REAL-FLUID FLOW TUTORIALS

Attendance to tutorials is very strongly advised. Repeated absences by some individuals will be noted and these would demonstrate some disappointing responsible behaviour.

Past course results demonstrated a **very strong correlation** between the performances at the end-of-semester examination, the attendance of tutorials during the semester and the overall course result.

Chapter 1

Exercise No. 1

Let us consider the following fluid flows. In each case, calculate the critical range of flow velocities above which the flow becomes turbulent.

Fluid	Density kg/m^3	Dynamic viscosity Pa.s	Channel dimensions	V m/s
Water	998.2	1.005 E-3	∅ = 0.15 m	
Air	1.2	1.7 E-5	Wind tunnel (3 m × 2 m)	
Bentonite suspension (mud)	1100	0.15	Rectangular open channel (0.35 m wide × 0.10 m depth)	
Blood	1050	4 E-3	∅ = 2.2 mm	

Solutions: V = 6.7 mm/s to 6.7 cm/s (water); V = 6 mm/s to 6 cm/s (air); V = 0.54 to 54 m/s (bentonite); V = 1.7 to 17 m/s (blood)

Exercise No. 2

(a) Considering a plane Couette flow, the gap between the plate is 1 mm. One plate is fixed and the other is moving at 10 cm/s. If the measured shear stress is 1.34 Pa, calculate the fluid viscosity ?

(b) For the same experiment and fluid, what the maximum plate speed to ensure a laminar Couette flow motion.

The fluid density is 960 kg/m^3.

Solutions: (a) μ = 0.013 Pa.s; (b) V_o = 41 m/s.

Exercise No. 3

A rotating viscometer consists of two 0.6 m long co-axial cylinders with ∅ = 0.3 m and 0.32 m respectively. (The gap between the cylinders is 1 cm.) The external cylinder is fixed and the outer cylinder is rotating at 2.5 rpm.

(a) Calculate the shear stress on the cylinder walls.

(b) What is the shear stress in the fluid at 5 mm from the walls ?

(c) Calculate the power required to drive the rotating viscometer.

The fluid is a SAE40 oil (density: 871 kg/m^3, viscosity: 0.6 Pa.s).

Remember, the power equals the product of the angular velocity time the torque between the fluid and the moving cylinder.

Solutions: (a) τ_o = 2.43 Pa; (b) τ_o = 2.43 Pa (Remember: the Couette flow is characterised by a constant shear stress distribution); (c) 0.06 W (Remember: the power equals the force times the velocity, or the torque times the angular velocity)

Exercise No. 4

A blood solution is tested in a cylindrical Coutte viscosimeter. The apparatus is 0.100 m high. The inner, rotating cylinder has an outer diameter of 40.0 mm and the outer (fixed) cylinder has an inner diameter of 40.4 mm. The rotation speed is 6.1 rpm.

(a) Calculate the shear stress on the outer cylinder wall.

(b) What is the shear stress in the fluid at 0.2 mm from the walls?

(c) Sketch the velocity profile between the cylinders.

(d) Calculate the torque on the inner cylinder.

(e) Calculate the power required to drive the viscometer.

The blood density is 1051 kg/m^3 and its viscosity is 4.1 E-3 Pa.s.

Solution

(a) The shear stress distribution is uniform between the two cylinders. Assuming a quasi-two-dimensional Couette flow:

$$\tau = \tau_o = \mu \times V/D = 0.262 \text{ Pa}$$

where V = R×ω = 0.0128 m/s and D = 0.2 mm.

(d) Torque = 6.6 E-5 N/m

(e) Power = 4.2 E-5 W

Exercise No. 5

Considering a plane flow between two plates, one plate is at rest while the other is moving at speed V_o = 0.75 m/s. The gap between the plates is 5 mm and the fluid is a viscous oil (ρ = 1,050 kg/m^3, μ = 0.011 Pa.s). Calculate the shear stress on the plate at rest and the flow rate per unit width between the plates.

Solution

The Reynolds number is: $\rho \times V_o \times D/\mu$ = 360 (laminar flow motion)

$\tau_o = \mu \times V_o/D$ = 1.65 Pa

$$q = \int_0^D V \times dy = 1.875 \times 10^{-3} \text{ m}^2/\text{s}$$

Chapter 2

Exercise No. 1

Turbulent velocity measurements were conducted in Eprapah Creek at 0.2 m above the creek bed. The data were recorded mid-estuary on 16 May 2005.

Time (s)	v_x (cm/s)	Time (s)	v_x (cm/s)	Time (s)	v_x (cm/s)	Time (s)	v_x (cm/s)
66030.98	0.0548	66033.34	0.0566	66035.82	0.0537	66038.42	0.0583
66031.02	0.0559	66033.38	0.0576	66035.86	0.0554	66038.46	0.0554
66031.06	0.0532	66033.42	0.054	66035.9	0.0593	66038.5	0.059
66031.1	0.0588	66033.46	0.0578	66035.94	0.0548	66038.54	0.0559
66031.14	0.0588	66033.5	0.0546	66035.98	0.0616	66038.58	0.0506
66031.18	0.0568	66033.54	0.0593	66036.02	0.0544	66038.62	0.0629
66031.22	0.0585	66033.58	0.06	66036.06	0.051	66038.66	0.0591
66031.26	0.057	66033.62	0.0546	66036.1	0.0542	66038.7	0.0534
66031.3	0.0592	66033.66	0.061	66036.14	0.0575	66038.74	0.062
66031.34	0.0596	66033.7	0.0608	66036.18	0.0496	66038.78	0.0591
66031.38	0.0602	66033.74	0.0615	66036.22	0.0517	66038.82	0.0549
66031.42	0.0603	66033.78	0.0616	66036.26	0.0538	66038.86	0.0642
66031.46	0.0618	66033.82	0.064	66036.3	0.0547	66038.9	0.0649
66031.5	0.0622	66033.86	0.0563	66036.34	0.0506	66038.94	0.0653
66031.54	0.0643	66033.9	0.0606	66036.38	0.0487	66038.98	0.0687
66031.58	0.0597	66033.94	0.0615	66036.42	0.0535	66039.02	0.0606
66031.62	0.0646	66033.98	0.06	66036.46	0.0516	66039.06	0.0639
66031.66	0.0641	66034.02	0.0595	66036.5	0.0513	66039.1	0.0609
66031.7	0.059	66034.06	0.0589	66036.54	0.0494	66039.14	0.0662
66031.74	0.0612	66034.1	0.0633	66036.58	0.0497	66039.18	0.0708
66031.78	0.0589	66034.14	0.0589	66036.62	0.0538	66039.22	0.0702
66031.82	0.0563	66034.18	0.0578	66036.66	0.0503	66039.26	0.0689
66031.86	0.0549	66034.22	0.0544	66036.7	0.0523	66039.3	0.0617
66031.9	0.0582	66034.26	0.0548	66036.74	0.0554	66039.34	0.0639
66031.94	0.0546	66034.3	0.0638	66036.78	0.0567	66039.38	0.0666
66031.98	0.0567	66034.34	0.0632	66036.82	0.057	66039.42	0.0616
66032.02	0.0551	66034.38	0.0615	66036.86	0.0601	66039.46	0.064
66032.06	0.0542	66034.42	0.0626	66036.9	0.0563	66039.5	0.0686
66032.1	0.0505	66034.46	0.0637	66036.94	0.0588	66039.54	0.0649
66032.14	0.0517	66034.5	0.0609	66036.98	0.0584	66039.58	0.0618
66032.18	0.0505	66034.54	0.0677	66037.02	0.0525	66039.62	0.0634
66032.22	0.0541	66034.58	0.0671	66037.06	0.0596	66039.66	0.0659
66032.26	0.0506	66034.62	0.0684	66037.1	0.0556	66039.7	0.0668
66032.3	0.0541	66034.66	0.0651	66037.14	0.0543	66039.74	0.0578
66032.34	0.0492	66034.7	0.0664	66037.18	0.0584	66039.78	0.0537
66032.38	0.0515	66034.74	0.0668	66037.22	0.056	66039.82	0.0614
66032.42	0.0478	66034.78	0.0687	66037.26	0.0552	66039.86	0.0609
66032.46	0.0568	66034.82	0.0648	66037.3	0.0575	66039.9	0.0647
66032.5	0.0511	66034.86	0.0706	66037.34	0.0555	66039.94	0.0666
66032.54	0.0531	66034.9	0.0618	66037.38	0.0559	66039.98	0.0656
66032.58	0.0535	66034.94	0.0629	66037.42	0.0576	66040.02	0.0643
66032.62	0.0581	66034.98	0.0677	66037.46	0.0603	66040.06	0.0645
66032.66	0.0507	66035.02	0.0673	66037.5	0.0599	66040.1	0.0659
66032.7	0.0549	66035.06	0.066	66037.54	0.062	66040.14	0.0564
66032.74	0.0533	66035.1	0.0658	66037.58	0.0579	66040.18	0.0566
66032.78	0.0449	66035.14	0.067	66037.62	0.052	66040.22	0.0643
66032.82	0.0572	66035.18	0.0667	66037.66	0.0558	66040.26	0.0637

66032.86	0.051	66035.22	0.0607	66037.7	0.0515	66040.3	0.0653
66032.9	0.0599	66035.26	0.0633	66037.74	0.0499	66040.34	0.0626
66032.94	0.0568	66035.3	0.0669	66037.78	0.0551	66040.38	0.064
66032.98	0.0579	66035.34	0.0617	66037.82	0.0521	66040.42	0.0626
66033.02	0.0507	66035.38	0.0591	66037.86	0.0508	66040.46	0.0674
66033.06	0.0521	66035.42	0.0617	66037.9	0.0584	66040.5	0.0638
66033.1	0.0554	66035.46	0.0616	66037.94	0.0503	66040.54	0.0652
66033.14	0.0558	66035.5	0.0551	66037.98	0.0558	66040.58	0.0648
66033.18	0.0581	66035.54	0.0593	66038.02	0.0521	66040.62	0.064
66033.22	0.0579	66035.58	0.0537	66038.06	0.0579	66040.66	0.0642
66033.26	0.0551	66035.62	0.0597	66038.1	0.0551	66040.7	0.0624
66033.3	0.0594	66035.66	0.0628	66038.14	0.054	66040.74	0.0669
		66035.7	0.0583	66038.18	0.0569	66040.78	0.0636
		66035.74	0.0583	66038.22	0.0576	66040.82	0.0705
		66035.78	0.0554	66038.26	0.0567	66040.86	0.065
				66038.3	0.0535	66040.9	0.0706
				66038.34	0.0581	66040.94	0.0691
				66038.38	0.0591		

(a) For the following 10 s record, plot the instantaneous velocity as a function of time.

(b) Calculate the time-average, standard deviation, skewness and kurtosis of the longitudinal velocity.

(c) Calculate the integral time scale and the dissipation time scale.

(d) calculate and plot the probability distribution function of the longitudinal velocity.

For the calculation of the dissipation time scale, compare the results obtained using a parabolic approximation of the auto-correlation function and the method of HALLBACK et al. 1989). The data were collected by TREVETHAN et al. (2006).

Solutions:

V_{avg} =	0.059 m/s	time-averaged
V_{med} =	0.059 m/s	median value
V_{std} =	0.00534 m/s	standard deviation
Skew =	0.0812	skewness
Kurt =	-0.669	kurtosis
T_E =	0.3985 s	integral time-scale
τ_E =	***-0.00332*** s	dissipation time-scale (**meaningless** !)

Exercise No. 2

The following data set was recorded in a steady open channel flow at 0.057 m above the bed.

Time (s)	V_x (cm/s)	Time (s)	V_x (cm/s)	Time (s)	V_x (cm/s)	Time (s)	V_x (cm/s)
4.00	108.1	4.5	108.7	5.00	114.2	5.5	110.2
4.02	107.3	4.52	110.2	5.02	111.9	5.52	106.7
4.04	107.4	4.54	109.3	5.04	110.9	5.54	108.71
4.06	107.9	4.56	110.2	5.06	109.7	5.56	109.9
4.08	109.4	4.58	108.4	5.08	111.4	5.58	109.5
4.1	108.3	4.6	110.1	5.1	111.1	5.6	109.2
4.12	110.5	4.62	110.1	5.12	110.4	5.62	110.6
4.14	111.7	4.64	109.4	5.14	110.0	5.64	112.8
4.16	110.8	4.66	111.5	5.16	111.3	5.66	112.2

4.18	110.2	4.68	111.8	5.18	108.7	5.68	112.6
4.2	110.1	4.7	111.2	5.2	112.4	5.7	111.6
4.22	109.2	4.72	112.2	5.22	108.4	5.72	111.8
4.24	108.9	4.74	110.8	5.24	112.2	5.74	109.8
4.26	110.8	4.76	109.6	5.26	111.9	5.76	108.1
4.28	109.8	4.78	111.1	5.28	109.2	5.78	109.8
4.3	109.1	4.8	108.0	5.3	111.1	5.8	110.5
4.32	108.9	4.82	107.6	5.32	109.1	5.82	109.8
4.34	108.0	4.84	108.9	5.34	107.6	5.84	110.4
4.36	109.3	4.86	110.2	5.36	112.0	5.86	108.4
4.38	109.6	4.88	109.3	5.38	109.5	5.88	109.1
4.4	110.9	4.9	107.8	5.4	109.9	5.9	107.7
4.42	110.5	4.92	109.4	5.42	112.1	5.92	109.8
4.44	111.4	4.94	110.5	5.44	111.5	5.94	111.4
4.46	110.4	4.96	112.0	5.46	111.5	5.96	112.1
4.48	110.1	4.98	113.0	5.48	112.1	5.98	110.9

(a) For the data set, plot the instantaneous velocity as a function of time.

(b) Calculate the time-average and standard deviation of the longitudinal velocity. What is the turbulence intensity defined as Tu = $\sqrt{\overline{v^2}}/\overline{V}$?

Solutions

V_{avg} =	110.14 cm/s	time-averaged
V_{med} =	110.10 cm/s	median value
V_{std} =	1.489 cm/s	standard deviation
Skew =	0.0296	skewness
Kurt =	-0.419	kurtosis
Tu =	1.35%	turbulence intensity
T_E =	0.028 s	integral time-scale
τ_E =	0.0028 s	dissipation time-scale

Exercise No. 3

The steady open channel flow situation, analysed in Exercise 2, is suddenly affected by the passage of a tidal bore. Both the instantaneous and time-averaged velocity data are listed below. The data was recorded at 0.057 m above the bed. (Note that the flow is unsteady and the time-averaged velocity is a variable time average.)

Instantaneous longitudinal velocity

Time (s)	V_x (cm/s)	Time (s)	V_x (cm/s)	Time (s)	V_x (cm/s)	Time (s)	V_x (cm/s)
12	67.5026	12.5	60.0794	13	79.6718	13.5	86.1862
12.02	69.7268	12.52	65.5508	13.02	80.8693	13.52	89.5882
12.04	64.5371	12.54	59.4951	13.04	79.989	13.54	86.3987
12.06	66.7903	12.56	63.3117	13.06	78.3692	13.56	88.8158
12.08	66.3739	12.58	69.5743	13.08	76.1492	13.58	86.9115
12.1	66.5177	12.6	69.9598	13.1	77.1707	13.6	89.9447
12.12	69.6151	12.62	64.1634	13.12	82.6693	13.62	85.8272
12.14	67.807	12.64	64.6971	13.14	84.1862	13.64	86.1953

12.16	66.1124	12.66	65.1664	13.16	82.9088	13.66	89.7416
12.18	66.3661	12.68	63.1063	13.18	83.9595	13.68	86.5472
12.2	67.3087	12.7	63.2548	13.2	82.643	13.7	90.1726
12.22	60.6616	12.72	64.2278	13.22	84.8607	13.72	91.3913
12.24	61.6533	12.74	70.6123	13.24	85.4088	13.74	88.1052
12.26	62.1375	12.76	75.9225	13.26	85.1977	13.76	86.5433
12.28	61.298	12.78	76.8025	13.28	86.2025	13.78	90.6055
12.3	67.3146	12.8	71.0797	13.3	84.7838	13.8	90.0265
12.32	68.2022	12.82	71.7941	13.32	85.5499	13.82	91.2321
12.34	68.7461	12.84	67.3993	13.34	83.6273	13.84	89.5507
12.36	63.1782	12.86	72.2021	13.36	80.6722	13.86	90.2612
12.38	67.9926	12.88	72.7026	13.38	80.7049	13.88	86.9007
12.4	70.3569	12.9	74.1936	13.4	84.4192	13.9	81.6948
12.42	66.5494	12.92	76.7423	13.42	84.3725	13.92	81.8264
12.44	65.036	12.94	79.0459	13.44	83.2162	13.94	85.5667
12.46	60.316	12.96	78.1772	13.46	86.4487	13.96	87.0048
12.48	58.4519	12.98	76.7767	13.48	83.8549	13.98	81.9291

"Time-averaged velocity " (variable time average)

Time (s)	V_x (cm/s)	Time (s)	V_x (cm/s)	Time (s)	V_x (cm/s)	Time (s)	V_x (cm/s)
12	68.59311	12.5	66.13795	13	77.34149	13.5	85.44806
12.02	68.44279	12.52	66.27083	13.02	77.86089	13.52	85.65064
12.04	68.27196	12.54	66.4389	13.04	78.36785	13.54	85.84535
12.06	68.08396	12.56	66.64473	13.06	78.8607	13.56	86.02884
12.08	67.88306	12.58	66.89004	13.08	79.33774	13.58	86.19742
12.1	67.6742	12.6	67.17561	13.1	79.79732	13.6	86.34727
12.12	67.4626	12.62	67.50117	13.12	80.23791	13.62	86.47462
12.14	67.25348	12.64	67.86545	13.14	80.65817	13.64	86.57596
12.16	67.0517	12.66	68.26624	13.16	81.05708	13.66	86.64817
12.18	66.86144	12.68	68.70055	13.18	81.43394	13.68	86.68868
12.2	66.68604	12.7	69.1648	13.2	81.78855	13.7	86.69551
12.22	66.5278	12.72	69.65499	13.22	82.12116	13.72	86.66734
12.24	66.38803	12.74	70.16697	13.24	82.43256	13.74	86.60349
12.26	66.26709	12.76	70.69659	13.26	82.72404	13.76	86.50391
12.28	66.16457	12.78	71.23991	13.28	82.9974	13.78	86.36909
12.3	66.07955	12.8	71.79326	13.3	83.2548	13.8	86.2
12.32	66.01086	12.82	72.35341	13.32	83.49874	13.82	85.99802
12.34	65.95744	12.84	72.91752	13.34	83.73186	13.84	85.76487
12.36	65.91856	12.86	73.48317	13.36	83.95681	13.86	85.50253
12.38	65.8941	12.88	74.04832	13.38	84.17605	13.88	85.21317
12.4	65.88468	12.9	74.61121	13.4	84.39172	13.9	84.89915
12.42	65.89173	12.92	75.17034	13.42	84.60543	13.92	84.56292
12.44	65.91749	12.94	75.72429	13.44	84.81812	13.94	84.20699
12.46	65.96483	12.96	76.27171	13.46	85.02999	13.96	83.83385
12.48	66.03713	12.98	76.81126	13.48	85.24045	13.98	83.44594

(a) For the data set, plot the instantaneous velocity and variable time average velocity as functions of time.

(b) Calculate the standard deviation and turbulence intensity defined as Tu = $\sqrt{\overline{v^2}}/\overline{V}$ of the longitudinal velocity.

(c) Compare the results with the steady flow conditions (Exercise 2).

All the velocities are positive in the downstream direction. Data obtained by KOCH and CHANSON (2005).

Solutions:

V_{avg} =	76.29	cm/s	time-averaged
V_{std} =	3.113	cm/s	standard deviation
Skew =	-0.459		skewness
Kurt =	-0.5176		kurtosis
Tu =	4.1 %		turbulence intensity
T_E =	0.024	s	integral time-scale

A comparison with the steady flow results obtained in the same channel prior to the arrival of the tidal bore (Exercise No. 2) shows that the passage of the undular bore is associated with higher turbulence levels (3 times higher) and smaller integral turbulent time scale.

Exercise No. 4

For the following data set:

(a) Plot the data, and

(b) Calculate the time-averaged and standard deviation of the normal and tangential stresses.

Time (s)	v_x (cm/s)	v_z (cm/s)	Time (s)	v_x (cm/s)	v_z (cm/s)	Time (s)	v_x (cm/s)	v_z (cm/s)
0.02	-1.54	3.19	0.82	-2.46	3.37	1.62	-2.14	2.92
0.06	-1.97	3.16	0.86	-2.27	3.01	1.66	-2.47	2.88
0.1	-1.95	3.28	0.9	-3.45	2.62	1.7	-2.35	3.45
0.14	-2.28	3.06	0.94	-2.82	2.62	1.74	-1.77	2.87
0.18	-1.83	3.12	0.98	-2.13	2.42	1.78	-1.8	2.75
0.22	-2.3	3.16	1.02	-2.47	3.01	1.82	-2.38	2.86
0.26	-1.5	2.75	1.06	-2.6	3.27	1.86	-2.76	2.88
0.3	-1.92	3.71	1.1	-2.21	2.68	1.9	-1.68	3.09
0.34	-2.49	3.55	1.14	-2.62	2.56	1.94	-2.36	3.05
0.38	-2.67	2.77	1.18	-2.63	2.93	1.98	-2.7	2.92
0.42	-2.41	3.08	1.22	-4.61	2.9	2.02	-2.71	3.11
0.46	-2.83	3.04	1.26	-2.72	3.03	2.06	-2.21	3.1
0.5	-2.21	2.92	1.3	-2.29	2.79	2.1	-2.58	2.86
0.54	-2	2.83	1.34	-2.84	2.4	2.14	-2.6	3.61
0.58	-1.7	2.97	1.38	-2.8	2.55	2.18	-1.61	2.87
0.62	-4.05	3.32	1.42	-2.39	2.61	2.22	-2.04	3.29
0.66	-2.59	2.99	1.46	-3.12	1.95	2.26	-1.76	3.49
0.7	-2.68	2.84	1.5	-2.41	3.14	2.3	-1.64	3.76
0.74	-2.41	3.41	1.54	-1.88	3.29	2.34	-1.72	3.32
0.78	-2.24	3.09	1.58	-2.08	2.61	2.38	-2.14	2.93

The data set was collected in Eprapah Creek estuary on 4 April 2004 (CHANSON 2003). The water density was about 1015 kg/m^3 (for brackish waters). v_x is positive upstream and the transverse velocity v_z is positive towards the right bank.

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Exercise No. 5

Considering a turbulent Couette flow between two parallel plates (Fig. 1-7), one plate moving at speed V_1 and the other moving at speed V_2, express the velocity distribution between the plate.
Assume that the "eddy viscosity" may be estimated by a parabolic distribution:

$$\nu_T = K \times V_* \times y \times \left(1 - \frac{y}{D}\right)$$

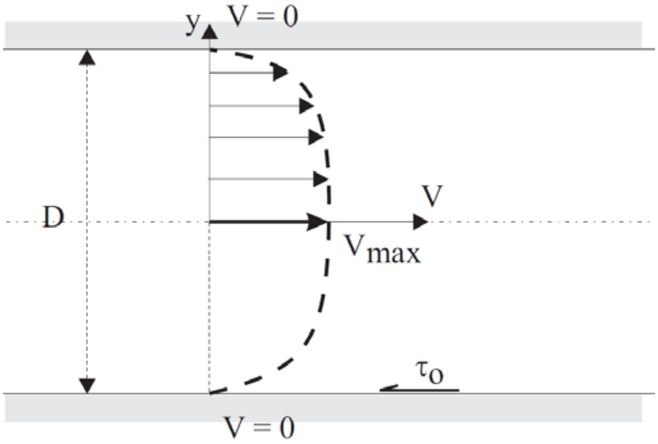
where K is the von Karman constant (K = 0.40) and D is the distance between plates (Chapter II-1).

Solution
For $V_1 = 0$ and $V_2 = V_o$, the solution is developed in Chapter II-1. It may be extended easily for $V_1 \neq 0$.

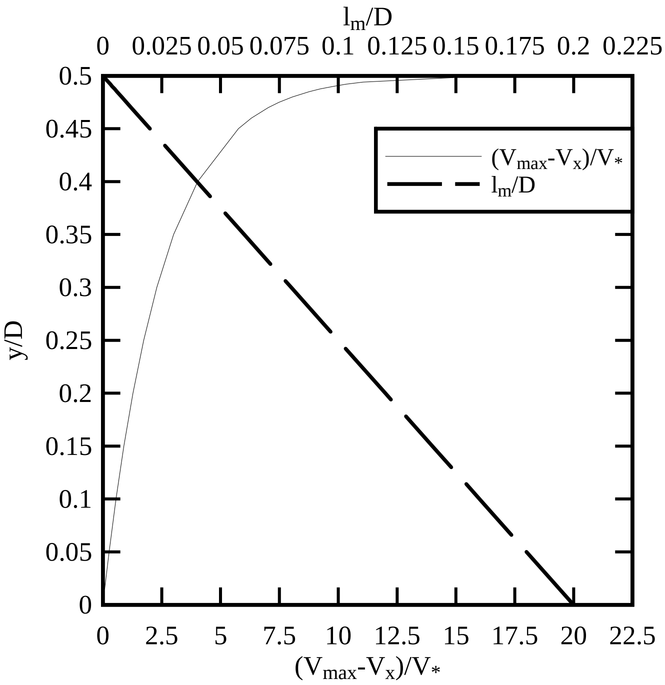
Exercise No. 6

In a steady turbulent flow between two flat plates (Fig. E-II-2-1), find a suitable Prandtl mixing length distribution. Derive the velocity profile. Discuss the result.

Fig. E-II-2-1 - Turbulent flow between plates
(A) Definition sketch



(B) Dimensionless velocity and mixing length distribution



Solution

Let us define y the distance from the channel centreline. The mixing length may be taken proportional the distance from the plate:

$$l_m = K \times \left(\frac{D}{2} - y\right) \qquad \text{for } y \geq 0$$

Replacing into Equation (2-18),

$$\frac{V_{max} - V_x}{V_*} = \frac{1}{K} \times \mathrm{Ln}\left(\frac{1}{1 - 2\times\frac{y}{D}}\right) \qquad \text{for } y \geq 0$$

where V_* is the shear velocity ($V_* = (\tau_o/\rho)^{1/2}$) and V_{max} is the centreline velocity. The results are presented in Figure E-II-2-1B.

Note that, on the channel centreline (y = 0), the velocity derivative is not continuous

Chapter 3

Exercise No. 1

In a developing boundary layer, the velocity distribution follows:

y (mm)	v_x (m/s)
0.5	0.271
1	0.518
2	1.113
3.5	1.93
6	3.137
8	4.157
10.2	5.32
14	7.28
18	9.22
21	10.79
35	11.3
41	11.52
58	11.55
60.5	11.51
66	11.5
70	11.6
90	11.59
100	11.50

Calculate the boundary layer thickness, the displacement thickness, the momentum thickness and the energy thickness.

Solution: δ = 37.3 mm; δ_1 = 11.6 mm; δ_2 = 4.3 mm; δ_3 = 6.7 mm.

Exercise No. 2

Considering a laminar boundary layer.

(a) Using a power series, demonstrate the asymptotic solutions of the velocity distribution:

$$\frac{v_x}{V_o} = 0.332 * \frac{y}{x} \times \sqrt{Re_x} \qquad \text{for } \frac{y}{x} \times \sqrt{Re_x} << 1$$

$$\frac{v_x}{V_o} = 1 \qquad \text{for } \frac{y}{x} \times \sqrt{Re_x} > 6$$

(b) For the developing boundary layer, calculate the boundary shear stress at a distance x from the plate leading edge:

$$\tau_o = \mu \times \left(\frac{\partial v_x}{\partial y}\right)_{y=0}$$

(c) Based upon the above result, calculate the shear force acting on a plate of length L and width B.

Exercise No. 3

Let us consider a plane developing boundary layer with zero pressure gradient and a free-stream velocity V_o = 0.1 m/s.

(a) Plot the velocity profile at x = 0.12 m. *First you must assess if the flow motion is laminar or turbulent.*

(b) Calculate the boundary layer thickness, the displacement thickness, and the momentum thickness at x = 0.12 m.

(c) Calculate and plot the boundary shear stress for $0 \le x \le 0.2$ m.

(d) Compute the overall friction force on the 0.2 m long 0.3 m wide plate.

The fluid is blood (density: 1050 kg/m^3, viscosity: 4 E-3 Pa.s).

Solutions: (b) δ = 0.010 m, δ_1 = 3.7 mm, δ_2 = 1.4 mm. (d) F_{shear} = 0.0058 N

Exercise No. 4

Velocity measurements were conducted at 3 locations along a developing laminar boundary layer.

y		v_x	
mm	cm/s	cm/s	cm/s
x (m) =	0.05	0.10	0.20
0.5	0.52	0.45	0.44
1	1.00	0.58	0.47
2	1.62	1.02	0.83
3.5	2.80	1.87	1.31
6	4.51	3.21	2.17
8	5.88	4.21	3.10
10.2	7.53	5.13	3.67
14	10.39	7.24	5.26
18	11.47	9.09	6.69
21	11.55	10.71	7.49
35	11.62	11.46	10.55
41	11.54	11.69	11.45
58	11.60	11.52	11.55
60.5	11.67	11.69	11.67
66	11.58	11.70	11.67
70	11.66	11.57	11.66
90	11.58	11.54	11.69
100	11.69	11.53	11.67

(a) Using the momentum integral equation, calculate the boundary shear stress at x = 0.1 m.

(b) Based upon the momentum integral equation, integrate numerically the boundary shear stress to estimate the friction force per unit width on the 0.2 m long plate.

(c) Compare your results with the theoretical calculations (Blasius equation) and with the approximate solution assuming a quadratic velocity distribution. *Check that the flow laminar before conducting the comparison.*

The fluid is a bentonite suspension (density: 1115 kg/m^3, viscosity: 0.19 Pa.s, mass concentration: 17%).

Solutions:

(a) The free-stream velocity is about 11.6 cm/s.

(b)

	Momentum integral equation	Blasius solution
τ_o (x=0.1 m) (Pa) =	0.41	0.6
F_{shear}/B (N/m) =	0.083	1.47

Exercise No. 5

Assuming that the velocity profile in a laminar boundary layer satisfies a polynomial of fourth degree, apply the momentum integral equation.

(a) Derive the expression of the velocity profile. *Write carefully the boundary conditions.*

(b) Derive mathematically the expression of the boundary layer thickness, bed shear stress and total shear force.

Exercise No. 6

Assuming that the velocity profile in a laminar boundary layer satisfies a polynomial of third degree, apply the momentum integral equation.

(a) Derive the expression of the velocity profile. *Write carefully the boundary conditions.*

(b) Derive mathematically the expression of the boundary layer thickness, displacement thickness and momentum thickness.

(c) Derive the expressions of the bed shear stress and total shear force.

(d) Compare your results with the Blasius solution.

Solution

(a) Let us assume that the velocity profile above a flat plate may be expressed as:

$$\frac{v_x}{V_o} = a_0 + a_1 \times \frac{y}{\delta} + a_2 \times \left(\frac{y}{\delta}\right)^2 + a_3 \times \left(\frac{y}{\delta}\right)^3$$

where a_0, a_1, a_2 and a_3 are undetermined coefficients, The coefficients are determined from the boundary conditions: $v_x(y=0) = 0$, $v_x(y=\delta) = V_o$, $(\partial v_x/\partial y) = 0$ for $y = \delta$ and $(\partial^2 v_x/\partial y^2) = 0$ for $y = \delta$ The velocity distribution is found to be:

$$\frac{v_x}{V_o} = 3 \times \frac{y}{\delta} - 3 \times \left(\frac{y}{\delta}\right)^2 + \left(\frac{y}{\delta}\right)^3$$

(b) The von Karman momentum integral equation for a flat plate becomes:

$$V_o^2 \times \frac{\partial}{\partial x}(\delta_2) = \frac{\tau_o}{\rho}$$

For a velocity profile satisfying a polynomial of third degree, the momentum thickness equals:

$$\delta_2 = \int_0^{+\infty} \frac{v_x}{V_o} \times \left(1 - \frac{v_x}{V_o}\right) \times dy = \frac{3}{28} \times \delta$$

The bed shear stress is defined as:

$$\tau_o = \mu \times \left(\frac{\partial v_x}{\partial y}\right)_{y=0} = \frac{3 \times \mu \times V_o}{\delta}$$

The momentum integral equation yields:

$$\frac{3}{28} \times V_o^2 \times \frac{\partial \delta}{\partial x} = \frac{\mu}{\rho} \times \frac{3 \times V_o}{\delta}$$

The integration gives the expression of the boundary layer growth:

$$\delta = \sqrt{14} \times \frac{x}{\sqrt{Re_x}}$$

The displacement thickness and momentum thickness equal:

$$\delta_1 = \frac{\delta}{4} = \sqrt{\frac{7}{2}} \times \frac{x}{\sqrt{Re_x}}$$

$$\delta_2 = \frac{3}{28} \times \delta = \sqrt{\frac{9}{56}} \times \frac{x}{\sqrt{Re_x}}$$

(c) The boundary shear stress is deduced from the momentum integral equation:

$$\frac{\tau_o}{\frac{1}{2} \times \rho \times V_o^2} = 2 \times \frac{\partial}{\partial x}(\delta_2) = \sqrt{\frac{9}{56}} \times \frac{1}{\sqrt{Re_x}}$$

The dimensionless boundary shear force per unit width equals:

$$\frac{\int_{x=0}^{L} \tau_o \times dx}{\frac{1}{2} \times \rho \times V_o^2 \times L} = \sqrt{\frac{9}{14}} \times \frac{1}{\sqrt{Re_L}}$$

(d) The results are compare with the Blasius analytical solution below:

Boundary layer parameter	Approximate solution	Theoretical solution
Velocity distribution:	$\frac{v_x}{V_o} = 3 \times \frac{y}{\delta} - 3 \times \left(\frac{y}{\delta}\right)^2 + \left(\frac{y}{\delta}\right)^3$	Blasius equation
$\delta =$	$3.74 \times \frac{x}{\sqrt{Re_x}}$	$4.91 \times \frac{x}{\sqrt{Re_x}}$
$\delta_1 =$	$1.87 \times \frac{x}{\sqrt{Re_x}}$	$1.72 \times \frac{x}{\sqrt{Re_x}}$
$\delta_2 =$	$0.40 \times \frac{x}{\sqrt{Re_x}}$	$0.664 \times \frac{x}{\sqrt{Re_x}}$
$\frac{\tau_o}{\frac{1}{2} \times \rho \times V_o^2}$	$\frac{0.40}{\sqrt{Re_x}}$	$\frac{0.664}{\sqrt{Re_x}}$
$\frac{\int_{x=0}^{L} \tau_o \times dx}{\frac{1}{2} \times \rho \times V_o^2 \times L}$	$\frac{0.80}{\sqrt{Re_L}}$	$\frac{1.328}{\sqrt{Re_L}}$

Exercise No. 7

Let us consider a laminar wake behind a 0.5 m long plate.

(a) Calculate the total drag force (per unit width) on the plate.

(b) Estimate at what distance, downstream of the plate, the velocity profile will recover (within 2% of the free-stream velocity)?

The free-stream velocity is 0.35 m/s and the fluid is a viscous SAE40 oil (density: 871 kg/m^3, viscosity: 0.6 Pa.s).

Solution

(a) Drag per unit width = 4.4 N/m

(b) x/L = 350 (x = 175 m !!!)

Exercise No. 8

Some fluid is injected in a vast container where the surrounding fluid is at rest. The nozzle height is 0.15 mm and the injected velocity is 1 cm/s.

(b) Calculate the maximum jet velocity at distances of 1.5 mm, 2 cm and 18 cm from the nozzle.

(b) At a distance of 18 cm from the nozzle, estimate the volume discharge of entrained fluid (per unit width).

Assume a two-dimensional jet.

The fluid is blood (density: 1050 kg/m^3, viscosity: 4 E-3 Pa.s).

Solution

	x = 1.5 mm	x = 2 cm	x = 18 cm
Maximum v_x/V_o =	15.5 %	6.5 %	3.1 %
$Q/(V_o \times D)$ =	--	--	48

Exercise No. 9

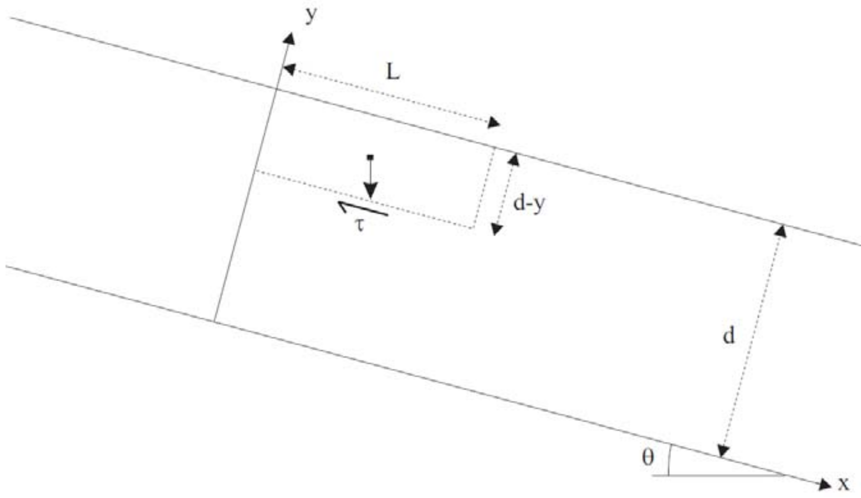
Let us consider a laminar flow down an inclined plane (Fig. EII-3-1) at uniform equilibrium.

(a) Derive the shear stress distribution in the direction normal to the plane.

(b) Express the pressure distribution in the direction normal to the plane.

(c) Deduce the laminar flow velocity.

Fig. II-E-3-1 - Laminar flow down an inclined plane



Solution

(a) The problem is solved by applying the momentum principle to the control volume sketched above. At uniform equilibrium down the slope, the velocity distribution and water depth d are independent of the distance x along the slope. The application of the momentum principle along the x-direction implies that the laminar shear stress τ along the lower interface amust equal exactly the control volume weight force component along the inclined plane:

$$\tau \times L = \rho \times g \times L \times (d - y) \times \sin\theta$$

where L is the control volume length. It yields:

$$\tau = \rho \times g \times (d - y) \times \sin\theta$$

(b) The pressure distribution is derived from the application of the momentum principle along the y-direction implying that the pressure force acting along the lower interface must equal the weight must equal exactly the control volume weight force component along the y-direction:

$$P \times L = \rho \times g \times L \times (d - y) \times \cos\theta$$

This gives the classical result in an open channel flow with hydrostatic pressure:

$$P = \rho \times g \times (d - y) \times \cos\theta$$

(c) In a laminar force, the shear stress is proportional to the shear rate:

$$\tau = \mu \times \frac{\partial V_x}{\partial y}$$

From the result derived in (a), the velocity distribution is:

$$V_x = \frac{\rho \times g \times \sin\theta}{\mu} \times y \times \left(d - \frac{y}{2} \right)$$

Since the boundary shear stress equals the fluid shear stress at y = 0, and introducing the shear velocity, the velocity distribution may be rewritten in dimensionless terms as:

$$\frac{V_x}{V_*} = \frac{\rho \times \sqrt{g \times \sin\theta \times d^3}}{\mu} \times \frac{y}{d} \times \left(1 - \frac{y}{2 \times d} \right)$$

Exercise No. 10

The frequency of oscillation between a circular cylinder follows approximately the following relationship:

$$St = \frac{\omega_{shedding} \times D}{V_o} = 0.2 \times \left(1 - \frac{20}{Re} \right) \qquad \text{for } 100 < Re < 10^5$$

where D is the cylinder diameter, V_o is the approach flow velocity and Re is the Reynolds number. Estimate the vortex shedding frequency of a 10 mm wire in a 35 m/s wind flow.

Solution

Re = 21.400; St = 0.1998; $\omega_{shedding}$ = 700 Hz

Exercise No. 11

We consider a developing laminar boundary layer along a 1-m long 0.5 m wide flat plate. The upstream velocity profile is uniform. The velocity profile in the laminar boundary layer may be approximated by the following expression:

$$V_x = a + b \times \sin\left(c \times \frac{y}{\delta} \right) \qquad \text{for } 0 \le c \times y/\delta \le \pi/2$$

(1) Based upon the momentum integral principle, derive the expression of the velocity profile.

(2) Give the expression of the boundary layer thickness, displacement thickness, momentum thickness, bed shear stress and total shear force as function of the distance x from the plate leading edge.

(3) Compare the above results with the Blasius solution.

(4) Application to a bentonite suspension

(4.1) Plot the longitudinal distribution of boundary shear stress along the plate. Include the Blasius solution for comparison.

(4.2) Calculate the friction force on the plate.

The bentonite suspension properties are: *ρ = 1,115 kg/m^3, μ = 0.19 Pa.s, mass concentration: 17%*. The free-stream velocity is: V_o = 0.10 m/s.

Mathematical Aids

$$\int \cos x(1-\cos x)dx = -\frac{1}{2}x + \sin x - \frac{1}{4}\sin(2x)$$

$$\int \cos x(1-\sin x)dx = \sin x + \frac{1}{2}\cos^2 x$$

$$\int \sin x(1-\cos x)dx = \frac{1}{2}(\cos x - 2)\cos x$$

$$\int \sin x(1-\sin x)dx = -\frac{1}{2}x - \cos x + \frac{1}{4}\sin(2x))$$

Solution

The velocity distribution may be rewritten as:

$$V_x = V_o \times \sin\left(\frac{\pi}{2} \times \frac{y}{\delta}\right)$$

The application of the momentum integral principle yields:

$$\delta = 4.795 \times \frac{x}{\sqrt{Re_x}}$$

$$\delta_2 = 0.6551 \times \frac{x}{\sqrt{Re_x}}$$

$$\frac{\tau_o}{\frac{1}{2} \times \rho \times V_o^2} = \frac{0.6551}{\sqrt{Re_x}}$$

$$\frac{\int_0^L \tau_o}{\frac{1}{2} \times \rho \times V_o^2 \times L} = \frac{1.31}{\sqrt{Re_L}}$$

Chapter 4

Exercise No. 1

A turbulent boundary layer develops over the Moreton Bay as a result of 35 m/s wind storm.

(a) Assuming a zero pressure gradient, predict the atmospheric boundary layer thickness, the displacement thickness and the momentum thickness at 1 and 5 km from the inception of the wind storm.

(b) Plot on graph paper the vertical distribution of the longitudinal velocity.

(c) Calculate the friction force per unit width on the 5 km long water-boundary layer interface.

Assuming that the water free-surface is equivalent to a smooth boundary.

Solutions

(a) Turbulent boundary layer calculations since $Re_x > 1$ E+9

x (m) =	1000	5000
δ (m) =	4.89	17.73
δ_1 (m) =	0.61	2.21
δ_2 (m) =	0.48	1.73

(c) $F_{shear}/B = 2500$ N/m

Exercise No. 2

A gust storm develops in a narrow, funnel shaped valley. The free-stream wind speed is 15 m/s at the start of that valley and it reaches 25 m/s at 5 km inside the valley.

(a) Calculate the atmospheric boundary layer growth in the first 5 km of the valley. Assume that the boundary layer growth to initiate of the start of valley.

(b) Plot the longitudinal profile of the boundary layer thickness.

Comment: Assuming a linear increase in free-stream velocity and a smooth boundary, the momentum integral equation is applied. The velocity distribution is assumed to be a 1/7 power law (smooth turbulent flow). A numerical integration of Equation (4-7) with a spreadsheet may be compared with the analytical solutions for a developing boundary layer on a smooth plate with V_o = 15 & 25 m/s. At the beginning of the valley, the numerical solution must be close to that of a developing boundary layer solution for V_o = 15 m/s.

Exercise No. 3

Turbulent velocity measurements were conducted in a developing boundary layer above a rough surface. The results at x = 0.98 m are listed below.

y mm	$\overline{V_x}$ m/s	$\sqrt{\overline{v_x^2}}/\overline{V_x}$
2	6.76	19.0
3	7.11	18.1
4	7.48	17.0
5	7.79	16.2
6	8.03	15.8
7	8.30	15.4
8	8.49	15.1
9	8.72	14.4
10	8.91	14.1
15	9.70	12.8
20	10.16	13.2
25	10.44	12.5
30	10.53	11.8
40	10.71	11.1
50	10.76	10.5
60	10.81	10.5
80	10.90	9.3
100	11.0	9.2
150	11.07	8.5
200	11.1	7.9
280	11.02	7.3

(a) Plot the vertical distribution of time-averaged velocity and turbulence intensity.

(b) Calculate the boundary layer thickness, the displacement thickness and the momentum thickness.

(c) Deduce the boundary shear stress and shear velocity from the best fit with the law of the wall (i.e. log-law).

(d) From the best fir with the law of the wall, deduced the value of the constant D_2 and the equivalent roughness height of the plate rugosity.

The test were performed in an environmental wind tunnel at atmospheric pressure and ambient temperature.

Solutions:

The analysis of the velocity profile yields:

V_o =	11.05	m/s
δ =	87.4	mm
δ_1 =	6.85	mm
δ_2 =	4.99	mm

Since the velocity profile was measured above a rough plate, the data are compared with Equation (4-20). The slope of the curve $v_x/V_* = f(Ln(y \times V_*/\nu))$ equals $1/K$ where $K = 0.4$ for $V_* = 0.60$ m/s ($\tau_o = 0.43$ Pa). A comparison between the data and Equation (4-20) gives $k_s = 0.8$ mm.

Exercise No. 4

In a water tunnel, turbulent velocity distributions were conducted in a developing boundary layer along a flat plate in absence of pressure gradient.

y	$\overline{V_x}$	$\overline{V_x}$	$\overline{V_x}$
mm	m/s	m/s	m/s
x (m) =	0.25	0.50	0.75
0.5	5.57	5.04	5.34
1	6.03	5.36	5.52
2	6.10	6.57	6.19
3.5	7.30	6.75	5.79
6	8.00	7.09	6.30
8	7.80	7.20	6.83
10.2	7.70	7.52	7.07
14	7.54	7.61	7.20
18	7.65	8.37	7.38
21	7.68	7.56	8.07
35	7.54	7.67	7.62
41	7.69	7.70	7.51
58	7.60	7.69	7.56
60.5	7.67	7.62	7.62
66	7.60	7.70	7.68
70	7.67	7.70	7.60
90	7.56	7.62	7.69
100	7.70	7.63	7.58

(a) Plot the vertical distributions of velocity.

(b) Calculate the boundary layer thickness, displacement thickness and momentum thickness at x = 0.25 m, 0.5 m and 0.75 m.

(c) Using the momentum integral equation, calculate (c1) the boundary shear stress distribution along the plate and (c2) the total force acting on the 0.75 m, 0.5 m wide plate.

Solutions:

x =	m	0.25	0.5	0.75
V_o =	m/s	7.72	7.7	7.71
δ =	mm	5.55	14.26	19.55
δ_1 =	mm	0.764	1.695	2.759
δ_2 =	mm	0.488	1.190	2.0340

The total shear force acting on the 1 m long, 0.5 m wide plate is about 0.9 N.

Exercise No. 5

Let us consider an outfall in the sea. The turbulent water jet is issued from a rectangular nozzle (0.1 m by 2 m). The jet velocity at the nozzle is 2.1 m/s. At the distance x = 0.3 m and 3.5 m, calculate velocity distributions. Plot your results in graph paper.

For seawater, the fluid density, dynamic viscosity and surface tension are respectively: ρ = 1024 kg/^{3}, μ = 1.22 E-3 Pa.s, and σ = 0.076 N/m (Pacifc Ocean waters off Japan).

Assume a plane jet (opening 0.1 m) with a width of 2 m.

Comment: At x/D = 3, the jet is not fully-developed. There is an ideal-fluid flow core with V(y=0) = V_o and two developing free shear layers. At x/D = 35, the jet flow is fully-developed and the centreline velocity equals V(y=0) = 0.95 m/s.

Exercise No. 6

A thin plate (0.7 m wide by 2 m long) is towed through water at a velocity of 1.1 m/s. Calculate the drag force on both sides of the submerged plate assuming that (a) the boundary layer remains laminar, and (b) the boundary layer becomes turbulent at the leading edge.

Solution

The Reynolds number defined in terms of the plate length equals: Re_L = 2.2 E+6. The boundary layer flow at the end of the plate would be expected to be turbulent.

	Laminar boundary layer	Turbulent boundary layer
δ (m) at x = 2 m	0.0066	0.040
Total drag force (N) =	1.5	6.6

Note: The total drag equals the force on both sides of the submerged plate

Exercise No. 7

Considering a wind turbine (Fig. E4-2), the power taken from the wind, assuming no energy losses, is:

$$\text{Power} = 2 \times \pi \times R^2 \times \rho \times V^3 \times \alpha \times (1 - \alpha)^2$$

where R is the turbine radius, ρ is the air density, V is the mean wind speed and α is the interference factor (Fig. E4-2A). As the wind passes through the turbine, it is decelerated down to $V \times (1-\alpha)$ at the turbine disk. The efficiency of the wind turbine is:

$$\eta = 4 \times \alpha \times (1 - \alpha)^2$$

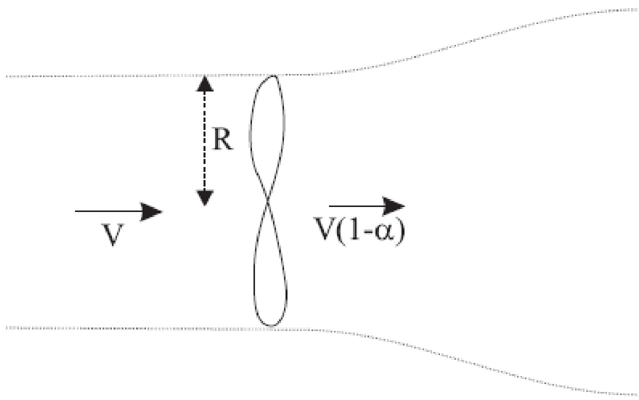
(a) Calculate the maximum efficiency.

(b) A wind turbine is located at 9.4 km from the coastline. The rotor diameter is 45 m. Assuming that the free-surface-stream velocity is 12 m/s, and that the boundary layer develops at the shoreline, calculate the optimum mast elevation to take 900 kW from the wind.

Assume maximum wind turbine efficiency. Assume the main wind direction perpendicular to the coastline.

Fig. E4-2 - Wind turbines

(A) Control volume and definition sketch of the wind flow pas a turbine



(B) Wind turbines in Plouarzel (France) on 1 March 2004 -The wind farm consists of 5 wind turbines of 750 kW - The wind farm is located 2.6 km from the coastline and each mast is 38.4 m high

Solution

(a) $\eta = 0.593$ for $\alpha = 1/3$

(b) $y = 30$ m (calculations performed assuming a smooth turbulent boundary layer and a 1/7-th power law velocity distribution).

Note the approximate nature of the calculations since the velocity distribution is not uniform.

Exercise No. 8

Considering a turbulent free-shear layer (Fig. 4-14), obtain an expression of the variation of the shear stress across the shear layer. *Assume the mixing length hypothesis*.

Comments

The mixing length hypothesis assumes that the momentum exchange coefficient satisfies:

$$\nu_T = l_m^2 \times \frac{\partial V_x}{\partial y}$$

where l_m is the mixing length. The turbulent shear stress equals hence:

$$\tau = \rho \times \nu_T \times \left(\frac{\partial V_x}{\partial y}\right) = \rho \times l_m^2 \times \left(\frac{\partial V_x}{\partial y}\right)^2$$

Exercise No. 9

Considering a two-dimensional turbulent jet, give the expression of the discharge of entrained fluid in the jet flow. Compare the result with a two-dimensional laminar jet.

Comment

Let us remember that the discharge of entrained fluid in the jet flow per unit width equals:

$$\frac{Q}{B} = \int_{y=-\infty}^{+\infty} V_x \times dy$$

Physically, the jet entrains some surrounding fluid as momentum is exchanged from the high-velocity region to the surrounding fluid at rest. The volume discharge Q increases with increasing longitudinal distance x.

Exercise No. 10

The nozzle of a ventilation duct is placed 12 m above the floor of a sport hall and it is directed vertically downward. The outlet discharges 45 m^3/minute. Calculate the diameter of the nozzle if the maximum permissible velocity at a height of 1.5 m above the floor is 1.3 m/s?

Solution

For a circular jet, the length of the developing flow region is about 5 to 10×D where D is the jet diameter. Let us assume a conservative estimate: 10×D. For x > 10×D, the jet flow is full-developed and the maximum velocity is on the jet centreline (Eq. (4-35)).

The basic equations are:

$$Q = V_o \times \frac{\pi}{4} \times D^2 \qquad \text{Continuity equation}$$

$$\frac{V_{max}}{V_o} = \frac{5.745}{\frac{x}{D}} \qquad \text{Centreline velocity, } x/D > 10 \quad (4\text{-}35b)$$

where x = 10.5 m, Q = 45 m^3/minute (0.75 m^3/s) and V_{max} = 1.3 m/s.

The results yield: D = 0.402 m. Let us verify that x > 10×D: x/D = 26.1. Note the calculations are based upon the assumption that the floor has little effect on the jet flow at 1.5 m above it.

Comment

In the fully-developed flow region of circular jet, the jet centreline velocity decreases as 1/x (Table 4-2). Another reasoning may be based upon the following considerations:

$$Q = V_o \times \frac{\pi}{4} \times D^2 \qquad \text{Continuity equation}$$

$$\frac{V_{max}}{V_o} = \frac{10 \times D}{x} \qquad \text{Centreline velocity, } x/D > 10 \quad (4\text{-}35)$$

This simple approximation gives:

$$Q = V_{max} \times \frac{x}{10} \times \frac{\pi}{4} \times D$$

where x = 10.5 m, Q = 45 m^3/minute (0.75 m^3/s) and V_{max} = 1.3 m/s. It yields: D = 0.7 m. Note the difference with the earlier result which was based upon Equation (4-35). Which one would you choose to be conservative?

Exercise No. 11

A Pitot-Prandtl-Preston tube may be used to determine the shear stress at a wall in a turbulent boundary layer (Appendix G). The tube is in contact with the wall and the shear stress is read from a calibration curve between the velocity head and the boundary shear stress. On the basis of the velocity distribution in the inner wall region, justify the Pitot-Prandtl-Preston tube's method.

Solution

When a Pitot tube is lying on the wall (Appendix G), it measures the velocity V_x at a distance y_o from the wall equal to half the Pitot tube outer diameter. Assuming that the Pitot tube is within the inner wall layer, the time-averaged longitudinal velocity satisfies:

$$\frac{V_x}{V_*} = \rho \times \frac{V_* \times y_o}{\mu} \qquad (4\text{-}12b)$$

Replacing the shear velocity V_* by its expression in terms of the boundary shear stress τ_o, it becomes:

$$\tau_o = \mu \times \frac{V_x}{y_o}$$

Practically, Pitot-Prandtl-Preston tubes may be also used in the wall region and in rough turbulent boundary layer with appropriate calibration (Appendix G).

Exercise No. 12

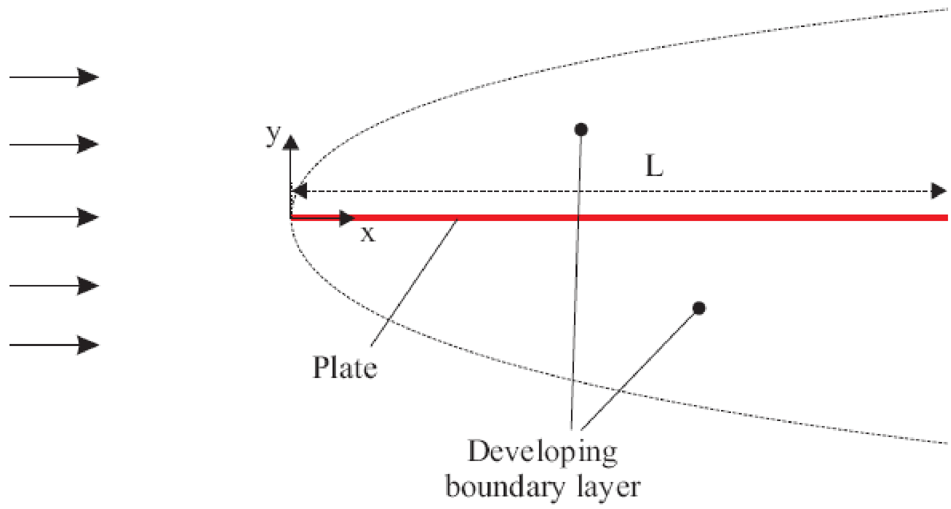
A 2 m long 1.5 m wide flat plate is placed in a water tunnel. The plate acts as a splitter (Fig. E4-3) and the flow is symmetrical around both sides of the plate. Detailed velocity measurements were conducted in the developing boundary layer and the data analysis yields the following velocity profiles:

x = 0.8 m	$V = 14.98 \times (y/0.021)^{0.161}$	$y < 0.021$ m
	V = 14.98 m/s	$y > 0.021$ m
x = 2 m	$V = 15.01 \times (y/0.036)^{0.154}$	$y < 0.036$ m
	V = 15.01 m/s	$y > 0.036$ m

(a) Calculate the boundary shear stress at x = 1.4 m.

(b) Calculate the total drag force on the plate.

Fig. E4-3 - Developing boundary layers on a splitter plate - Note that the boundary layers around the plate are not drawn to scale: the vertical scale is enlarged



Solution

The flow is turbulent since $Re_L = 3$ E+7.

The velocity distributions at x = 0.8 and 2 m follow a power law:

$$\frac{V_x}{V_o} = \left(\frac{y}{\delta}\right)^{1/N} \tag{4-21}$$

Hence the displacement thickness and the momentum thickness are:

$$\frac{\delta_1}{\delta} = \frac{1}{1+N}$$

$$\frac{\delta_2}{\delta} = \frac{N}{(1+N)\times(2+N)}$$

Using the integral momentum equation:

$$\tau_o = \rho \times \left(V_o{}^2 \times \frac{\partial\,\delta_2}{\partial x} + V_o \times \frac{\partial\,V_o}{\partial x} \times (2\times\delta_2 + \delta_1)\right) \tag{4-4}$$

the boundary shear stress equals 857 P and 119 Pa at x = 0.4 and 1.4 m respectively. The total drag force on the plate equals 2.5 kN.

Note that x = 0, $\delta = \delta_1 = \delta_2 = 0$.

Exercise No. 13

Velocity measurements in a developing, turbulent boundary layer along a smooth flat plate yield the data set given in the table below.

(a) On graph paper, plot $Ln(V_x)$ versus Ln(y) at x = 0.4 m.

(b) Estimate the shear velocity and the boundary shear stress at x = 0.4 m.

(c) Using the momentum integral equation, calculate the boundary shear stress and shear force between x = 0.4 and 0.6 m. *Compare your results with the shear velocity estimate and the Blasius formula for smooth turbulent flows. Discuss your findings.*

The fluid is air at 25 Celsius and standard pressure.

x mm	y mm	V_x m/s	v_x' m/s
400	2	8.111	1.33
	4	8.859	1.362
	6	9.68	1.502
	8	10.283	1.476
	10	10.714	1.297
	15	10.756	1.366
	20	10.77	1.426
	30	10.8	1.286
	40	10.799	1.301
	50	10.9	1.03
	70	10.789	0.965
	90	10.98	1.037
	110	10.81	0.923
	130	10.78	0.919
600	2	8.591	1.159
	4	8.508	1.342
	6	9.392	1.249
	8	10.241	1.184
	10	10.327	1.448
	15	10.814	1.368
	20	11.12	1.341
	30	11.23	1.301
	40	11.157	1.102
	50	11.162	1.168
	70	11.219	0.943
	90	11.239	0.969
	110	11.186	1.056
	130	11.359	0.927

Solution

At x = 0.400 m:

$\tau_o = 0.32$ Pa Log law

$\tau_o = 0.34$ Pa Blasius formula

Between x = 0.400 m and 0.600 m:

τ_o = 0.43 Pa Momentum integral equation

Overall the results are close.

Exercise No. 14

The nozzle of a ventilation duct is placed 9.5 m above the floor of the hydraulic laboratory and it is directed vertically downward. The outlet discharges 2,300 m^3/hour.

(a) For a circular duct, calculate the diameter of the nozzle if the maximum permissible velocity at a height of 1.5 m above the floor is 0.95 m/s?

(b) An alternative design uses a wide rectangular duct (10 m long), calculate the opening of the nozzle if the maximum permissible velocity at a height of 1.5 m above the floor is (b1) 0.95 m/s and (b2) 0.50 m/s ? Discuss your results.

Assume a quasi-two-dimensional flow pattern.

The fluid is air at 28 Celsius and standard pressure.

Solution

(a) D = 0.615 m (Circular jet)

(b) D = 0.004 m & 0.0145 m (Two-dimensional nozzle)

A velocity of 0.95 m/s is relatively fast and may induce unpleasant working conditions in the hydraulics laboratory. The preferred design option would be a 0.0145 m thick slot.

Revision exercise

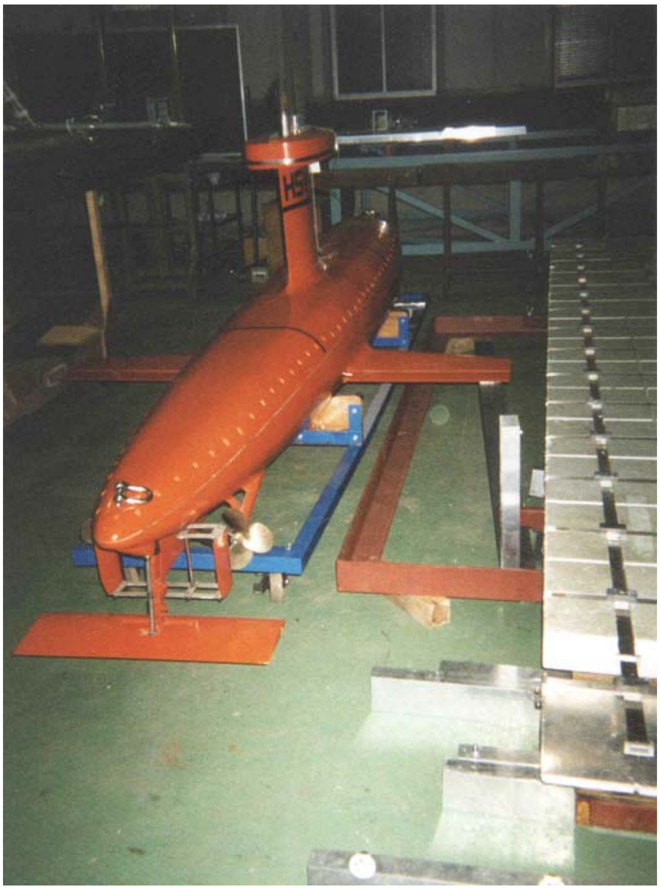
A large towing tank facility is used to conduct drag tests on a 1:40 scale model of a submerged freighter to be used for trans-oceanic shipping (Fig. R-1). The maximum speed of the prototype is expected to be 54 knots. Since it is impossible to achieve simultaneously both Froude and Reynolds similitudes, the tests are conducted at identical Froude number. The magnitude of the corrected surface drag for the prototype will be deduced by means of the boundary layer equations.

The prototype freighter has a total length of 140 m, a length of 32 m at the waterline, and a wetted area of 5,100 m^2. What would be the total drag on the prototype at maximum speed, if the corresponding model drag is 95 N.

For seawater, the fluid density, dynamic viscosity and surface tension are respectively: ρ *= 1024* kg/m^3, μ *= 1.22 E-3 Pa.s, and* σ *= 0.076 N/m.*

The hydraulic model tests are conducted in freshwater.

Fig. R-1 - Photograph of the 1:40 scale model of a submerged freighter tested at Hiroshima University in 2001



Solution: Surface resistance force = 9.3 E+5 N in prototype; Total drag force = 2.7 E+6 N.

Check the required thrust power = 76 MW (prototype freighter).

Remember: Thrust power = Force × V