; (b) Calculate the velocity magnitude at all vertical elevation; (c) Apply the Bernoulli principle.

Remember that the pressure gradient is hydrostatic far away upstream.

Application

This flow pattern is typical of the flow above standing wave bed forms, although these tend to be significantly larger (KENNEDY 1963, CHANSON 2000). The height of the wall undulations is closer to large ripples and small dunes (CHANSON 2204, pp. 151-155 & 223-231).

KENNEDY, J.F. (1963). "The Mechanics of Dunes and Antidunes in Erodible-Bed Channels." *Jl of Fluid Mech.*, Vol. 16, No. 4, pp. 521-544 (& 2 plates).

CHANSON, H. (2000). "Boundary Shear Stress Measurements in Undular Flows : Application to Standing Wave Bed Forms." *Water Res. Res.*, Vol. 36, No. 10, pp. 3063-3076 (ISSN 0043-1397).

{<http://espace.library.uq.edu.au/view.php?pid=UQ:11117>}

CHANSON, H. (2004). "The Hydraulics of Open Channel Flow : An Introduction." *Butterworth-Heinemann*, Oxford, UK, 2nd edition, 630 pages (ISBN 978 0 7506 5978 9).

Exercise 3.8

Solution

By definition:

$$\frac{\partial \Psi}{\partial y} = -V_x = -V \times \cos\alpha \times y + f_1(x)$$

$$\frac{\partial \Psi}{\partial x} = V_y = V \times \sin\theta \times x + f_2(y)$$

Hence:

$$\Psi = V \times \sin\theta \times x - V \times \cos\alpha \times y + \text{constant}$$

A streamline equation is:

$$y = \tan\theta \times x - \frac{\Psi}{V \times \cos\alpha}$$

which is a straight line equation.

The Cauchy-Riemann equations are:

$$\frac{\partial \phi}{\partial x} = \frac{\partial \Psi}{\partial y} \phi = -V \times \cos\alpha \times x + g_1(y)$$

$$\frac{\partial \phi}{\partial y} = -\frac{\partial \Psi}{\partial x} \phi = -V \times \sin\theta \times y + g_2(x)$$

Hence:

$$\phi = -V \times \sin\theta \times y - V \times \cos\alpha \times x + \text{constant}$$

$$V_x = V_0 \times \cos\alpha$$

$$V_y = V_0 \times \sin\alpha$$

$$\Psi = -V_0 \times (y \times \cos\alpha - x \times \sin\alpha)$$

Exercise 3.9

Solution

In laminar flows between two parallel plates, the application of the momentum principle in its integral form yields an expression of the longitudinal head loss:

$$\frac{\partial H}{\partial x} = -\frac{f}{D_H} \times \frac{V^2}{2 \times g}$$

where D_H is the equivalent pipe diameter (or hydraulic diameter), V is the cross-sectional averaged velocity and f is the Darcy-Weisbach friction factor:

$$f = \frac{64 \times \mu}{\rho \times V \times D_H}$$

Since $D_H = 2 \times \delta$, it yields:

$$V = -\frac{\rho \times \delta^2}{8 \times \mu} \times \frac{\partial H}{\partial x}$$

By analogy with the Darcy law, this gives a hydraulic conductivity K:

$$K = \frac{\rho \times \delta^2}{8 \times \mu} = 2.5 \times 10^{-4} \text{ m/s}$$

IDEAL-FLUID FLOW TUTORIALS**TUTORIAL 3**

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4. Two-Dimensional Flows (2) Basic flow patterns - Tutorials**4.1 Doublet in uniform flow (1)**

Select the strength of doublet needed to portray an uniform flow of ideal fluid with a 20 m/s velocity around a cylinder of radius 2 m.

4.2 Source and sink

A source discharging $0.72 \text{ m}^2/\text{s}$ is located at $(-1, 0)$ and a sink of twice the strength is located at $(+2, 0)$. For a remote pressure (far away) of 7.2 kPa, $\rho = 1,240 \text{ kg/m}^3$, find the velocity and pressure at $(0, 1)$ and $(1, 1)$.

Note : When some measurements are conducted with a Prandtl-Pitot tube, the pressure tapping at the leading edge of the tube gives the dynamic pressure, while the pressure tappings on the side give the piezometric pressure. Remember that, at the leading edge of the tube, stagnation occurs.

Remarks

The Pitot tube is named after the Frenchman Henri PITOT. The first presentation of the concept of the Pitot tube was made in 1732 at the French Academy of Sciences by Henri PITOT. The original Pitot tube included basically a total head reading. Ludwig PRANDTL improved the device by introducing a pressure (or piezometric head) reading. The modified Pitot tube is sometimes called a Pitot-Prandtl tube.

For many years, aeroplanes used Prandtl-Pitot tubes to estimate their relative velocity.

4.3 Flow pattern (2)

In two-dimensional flow we now consider a source, a sink and an uniform stream. For the pattern resulting from the combinations of a source (located at $(-L, 0)$) and sink (located at $(+L, 0)$) of equal strength Q in uniform flow (velocity $+V_0$ parallel to the x-axis) :

- (a) Sketch streamlines and equipotential lines;
- (b) Give the velocity potential and the stream function.

This flow pattern is called the flow past a Rankine body. W.J.M. RANKINE (1820-1872) was a Scottish engineer and physicist who developed the theory of sources and sinks. The shape of the body may be altered by varying the distance between source and sink (i.e. $2 \times L$) or by varying the strength of the source and sink. Other shapes may be obtained by the introduction of additional sources and sinks and RANKINE developed ship contours in this way.

- (c) What is the profile of the Rankine body (i.e. find the streamline that defines the shape of the body)?
- (d) What is the length and height of the body ?
- (e) Explain how the flow past a cylinder can be regarded as a Rankine body. Give the radius of the cylinder as a function of the Rankine body parameter.

4.4 Flow pattern (3)

In two-dimensional flow we consider again a source, a sink and an uniform stream. But. the source is located at $(+L, 0)$ and the sink is located at $(-L, 0)$ (i.e. opposite to a Rankine body flow pattern). They are of equal strength q in an uniform flow (velocity $+V_0$ parallel to the x-axis).

Derive the relationship between the discharge q , the length L and the flow velocity such that no flow injected at the source becomes trapped into the sink.

4.5 Doublet in uniform flow (2)

We consider the air flow ($V_0 = 9$ m/s, standard conditions) past a suspension bridge cable ($\varnothing = 20$ mm),

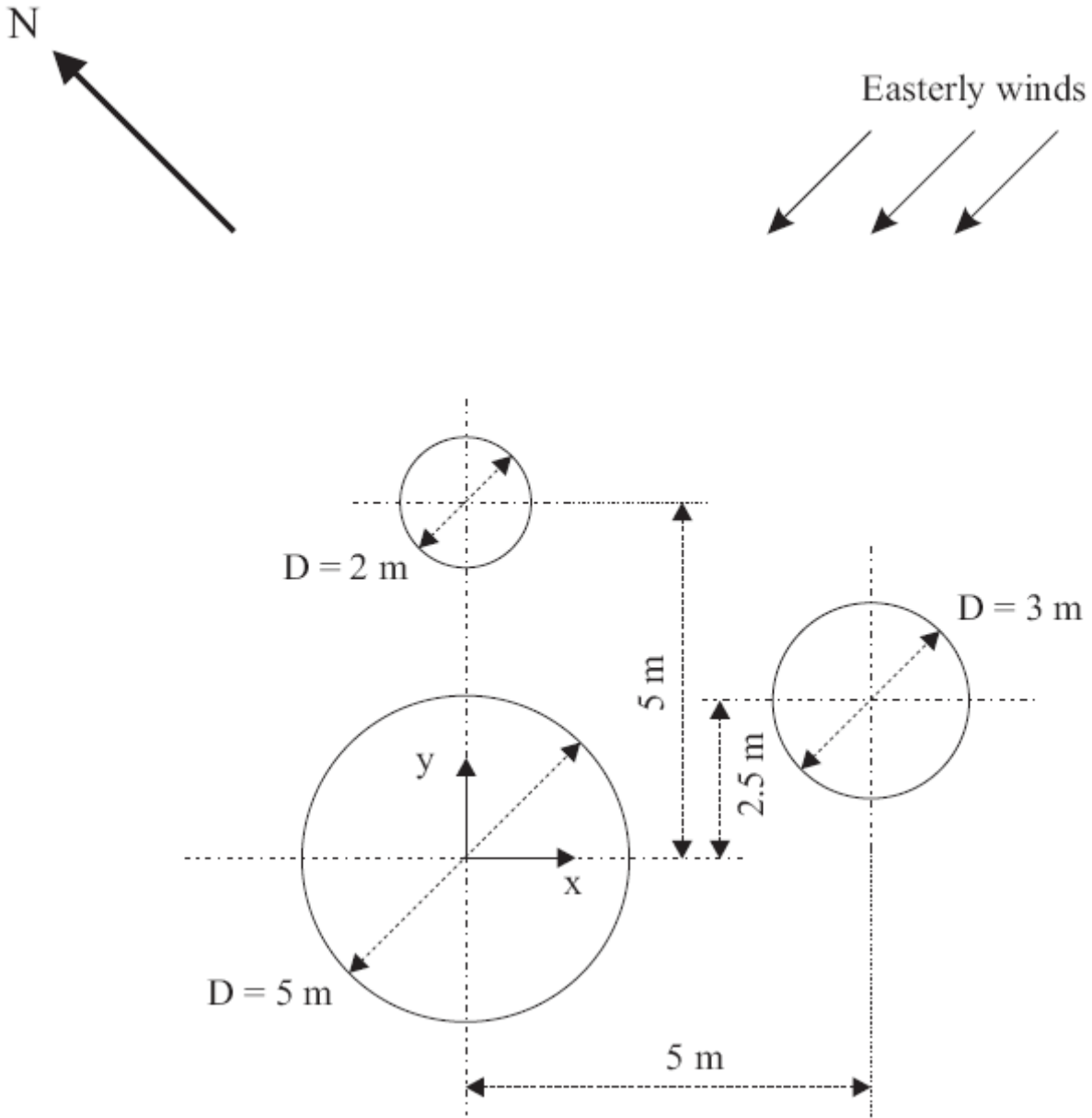
- (a) Select the strength of doublet needed to portray the uniform flow of ideal fluid around the cylindrical cable.
- (b) In real fluid flow, calculate the hydrodynamic frequency of the vortex shedding.

4.6 Flow past buildings - 2003 exam paper

Let us consider a new architectural landmark to be built at the Mt Cootha Lookout. The structure consists of three circular cylinders (Height : 25 m - Diameter : 2, 3 and 5 m). The landmark will be facing North-East, while the dominant winds are Easterlies (Fig. 1).

You will assume that the wind flow around the structure is a two-dimensional irrotational flow of ideal fluid. The atmospheric conditions are : $P = P_{\text{atm}} = 10^5$ Pa ; $T = 25$ Celsius.

Fig. 1 - Sketch of the Mt Cootha landmark - View in elevation



(1) On graph paper, sketch the flow net with the landmark for a 25 m/s Easterly wind. Indicate clearly on the graph the discharge between two streamlines, the x -axis and y -axis, their direction, and use the centre of the 5-m diameter cylinder as the origin of your system of coordinates (with x in the South-East direction and y in the North-East direction).

(2) The Brisbane City Council is concerned about wind velocities between the buildings that may blow down tourists and damage cars.

(a) From your flow net, compute the wind velocity and the pressure at :

$x = 1.5\text{ m}, y = 6\text{ m}$

$x = 5\text{ m}, y = 4.5\text{ m}$

$x = 2.1\text{ m}, y = 2.1\text{ m}$

These locations would be typical of tourists standing in front of the vertical cylinders.

- (b) Where is located the point of maximum velocity and minimum pressure ?
- (c) What is the maximum velocity and minimum pressure between the cylinders ? Indicate that location on your flow net.
- (d) Discuss your results. Do you think that this result is realistic ? Why ?

(3) Explain what standard flow patterns you would use to describe the flow around these three buildings.

(4) Write the stream function and the velocity potential as a function of the wind speed V_0 (25 m/s) and the two cylinder diameters D_1 (5 m), D_2 (3 m) and D_3 (2 m).

Do not use numbers. Express the results as functions of the above symbols.

(5) For a real fluid flow, what is (are) the drag force(s) on each cylinder (Height: 25 m) ?

4.7 Magnus effect (1)

Two 15-m high rotors 3 m in diameter are used to propel a ship. Estimate the total longitudinal force exerted upon the rotors when the relative wind velocity is 25 knots, the angular velocity of the rotors is 220 revolutions per minute and the wind direction is at 60° from the bow of the ship.

Perform the calculations for (a) an ideal fluid with irrotational motion and (b) a real fluid.

(c) What orientation of the vector of relative wind velocity would yield the greatest propulsion force upon the rotorship ? Calculate the magnitude of this force.

Assume a real fluid flow. The result is trivial for ideal fluid with irrotational flow motion.

(d) Determine how nearly into the wind the rotorship could sail. That is, at wind angle would the resultant propulsion force be zero ignoring the wind effect on the ship itself.

4.8 Magnus effect (2)

A infinite rotating cylinder ($R = 1.1$ m) is placed in a free-stream flow ($V_0 = 15$ knots) of water. The cylinder is rotating at 45 rpm and the ambient pressure far away is $1.1 \text{ E}+5$ Pa.

(a) Calculate and plot the pressure distribution on the cylinder surface.

(b) Find the maximum and minimum pressures on the cylinder surface.

(c) Find the location θ where the pressure on the cylinder surface is the fluid pressure far away from the cylinder.

4.9 Whirlpools

Whirlpools may be approximated by a series of vortices of same signs advected into an uniform flow.

(a) Consider two vortices of equal strength $K = +1$ located at $(-2,)$ and $(+2, 0)$. Estimate how far away the effect of the vortices is perceived to be that of an unique vortex. What would be the strength of that vortex ?

(b) Consider two vortices of equal strength $K = +1$ located at $(-2,)$ and $(+2, 0)$ in a horizontal uniform flow $V = +0.03$. What are the stream function and velocity potential of the resulting flow motion.

4.10 Magnus effect aircraft

Two rotating cylinders are used instead of conventional wings to provide the lift to an aircraft. Calculate the length of each cylinder wing for the following design conditions :

Cruise speed : 320 km/h

Cruise altitude : 2,000 m

Aircraft mass : 8 E+6 kg

Cylinder radius : 1.8 m

Cylinder rotation speed : 500 rpm

4.11 Wind force on a Nissen hut

A 45 km/h wind flows over a Nissen hut which has a 3.5 m radius and is 54.9 m long (Fig. E4-1). The upstream pressure and temperature are 1.013 E+5 Pa and 288.2 K respectively, and they are equal to the pressure and temperature inside the Nissen hut.

(a) Calculate the lift and drag forces on the building.

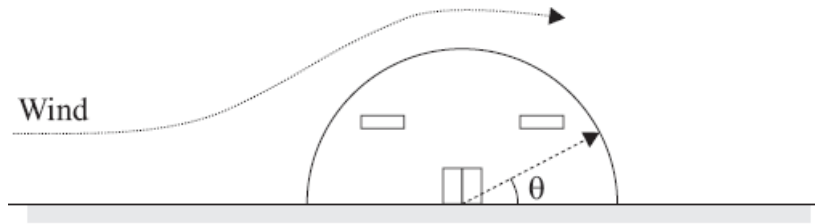
(b) Find the location θ on the building roof where the pressure is 1.013 E+5 Pa.

Assume an irrotational flow motion of ideal fluid in parts (a) and (b).

(c) Calculate the drag force for a real fluid flow.

Notes: The Nissen hut is a building made from a semi-circle of corrugated steel. A variant was the Quonset hut used extensively during World War 2 by the Commonwealth and US military for army camps and air bases. The design was named after Major Peter Norman NISSEN, 29th Company Royal Engineers who experimented with hut design and constructed three prototype semi-circular huts in April 1916. The building was not only economical but also portable.

Fig. E4-1 - Wind flow past a Nissen hut



4.12 Wind flow past columns (2008 Exam paper)

The facade of a 150 m tall building is hindered by two cylindrical columns (a service column and a structural column) (Fig. E4-2). There are concerns about the wind flow around the columns during storm conditions ($V_o = 35$ m/s).

The column dimensions are:

	Column 1	Column 2	Units
D	0.8	2.0	m
x (centre)	0	2.8	m
y (centre)	1.0	2.0	m

You will assume that the wind flow around the building is a two-dimensional irrotational flow of ideal fluid. The atmospheric conditions are: $P = P_{atm} = 105$ Pa; $T = 20$ Celsius.

(a) On graph paper, sketch the flow net for a 35 m/s wind flow. Indicate clearly on the graph the discharge between two streamlines, the x-axis and y-axis, and their direction.

(b) The developer is concerned about the maximum wind speeds next to the building facade that may damage the cladding and service column (column 1).

(b1) From your flow net, compute the wind velocity and the pressure at:

$x = 0$ m, $y = 0.4$ m

$x = 2.8$ m, $y = 1.0$ m

$x = 1.4$ m, $y = 0.8$ m

(b2) Where is located the point of maximum wind velocity and minimum ambient pressure ? Show that location on your flow net.

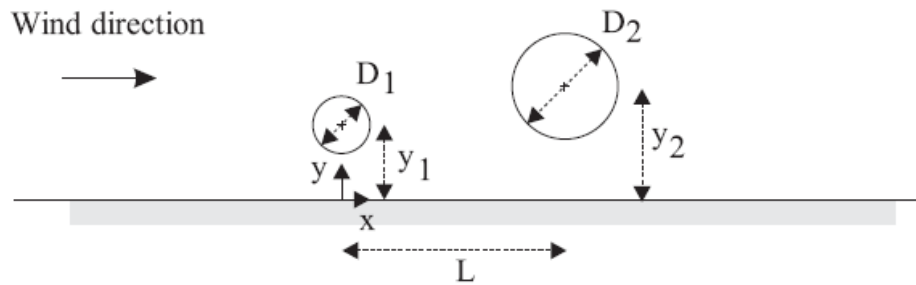
(b3) Discuss your results. Do you think that these results are realistic ? Why ?

(c) Explain what standard flow patterns you would use to describe the flow around these two columns.

(d) Write the stream function and the velocity potential as a function of the wind speed V_o (35 m/s), the two cylinder diameters D_1 and D_2 , and their locations (x_1, y_1) and (x_2, y_2) . Do not use numbers. Express the results as functions of the above symbols.

(e) For a real fluid flow, what is (are) the drag force(s) on each cylinder (Height: 150 m) neglecting interactions between the columns?

Fig. E4-2 - View in elevation of the building facade and cylindrical columns



More exercises in textbook pp. 74-80.

CHANSON, H. (2009). "Applied Hydrodynamics: An Introduction to Ideal and Real Fluid Flows." CRC Press, Taylor & Francis Group, Leiden, The Netherlands, 478 pages (ISBN: 978-0-415-49271-3 (Hardback); 978-0-203-87626-8 (eBook)) (PSE library: TC171 .C54 2009).

Exercise Solutions

Exercise 4.1

Solution

(a) A doublet and uniform flow is analogous to the flow past a cylinder of radius :

$$R = \sqrt{\frac{-\mu}{V_0}}$$

where μ is the strength of the doublet. Hence :

$$\mu = -V_0 \times R^2 = 80 \text{ m}^3/\text{s}$$

Remark

See Textbook (CHANSON 2009), Chapter I-4.

Exercise 4.3

Solution

The flow past a Rankine body is the pattern resulting from the combinations of a source and sink of equal strength in uniform flow (velocity $+V_0$ parallel to the x-axis) :

$$\phi = -V_0 \times r \times \cos\theta - \left(+\frac{q}{2 \times \pi} \times \text{Ln}\left(\frac{r_1}{r_2}\right) \right)$$

$$\psi = -V_0 \times r \times \sin\theta - \left(+\frac{q}{2 \times \pi} \times (\theta_1 - \theta_2) \right)$$

where the subscript 1 refers to the source, the subscript 2 to the sink and q is positive for the source located at $(-L, 0)$ and the sink located at $(+L, 0)$.

The profile of the Rankine body is the streamline $\psi = 0$:

$$\psi = -V_o \times r \times \sin\theta + \frac{q}{2 \times \pi} \times (\theta_1 - \theta_2) = 0$$

$$r = \frac{q \times (\theta_1 - \theta_2)}{2 \times \pi \times V_o \times \sin\theta}$$

The length of the body equals the distance between the stagnation points where :

$$V = V_o + \frac{q}{2 \times \pi \times r_1} - \frac{q}{2 \times \pi \times r_2} = V_o + \frac{q}{2 \times \pi} \times \left(\frac{1}{r_s - L} - \frac{1}{r_s + L} \right) = 0$$

and hence :

$$L_{\text{body}} = 2 \times r_s = 2 \times L \times \sqrt{1 + \frac{q}{\pi \times L \times V_o}}$$

The half-width of the body h is deduced from the profile equation at the point $(h, \pi/2)$:

$$h = \frac{q \times (\theta_1 - \theta_2)}{2 \times \pi \times V_o}$$

where : $\theta_1 = \alpha$ and $\theta_2 = \pi - \alpha$ and hence :

$$\alpha = \frac{\pi}{2} - \frac{\pi \times h \times V_o}{q}$$

But also :

$$\tan\alpha = \frac{h}{L}$$

So the half-width of the body is the solution of the equation :

$$h = L \times \cot\left(\frac{\pi \times V_o}{q} \times h\right)$$

Remark

See Textbook (CHANSON 2009), Chapter I-4.

Exercise 4.4

See Textbook (CHANSON 2009), Chapter I-4.

Exercise 4.5

Solution

(a) A doublet and uniform flow is analogous to the flow past a cylinder of radius :

$$R = \sqrt{\frac{-\mu}{V_o}}$$

where μ is the strength of the doublet. Hence :

$$\mu = -V_o \times R^2 = -9 \text{ E-4 m}^3/\text{s}$$

(b) The Reynolds number of the flow is $1.1 \text{ E-}4$. For that range of Reynolds number, the vortex shedding behind the cable is characterised by a well-defined von Karman street of vortex. The hydrodynamic frequency satisfies :

$$St = \frac{\omega \times 2 \times R}{V_o} \sim 0.2$$

It yields : $\omega = 90 \text{ Hz}$. If the hydrodynamic frequency happens to coincide with the natural frequency of the structure, the effects may be devastating : e.g., Tacoma Narrows bridge failure on 7 November 1940.

4.6 2003 exam paper

(2)

(a1) $x = 1.5 \text{ m}$, $y = 6 \text{ m}$	$V \sim 23.2 \text{ m/s}$	$P - P_{\text{atm}} = +52 \text{ Pa}$
(a2) $x = 5 \text{ m}$, $y = 4.5 \text{ m}$	$V \sim 28.5 \text{ m/s}$	$P - P_{\text{atm}} = -112 \text{ Pa}$
(a3) $x = 2.1 \text{ m}$, $y = 2.1 \text{ m}$	$V \sim 10.5 \text{ m/s}$	$P - P_{\text{atm}} = +309 \text{ Pa}$

(b) between the 5-m and 2-m diameter cylinders

(c) $V \sim 60 \text{ m/s}$, $P - P_{\text{atm}} = -1785 \text{ Pa}$

Such a large maximum wind velocity may cause a potential hazard for pedestrians and cyclists.

(d) The flow is turbulent : $\rho \times V \times D / \mu \sim 3.2 \text{ E}+6$ ($D = 2 \text{ m}$). Separation is likely to occur behind the cylinder. However, since the location of maximum velocity is likely to be outside of a wake region, the above results are very likely representative.

(3) Use 3 doublet patterns ($\mu = 100, 225, 625 \text{ m}^3/\text{s}$)

(5)

D (m)=	2	3	5
Re =	3.2 E+6	4.8 E+6	8 E+6
$C_D =$	0.7	0.75	0.75
Drag (N) =	1.9 E+4	2.1 E+4	3.5 E+4

Exercise 4.7

Solution

The tangential velocity of the rotors is :

$$R \times \omega = 2 \times \pi \times 220 / 60 \times 1.5 = 34.56 \text{ m/s}$$

The relative velocity of the wind is :

$$V_o = 25 \times 1852 / 3600 = 12.86 \text{ m/s}$$

Note : if the wind comes from starboard, the rotation of the rotor masts must be in the trigonometric positive direction to propel the ship forward.

(a) For an ideal fluid with irrotational motion :

$$\text{Total Lift} = 2 \times 15 \times 1.2 \times 12.86 \times 13.57 = 150.8 \text{ kN}$$

In the direction of flow motion, the total force is :

$$\text{Total force} = \text{Total Lift} \times \cos 30^\circ = 131 \text{ kN}$$

(b) The total lift and drag forces are :

$$\text{Total Lift} \sim 62.5 \text{ kN}$$

$$\text{Total Drag} \sim 17.9 \text{ kN}$$

In the direction of flow motion, the total force is :

$$\text{Total force} = 45.2 \text{ kN}$$

Exercise 4.8

Solution

(a) The flow pattern has two stagnation points.

(b) The minimum and maximum pressures at the cylinder surface are respectively $-7.24 \text{ E}+4$ and $+1.40 \text{ E}+5$ Pa for $\theta = 270^\circ$ and 20° (and 160°) respectively.

Notes : (1) The minimum pressure is sub-atmospheric and may lead to some cavitation. (2) There are two locations the pressure is maximum which correspond both the location of a stagnation point.

Exercise 4.9

Solution

Use 2D Flow Plus to assess the flow pattern.

A whirlpool is a vortex of vertical axis, with a downward velocity component near its centre. A good example is the bathtub vortex. VAN DYKE (1982, p. 59) presented a superb illustration. See also the Queensland Science Museum. A related example is the vortex dropshaft design. In coastal zones, whirlpools are produced by the interaction of rising and falling tides. They are often observed at the edges of straits with large tidal currents. (At Naruto, currents of up to 9 knots were observed.) The vortex (whirlpool) is a coherent structure typical of shear flows where there is a velocity difference across the shear layer. It affects the surrounding flow and water can be seen going back and forth across the shear layer between vortices. Notable oceanic whirlpools include those of Garofalo along the coast of Calabria in southern Italy, and of Messina in the strait between Sicily and peninsular Italy, the Maelstrøm (from Dutch for "whirling stream") located near the Lofoten Islands off the coast of Norway. Whirlpools near the Hebrides and Orkney islands, and in the Naruto strait between Awaji and Shikoku islands, are also well known.

See Textbook (CHANSON 2009), App. F.

Also : {<http://www.uq.edu.au/~e2hchans/whirlpl.html>}.

Exercise 4.10*Solution*

The air density at 2,000 m altitude is about 1.1 kg/m³.

The ideal fluid flow pattern is the superposition of an uniform flow ($V_o = 88.9$ m/s), a doublet (strength μ) and a vortex strength K .

The cylinder radius and doublet strength are linked as :

$$R = \sqrt{\frac{-\mu}{V_o}}$$

while the vortex strength and rotation speed ($\omega = 52.36$ rad/s) satisfy :

$$\omega = \frac{K}{2 \times \pi \times R^2} = \frac{K \times V_o}{2 \times \pi \times \mu}$$

The lift force per unit length of wing is

$$\text{Lift} = -\rho \times V_o \times K$$

The right wing must rotate in the anti-clockwise direction as seen by the pilot and ω must be negative to provide a positive lift.

The calculations for cruise conditions imply that each wing is 376 m long (!).

Note : The Magnus effect lift force decreases with decreasing speed and it becomes small at take-off and landing conditions.

Exercise 4.11*Solution*

(a) Using the method of images, the flow field is that a doublet in an uniform flow. The pressure on the roof is

$$P = P_o + \frac{1}{2} \times \rho \times V_o^2 \times (1 - 4 \times \sin^2\theta)$$

where $P_o = 1.013 \text{ E}+5$ Pa, $V_o = 12.5$ m/s and $\rho = 1.225$ kg/m³.

The flow direction is positive from left to right, and the doublet is centred at the origin. The lift and drag forces on the building are respectively:

$$\text{Lift} = - \int_0^\pi (P - P_o) \times R \times \sin\theta \times d\theta$$

$$\text{Drag} = - \int_0^\pi (P - P_o) \times R \times \cos\theta \times d\theta$$

The integration yields:

$$\text{Lift} = +2 \times \rho \times V_o^2 \times R = 1.3 \text{ kN/m}$$

$$\text{Drag} = 0$$

The total lift force on the 54.9 m long Nissen hut is 72 kN.

Notes

$$\int (1 - 4 \times \sin^2 \theta) \times d\theta = \sin(2 \times \theta) - \theta$$

$$\int (1 - 4 \times \sin^2 \theta) \times \sin \theta \times d\theta = 2 \times \cos(\theta) - \frac{1}{3} \times \cos(3 \times \theta)$$

$$\int (1 - 4 \times \sin^2 \theta) \times \cos \theta \times d\theta = \frac{1}{3} \times \sin(3 \times \theta)$$

(b) The pressure on the building roof is $1.013 \text{ E}+5 \text{ Pa}$ for $\theta = +30^\circ (\pi/6)$ and $+150^\circ (5 \times \pi/6)$.

(c) For a real fluid flow, separation takes place on the downstream of the building. The Reynolds number $\rho \times V_0 \times D / \mu$ equals $5.6 \text{ E}+6$, and the corresponding drag coefficient is about 0.8 (Fig. 4-13). The total drag force equals hence:

$$\text{Drag force} = \frac{1}{2} \times C_D \times \frac{1}{2} \times \rho \times V_0^2 \times D \times L = 14 \text{ kN}$$

where $D = 2 \times R$.