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14

Physical modelling of hydraulics

14.1 Introduction

Definition: the physical hydraulic model

A physical model is a scaled representation of a hydraulic flow situation. Both the boundary conditions (e.g. channel bed, sidewalls), the upstream flow conditions and the flow field must be scaled in an appropriate manner (Fig. 14.1).

Physical hydraulic models are commonly used during design stages to *optimize* a structure and to ensure a safe operation of the structure. They have an important further role to assist non-engineering people during the 'decision-making' process. A hydraulic model may help the decision-makers to visualize and to picture the flow field, before selecting a 'suitable' design.

In civil engineering applications, a physical hydraulic model is usually a smaller-size representation of the prototype (i.e. the *full-scale* structure) (e.g. Fig. 14.2). Other applications of model studies (e.g. water treatment plant, flotation column) may require the use of models larger than the prototype. In any case the model is investigated in a laboratory under controlled conditions.

Discussion

Hydraulic modelling cannot be disassociated from the basic theory of fluid mechanics. To be efficient and useful, experimental investigations require theoretical guidance which derives primarily from the basic principles (see Chapter 13) and the theory of similarity (see the next subsection).

In the present section, we will consider the physical modelling of hydraulic flows: i.e. the use of laboratory models (with controlled flow conditions) to predict the behaviour of prototype flow situations.

14.2 Basic principles

In a physical model, the flow conditions are said to be similar to those in the prototype if the model displays similarity of form (*geometric similarity*), similarity of motion (*kinematic similarity*) and similarity of forces (*dynamic similarity*).

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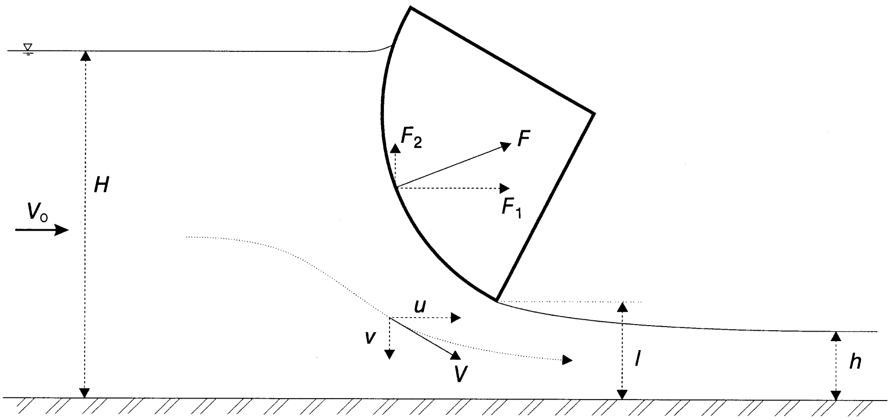

Fig. 14.1 Basic flow parameters.

Basic scale ratios

Geometric similarity implies that the ratios of prototype characteristic lengths to model lengths are equal:

$$L_r = \frac{l_p}{l_m} = \frac{d_p}{d_m} = \frac{H_p}{H_m} \qquad \text{Length} \tag{14.1}$$

where the subscripts p and m refer to prototype (full-scale) and model parameters respectively, and the subscript r indicates the ratio of prototype-to-model quantity. Length, area and volume are the parameters involved in geometric similitude.

Kinematic similarity implies that the ratios of prototype characteristic velocities to model velocities are the same:

$$V_r = \frac{V_p}{V_m} = \frac{(V_1)_p}{(V_1)_m} = \frac{(V_2)_p}{(V_2)_m} \qquad \text{Velocity} \tag{14.2}$$

Dynamic similarity implies that the ratios of prototype forces to model forces are equal:

$$F_r = \frac{F_{1_p}}{F_{1_m}} = \frac{F_{2_p}}{F_{2_m}} \qquad \text{Force} \tag{14.3}$$

Work and power are other parameters involved in dynamic similitude.

Notes

1. Geometric similarity is not enough to ensure that the flow patterns are similar in both model and prototype (i.e. kinematic similarity).
2. The combined geometric and kinematic similarities give the prototype-to-model ratios of time, acceleration, discharge, angular velocity.

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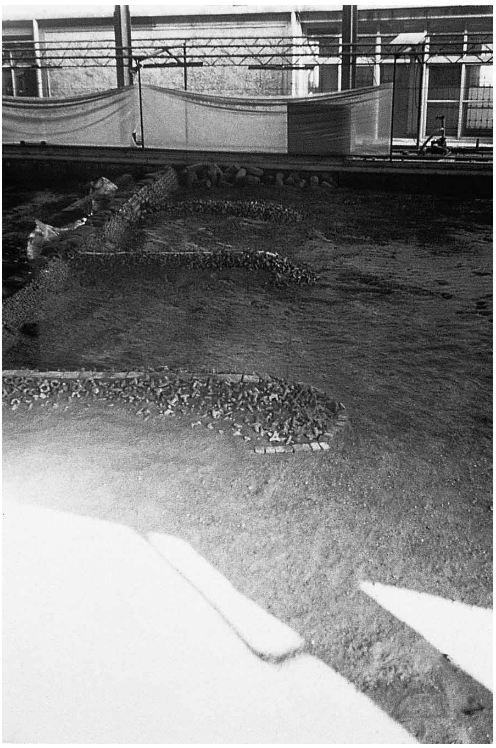
(a)



(b)

Fig. 14.2 Example of physical model: breakwater jetty for the Changhua Reclamation area, along the north-west coastline of Taiwan, Republic of China (January 1994). (a) Prototype breakwater jetty. (b) Model breakwater jetty in a wave flume (Tainan Hydraulic Laboratory).

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Subsequent scale ratios

The quantities L_r, V_r and F_r (defined in equations (14.1) to (14.3)) are the basic scale ratios. Several scale ratios can be deduced from equations (14.1) to (14.3):

$$M_r = \rho_r L_r^3 \qquad \text{Mass} \tag{14.4}$$

$$t_r = \frac{L_r}{V_r} \qquad \text{Time} \tag{14.5}$$

$$Q_r = V_r L_r^2 \qquad \text{Discharge} \tag{14.6}$$

$$P_r = \frac{F_r}{L_r^2} \qquad \text{Pressure} \tag{14.7}$$

where ρ is the fluid density. Further scale ratios may be deduced in particular flow situations.

Application

In open channel flows, the presence of the free-surface means that gravity effects are important. The Froude number $(Fr = V/\sqrt{gL})$ is always significant. Secondary scale ratios can be derived from the constancy of the Froude number[1] which implies:

$$V_r = \sqrt{L_r} \qquad \text{Velocity}$$

Other scale ratios are derived from the Froude similarity (e.g. Henderson 1966):

$$Q_r = V_r L_r^2 = L_r^{5/2} \qquad \text{Discharge}$$

$$F_r = \frac{M_r L_r}{T_r^2} = \rho_r L_r^3 \qquad \text{Force}$$

$$P_r = \frac{F_r}{L_r^2} = \rho_r L_r \qquad \text{Pressure}$$

14.3 Dimensional analysis

14.3.1 Basic parameters

The basic relevant parameters needed for any dimensional analysis (Fig. 14.1) may be grouped into the following groups.

(a) Fluid properties and physical constants (see Appendix A1.1). These consist of the density of water ρ (kg/m^3), the dynamic viscosity of water μ (N s/m^2), the surface tension of air and water σ (N/m), the bulk modulus of elasticity of water E_b (Pa), and the acceleration of gravity g (m/s^2).

[1] It is assumed that the gravity acceleration is identical in both the model and the prototype.

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(b) Channel (or flow) geometry. These may consist of the characteristic length(s) L (m).
(c) Flow properties. These consist of the velocity(ies) V (m/s) and the pressure difference(s) ΔP (Pa).

14.3.2 Dimensional analysis

Taking into account all basic parameters, dimensional analysis yields:

$$\mathscr{F}_1(\rho, \mu, \sigma, E_\mathrm{b}, g, L, V, \Delta P) = 0 \tag{14.8}$$

There are eight basic parameters and the dimensions of these can be grouped into three categories: mass (M), length (L) and time (T). The Buckingham Π-theorem (Buckingham 1915) implies that the quantities can be grouped into five ($5 = 8 - 3$) independent dimensionless parameters:

$$\mathscr{F}_2\left(\frac{V}{\sqrt{gL}}; \frac{\rho V^2}{\Delta P}; \frac{\rho V L}{\mu}; \frac{V}{\sqrt{\dfrac{\sigma}{\rho L}}}; \frac{V}{\sqrt{\dfrac{E_\mathrm{b}}{\rho}}}\right) \tag{14.9a}$$

$$\mathscr{F}_2(Fr; Eu; Re; We; Ma) \tag{14.9b}$$

The first ratio is the Froude number Fr, characterizing the ratio of the inertial force to gravity force. Eu is the Euler number, proportional to the ratio of inertial force to pressure force. The third dimensionless parameter is the Reynolds number Re which characterizes the ratio of inertial force to viscous force. The Weber number We is proportional to the ratio of inertial force to capillary force (i.e. surface tension). The last parameter is the Sarrau–Mach number, characterizing the ratio of inertial force to elasticity force.

Notes

1. The Froude number is used generally for scaling free surface flows, open channels and hydraulic structures. Although the dimensionless number was named after William Froude (1810–1879), several French researchers used it before: e.g. Bélanger (1828), Dupuit (1848), Bresse (1860), Bazin (1865a). Ferdinand Reech (1805–1880) introduced the dimensionless number for testing ships and propellers in 1852, and the number should really be called the Reech–Froude number.
2. Leonhard Euler (1707–1783) was a Swiss mathematician and physicist, and a close friend of Daniel Bernoulli.
3. Osborne Reynolds (1842–1912) was a British physicist and mathematician who expressed first the 'Reynolds number' (Reynolds 1883).
4. The Weber number characterizing the ratio of inertial force over surface tension force was named after Moritz Weber (1871–1951), German Professor at the Polytechnic Institute of Berlin.
5. The Sarrau–Mach number is named after Professor Sarrau who first highlighted the significance of the number (Sarrau 1884) and E. Mach who

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introduced it in 1887. The Sarrau–Mach number was once called the Cauchy number as a tribute to Cauchy's contribution to wave motion analysis.

Discussion

Any combination of the dimensionless numbers involved in equation (14.9) is also dimensionless and may be used to replace one of the combinations. It can be shown that one parameter can be replaced by the Morton number $Mo = (g\mu^4)/(\rho\sigma^3)$, also called the liquid parameter, since:

$$Mo = \frac{We^3}{Fr^2 Re^4} \qquad (14.10)$$

The Morton number is a function only of fluid properties and the gravity constant. For the same fluids (air and water) in both model and prototype, Mo is a constant (i.e. $Mo_p = Mo_m$).

14.3.3 Dynamic similarity

Traditionally model studies are performed using geometrically similar models. In a geometrically similar model, true dynamic similarity is achieved if and only if each dimensionless parameter (or Π-terms) has the same value in both model and prototype:

$$Fr_p = Fr_m; \quad Eu_p = Eu_m; \quad Re_p = Re_m; \quad We_p = We_m; \quad Ma_p = Ma_m \qquad (14.11)$$

Scale effects will exist when one or more Π-terms have different values in the model and prototype.

Practical considerations

In practice, hydraulic model tests are performed under controlled flow conditions. The pressure difference ΔP may usually be controlled. This enables ΔP to be treated as a dependent parameter. Further compressibility effects are small in clear-water flows[2] and the Sarrau–Mach number is usually very small in both model and prototype. Hence, dynamic similarity in most hydraulic models is governed by:

$$\frac{\Delta P}{\rho V^2} = \mathscr{F}_3\left(\frac{V}{\sqrt{gL}}; \frac{\rho V L}{\mu}; \frac{V}{\sqrt{\dfrac{\sigma}{\rho L}}}\right) \qquad (14.12a)$$

$$Eu = \mathscr{F}_3(Fr; Re; We) \qquad \text{Hydraulic model tests} \qquad (14.12b)$$

There are a multitude of phenomena that might be important in hydraulic flow situations: e.g. viscous effects, surface tension, gravity effect. The use of the same fluid on both prototype and model prohibits simultaneously satisfying the Froude, Reynolds and Weber number scaling criteria (equation (14.12)) because the Froude

[2] This statement is not true in air–water flows (e.g. free-surface aerated flows) as the sound celerity may decrease to about 20 m/s for 50% volume air content (e.g. Cain 1978, Chanson 1997).

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number similarity requires $V_r = \sqrt{L_r}$, the Reynolds number scaling implies that $V_r = 1/L_r$ and the Weber number similarity requires: $V_r = 1/\sqrt{L_r}$.

In most cases, only the most dominant mechanism is modelled. Hydraulic models commonly use water and/or air as flowing fluid(s). In *fully-enclosed flows* (e.g. pipe flows), the pressure losses are basically related to the Reynolds number *Re*. Hence, a Reynolds number scaling is used: i.e. the Reynolds number is the same in both model and prototype. In *free-surface flows* (i.e. flows with a free surface), gravity effects are always important and a Froude number modelling is used (i.e. $Fr_m = Fr_p$) (e.g. Fig. 14.2).

Discussion

When inertial and surface tension forces are dominant, a Weber number similarity must be selected. Studies involving air entrainment in flowing waters (i.e. white waters), de-aeration in shaft or bubble plumes are often based upon a Weber number scaling.

The Euler number is used in practice for the scaling of models using air rather than water: e.g. hydraulic models in wind tunnels, or a manifold system with water flow which is scaled at a smaller size with an air flow system.

14.3.4 Scale effects

Scale effects may be defined as the distortions introduced by effects (e.g. viscosity, surface tension) other than the dominant one (e.g. gravity in free-surface flows). They take place when one or more dimensionless parameters (see Section 14.3.3) differ between model and prototype.

Scale effects are often small but they are not always negligible altogether. Considering an overflow above a weir, the fluid is subjected to some viscous resistance along the invert. However the flow above the crest is not significantly affected by resistance, the viscous effects are small and the discharge–head relationship can be deduced as for ideal-fluid flow.

In free-surface flows, the gravity effect is dominant. If the same fluid (i.e. water) is used in both the model and the prototype, it is impossible to keep both the Froude and Reynolds numbers in the model and full-scale. Indeed it is elementary to show that a Froude similitude implies $(Re)_r = L_r^{3/2}$, and the Reynolds number becomes much smaller in the model than in the prototype (if $L_r < 1$).

Note that different fluids may be used to have the same Reynolds and Froude numbers in both the model and prototype, but this expedient is often not practical nor economical.

Some examples of scale effects

Example No. 1

Considering the drag exerted on two-dimensional bodies, Fig. 14.3 shows the effects of the Reynolds number on the drag coefficient. Dynamic similarity

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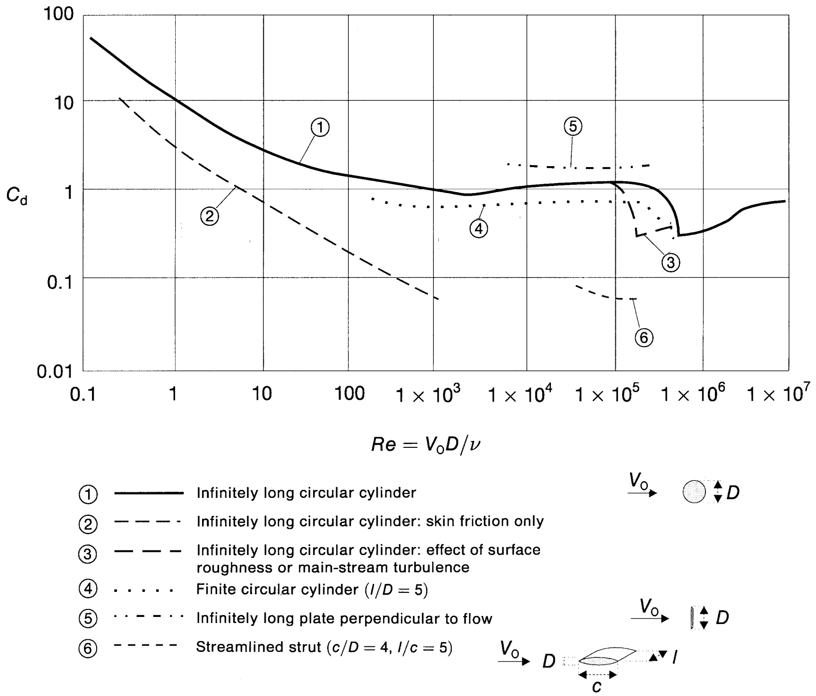

Fig. 14.3 Drag coefficient on two-dimensional bodies.

(equation (14.12)) requires the drag coefficient to be the same in the model and prototype. If the Reynolds number is smaller in the model than at full-scale (in most practical cases), Fig. 14.3 suggests that the model drag coefficient would be larger than that of the prototype and dynamic similarity could not be achieved. Moreover, the drag force comprises the form drag and the surface drag (i.e. skin friction). In small-size models, the surface drag might become predominant, particularly if the model flow is not fully-rough turbulent or the geometrical scaling of roughness height is not achievable.

In practice, an important rule, in model studies is that the model Reynolds number Re_m should be kept as large as possible, so that the model flow is fully-rough turbulent (if prototype flow conditions are fully-rough turbulent).

Example No. 2

Another example is the effect of the corner radius on the drag force on two-dimensional bodies (Fig. 14.4). Figure 14.4 shows significant differences in the Reynolds number–drag coefficient relationships depending upon the relative radius r/D. When the corner radius on the prototype is small and L_r is large, it is impossible to have the same ratio of corner radius to body size in the model and prototype because the model cannot be manufactured with the required accuracy. In such cases, the drag force is not scaled adequately.

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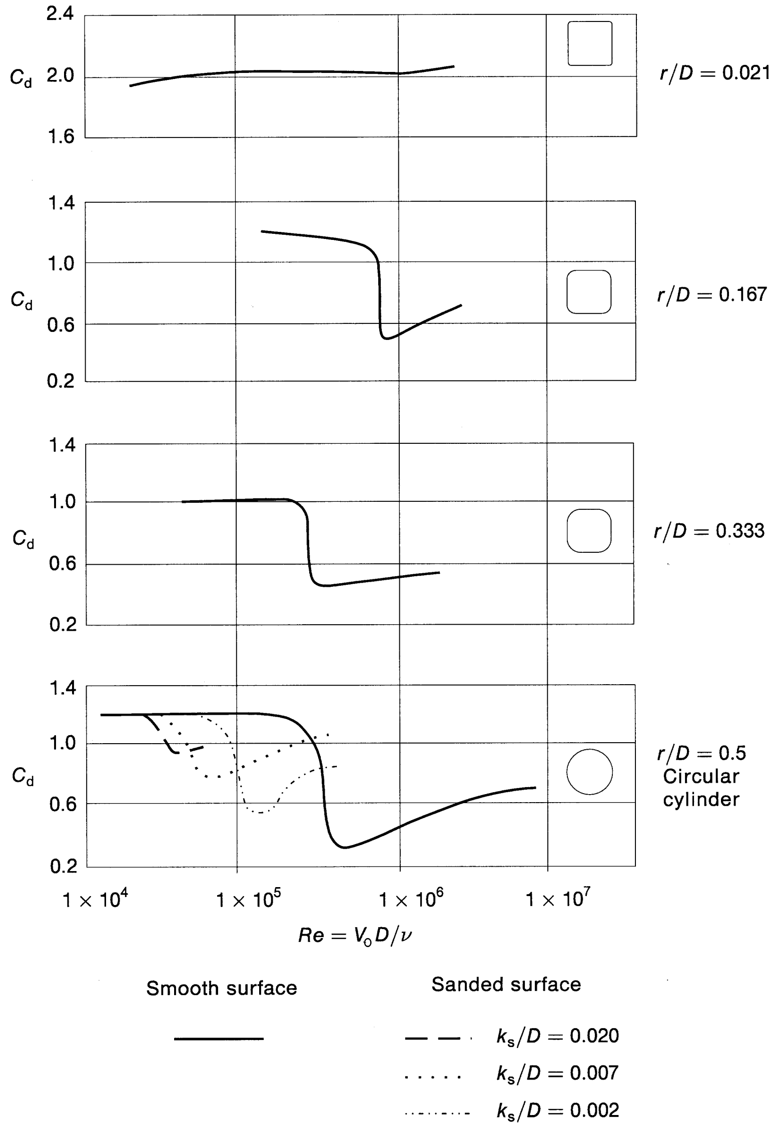

Fig. 14.4 Effect of corner radius and surface roughness on the drag coefficient of two-dimensional bodies.

Example No. 3

A different example is the flow resistance of bridge piers. Henderson (1966) showed that the resistance to flow of normal bridge pier shapes is such that the drag coefficient is about or over unity, implying that the form drag is a significant component of the total drag. In such a case, the viscous effects are relatively small, and dynamic similarity is achievable, provided that model viscous effects remain negligible.

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Discussion

If scale effects would become significant in a model, a smaller prototype-to-model scale ratio L_r should be considered to minimize the scale effects. For example, in a 100:1 scale model of an open channel, the gravity effect is predominant but viscous effects might be significant. A geometric scale ratio of 50:1 or 25:1 may be considered to reduce or eliminate viscous scale effects.

Another example is the entrainment of air bubbles in free-surface flows. Gravity effects are predominant but it is recognized that surface tension scale effects can take place for $L_r > 10$ to 20 (or $L_r < 0.05$ to 0.1) (e.g. Wood 1991, Chanson 1997).

At the limit, no scale effect is observed at full-scale (i.e. $L_r = 1$) as all the Π-terms (equation (14.11)) have the same values in the prototype and model when $L_r = 1$.

14.4 Modelling fully-enclosed flows

14.4.1 Reynolds models

Fully-enclosed flow situations include pipe flows, turbomachines and valves. For such flow situations, viscosity effects on the solid boundaries are important. Physical modelling is usually performed with a Reynolds similitude: i.e. the Reynolds number is kept identical in both the model and prototype:

$$Re_p = Re_m \tag{14.13}$$

If the same fluid is used in both the model and prototype, equation (14.13) implies:

$$V_r = 1/L_r \qquad \text{(Reynolds similitude)}$$

For $L_r > 1$, the model velocity must be larger than that in the prototype.

Discussion

For example, if the model scale is 10:1 (i.e. $L_r = 10$), the velocity in the model must be ten times that in the prototype. By using a different fluid in the model, the ratio (μ_r/ρ_r) becomes different from unity and V_m can be reduced.

14.4.2 Discussion

Flow resistance in pipe flows

For pipe flows, the Darcy equation relates the pressure losses to the pipe geometry (diameter D, length L) and to the flow velocity V:

$$\Delta P = f \frac{L}{D} \frac{\rho V^2}{2} \tag{14.14}$$

where f is the Darcy–Weisbach friction factor. After transformation and combining with equation (14.10), it leads:

$$\frac{fL}{2D} = Eu = \mathscr{F}_4(Fr; Re; We; Ma; \ldots) \tag{14.15}$$

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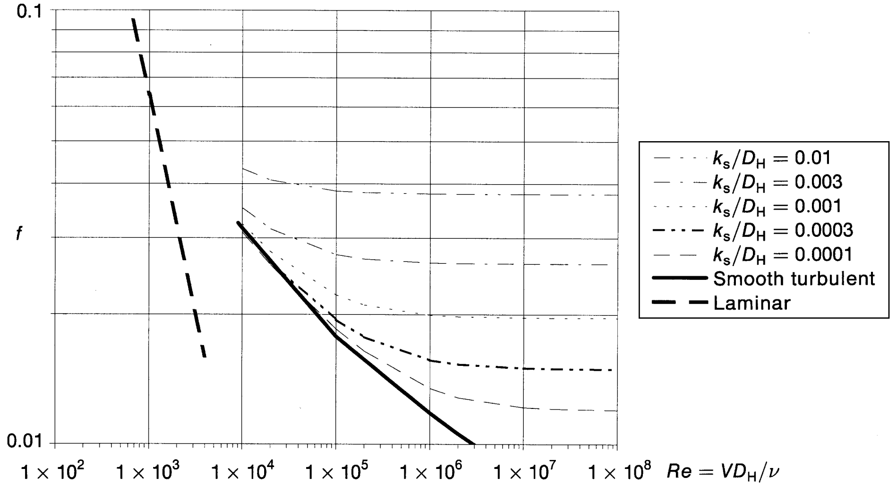

Fig. 14.5 Friction factor versus Reynolds number in pipe flows.

In pipe flows, gravity and surface tension have no effect on the pressure losses. For steady liquid flows, the compressibility effects are negligible. The roughness height k_s is, however, an additional characteristic length. For a uniformly distributed-roughness, equation (14.15) becomes:

$$\frac{fL}{2D} = \mathcal{F}_4\left(Re; \frac{k_s}{D}\right) \tag{14.16}$$

Equation (14.16) expresses the dimensionless relationship between friction losses in pipes, the Reynolds number and relative roughness. An illustration is the Moody diagram (Fig. 14.5).

Skin friction and form drag

Considering the drag on a body (e.g. Figs 14.3 and 14.4), the pressure losses associated with the modification of the flow field caused by the presence of the body are usually expressed in terms of the drag force F_d on the body. The Euler number is rewritten as: $Eu = F_d/(\rho V^2 A)$, where A is the projection of the body in the plane normal to the flow direction. F_d/A is equivalent to the pressure loss ΔP.

Equations (14.10) and (14.15) may be combined to relate the drag coefficient C_d to the Π-terms:

$$C_d = \frac{Eu}{2} = \frac{F_d}{\frac{\rho}{2} V^2 A} = \mathcal{F}_5(Fr; Re; We; Ma; \ldots) \tag{14.17}$$

In equation (14.17), the Reynolds number Re is related to the *skin friction drag* due to viscous shear as well as to *form drag* resulting from the separation of the flow streamlines from the body.

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14.4.3 Practical considerations in modelling fully-enclosed flows

The flow regime in pipes is either laminar or turbulent. In industrial applications, it is commonly accepted that the flow becomes turbulent for Reynolds numbers larger than 2000 to 3000, the Reynolds number being defined in terms of the equivalent pipe diameter D and of the mean flow velocity V.

For turbulent flows, the flow regime can be sub-divided into three categories: smooth, transition and fully rough. Each category can be distinguished as a function of a shear Reynolds number defined as:

$$Re_* = \rho \frac{V_* k_s}{\mu} \tag{14.18}$$

where V_* is the shear velocity. The transition between smooth turbulence and fully-rough turbulence is approximately defined as:

Flow situation Ref.	Open channel flow (Henderson 1966)	Pipe flow (Schlichting 1979)
Smooth turbulent	$Re_* < 4$	$Re_* < 5$
Transition	$4 < Re_* < 100$	$5 < Re_* < 75$
Fully rough turbulent	$100 < Re_*$	$75 < Re_*$

Dynamic similarity of fully-enclosed flows implies the same resistance coefficient in both the model and the prototype. This can be achieved with the Reynolds number being the same in the model and prototype, or with both flows in the model and prototype being fully-rough turbulent (Fig. 14.5).

If the full-scale flow is turbulent, it is extremely important to ensure that the model flow is also turbulent. Laminar and turbulent flows exhibit very important basic differences. In most cases, turbulent flows should not be scaled with laminar flow models.

Note
The Reynolds number can be kept constant by changing the flowing fluid. For example the atmospheric wind flow past a tall building could be modelled in a small-size water tunnel at the same Reynolds number.

14.5 Modelling free-surface flows

14.5.1 Presentation

In free-surface flows (e.g. rivers, wave motion), gravity effects are predominant. Model-prototype similarity is performed usually with a Froude similitude:

$$Fr_p = Fr_m \tag{14.19}$$

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If the gravity acceleration is the same in both the model and prototype, a Froude number modelling implies:

$$V_r = \sqrt{L_r} \qquad \text{(Froude similitude)}$$

Note that the model velocity is less than that in the prototype for $L_r > 1$ and the time scale equals $t_r = \sqrt{L_r}$.

Remarks

Froude number modelling is typically used when friction losses are small and the flow is highly turbulent: e.g. spillways, overflow weirs, flow past bridge piers. It is also used in studies involving large waves: e.g. breakwater or ship models.[3]

A main concern is the potential for scale effects induced by viscous forces. Scale effects caused by surface tension effects are another concern, in particular when free-surface aeration (i.e. air entrainment) takes place.

14.5.2 Modelling hydraulic structures and wave motion

In hydraulic structures and for wave motion studies (Fig. 14.2), the gravity effect is usually predominant in the prototype. The flow is turbulent, and hence viscous and surface tension effects are negligible in the prototype if the flow velocity is reasonably small. In such cases a Froude similitude must be selected.

The most economical strategy is:

1. to choose a geometric scale ratio L_r such as to keep the model dimensions small, and
2. to ensure that the model Reynolds number Re_m is large enough to make the flow turbulent at the smallest test flows.

14.5.3 Modelling rivers and flood plains

In river modelling, gravity effects and viscous effects are basically of the same order of magnitude. For example, in uniform equilibrium flows (i.e. normal flows), the gravity force component counterbalances exactly the flow resistance and the flow conditions are deduced from the continuity and momentum equations.

In practice, river models are scaled with a Froude similitude (equation (14.19)) and viscous scale effects must be minimized. The model flow must be turbulent, and possibly fully-rough turbulent with the same relative roughness as for the prototype:

$$Re_m > 5000 \qquad (14.20)$$

$$(k_s)_r = L_r \qquad (14.21)$$

[3] The testing of ship models is very specialized. Interestingly, F. Reech and W. Froude were among the first to use the Froude similitude for ship modelling.

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where the Reynolds number is defined in terms of the hydraulic diameter (i.e. $Re = \rho V D_H / \mu$).

Distorted models

A distorted model is a physical model in which the geometric scale is different between each main direction. For example, river models are usually designed with a larger scaling ratio in the horizontal directions than in the vertical direction: $X_r > Z_r$. The scale distortion does not distort seriously the flow pattern and it usually gives good results.

A classical example of a distorted model is that of the Mississippi river, built by the US Army Corps of Engineers. The Mississippi basin is about 3 100 000 km^2 and the river is nearly 3800 km long. An outdoor model was built with a scale of 2000:1. If the same scaling ratio was applied to both the vertical and horizontal dimensions, prototype depths of about 6 m would imply model depths of about 3 mm. With such small flow depths, surface tension and viscous effects would be significant. The Mississippi model was built, in fact, with a distorted scale: $Z_r = 100$ and $X_r = 2000$. Altogether the model size is about 1.5 km per 2 km!

A distorted model of rivers is designed with a Froude similitude:

$$Fr_p = Fr_m \tag{14.19}$$

where the Froude number scaling ratio is related to the vertical scale ratio:

$$Fr_r = \frac{V_r}{\sqrt{Z_r}} \tag{14.22}$$

As for an undistorted model, the distorted model flow must be turbulent (equation (14.20)), and preferably fully-rough turbulent with the same relative roughness as for the prototype:

$$(k_s)_r = Z_r \tag{14.23}$$

The Froude similitude (equation (14.22)) implies:

$$V_r = \sqrt{Z_r} \qquad \text{Velocity} \tag{14.24}$$

$$Q_r = V_r X_r Z_r = Z_r^{3/2} X_r \qquad \text{Discharge} \tag{14.25}$$

$$t_r = \frac{X_r}{V_r} = \frac{X_r}{\sqrt{Z_r}} \qquad \text{Time} \tag{14.26}$$

$$(\tan\theta)_r = \frac{Z_r}{X_r} \qquad \text{Longitudinal bed slope} \tag{14.27}$$

where θ is the angle between the channel bed and the horizontal.

With a distorted scale model, it is possible to select small physical models (i.e. X_r large). In addition to the economical and practical benefits, distorted models also have the following advantages compared with non-distorted models:

- the flow velocities and turbulence in the model are larger (equation (14.24)),
- the time scale is reduced (equation (14.26)),
- the model Reynolds number is larger, improving the prototype-to-model dynamic similarity, and

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- the larger vertical scale (i.e. $Z_r < X_r$) allows a greater accuracy on the flow depth measurements.

Discussion

Practically it is recommended that the model distortion (i.e. the ratio X_r/Z_r) should be less than 5 to 10. Some disadvantages of distorted models may be mentioned for completeness: the velocity directions are not always reproduced correctly, and some observers might be distracted unfavourably by the model distortion leading to inaccurate or incorrect judgements.

Movable-bed models

Movable-bed hydraulic models are some of the most difficult types of models and they often give unsatisfactory results.

The primary difficulty is to scale both the sediment movement and the fluid motion. Furthermore, the bed roughness becomes a function of the bed geometry and of the sediment transport. Early movable bed model studies on the River Mersey (England) and Seine River (France) in the 1880s showed that the time scale governing the fluid flow differs from the time scale governing sediment motion (see Appendix A3.1).

A detailed analysis of sediment transport modelling is developed in Appendix A3.1. Several authors (e.g. Henderson 1996, pp. 497–508, Graf 1971, pp. 392–398) also discussed various methods for 'designing' a movable-bed model.

The most important point is the need to verify and to calibrate a movable-bed model before using it as a prediction tool.

14.5.4 Resistance scaling

The modelling of flow resistance is not a simple matter. Often the geometric similarity of roughness height and spacing is not enough. For example, it is observed sometimes that the model does not reproduce the flow patterns in the prototype because the model is too 'smooth' or too 'rough'. In some cases (particularly with a large scale ratio L_r), the model flow is not as turbulent as the prototype flow. A solution is to use roughness elements (e.g. mesh, wire, vertical rods) to enhance the model flow turbulence, hence to simulate more satisfactorily the prototype flow pattern.

Another aspect is the scaling of the resistance coefficient. The flow resistance can be described in terms of the Darcy friction factor or an empirical resistance coefficient (e.g. Chézy or Gauckler–Manning coefficients).

In uniform equilibrium flows, the momentum equation implies:

$$V_r = \sqrt{L_r} = \sqrt{\frac{(D_H)_r(\sin\theta)_r}{f_r}} \tag{14.28}$$

For an undistorted model, a Froude similitude (equation (14.19) and (14.28)) implies that the model flow resistance will be similar to that in the prototype:

$$f_r = 1 \tag{14.29}$$

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Most prototype flows are fully-rough turbulent and the Darcy friction factor is primarily a function of the relative roughness.

Another approach is based upon the Gauckler–Manning coefficient. The Chézy equation implies that, in gradually-varied and uniform equilibrium flows, the following scaling relationship holds:

$$V_r = \sqrt{L_r} = \frac{1}{(n_{\text{Manning}})_r}((D_H)_r)^{2/3}\sqrt{(\sin\theta)_r} \tag{14.30}$$

For an undistorted scale model, equation (14.30) becomes:

$$(n_{\text{Manning}})_r = L_r^{1/6} \tag{14.31}$$

Equation (14.31) indicates that the notion of complete similarity is applied both to the texture of the surface and to the shape of its general outline (Henderson 1966). In practice, the lowest achievable value of n_{Manning} is about 0.009 to 0.010 s/m$^{1/3}$ (i.e. for glass). With such a value, the prototype resistance coefficient $(n_{\text{Manning}})_p$ and the Gauckler–Manning coefficient similarity $(n_{\text{Manning}})_r$ could limit the maximum geometrical similarity ratio L_r. If L_r is too small (typically less than 40), the physical model might not be economical nor convenient.

In summary, a physical model (based upon a Froude similitude) has proportionally more resistance than the prototype. If the resistance losses are small (e.g. at a weir crest), the resistance scale effects are not considered. In the cases of river and harbour modelling, resistance is significant. The matter may be solved using distorted models.

Distorted models

With a distorted scale model, equations (14.28) and (14.30) become respectively:

$$V_r = \sqrt{Z_r} = \sqrt{\frac{(D_H)_r(\sin\theta)_r}{f_r}} \tag{14.32}$$

$$V_r = \sqrt{Z_r} = \frac{1}{(n_{\text{Manning}})_r}((D_H)_r)^{2/3}\sqrt{(\sin\theta)_r} \tag{14.33}$$

For a wide channel (i.e. $(D_H)_r = Z_r$) and a flat slope (i.e. $(\sin\theta)_r = (\tan\theta)_r$), the scaling of flow resistance in distorted models implies:

$$f_r = \frac{Z_r}{X_r} \quad \text{wide channel and flat slope} \tag{14.34}$$

$$(n_{\text{Manning}})_r = \frac{Z_r^{2/3}}{\sqrt{X_r}} \quad \text{wide channel and flat slope} \tag{14.35}$$

Discussion

In practice $Z_r/X_r < 1$ and equation (14.34) would predict a model friction factor lower than that in the prototype. But equation (14.35) could imply a model

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resistance coefficient larger or smaller than that in the prototype depending upon the ratio $Z_r^{2/3}/\sqrt{X_r}$!

14.6 Design of physical models

14.6.1 Introduction

Before building a physical model, engineers must have the appropriate topographic and hydrological field information. The type of model must then be selected, and a question arises:

Which is the dominant effect: e.g. viscosity, gravity or surface tension?

14.6.2 General case

In the general case, the engineer must choose a proper geometric scale. The selection procedure is an iterative process.

Step 1. Select the smallest geometric scale ratio L_r to fit within the constraints of the laboratory.

Step 2. For L_r, and for the similitude criterion (e.g. Froude or Reynolds), check if the discharge can be scaled properly in the model, based upon the maximum model discharge $(Q_m)_{max}$.

For L_r and the similitude criterion, is the maximum model discharge large enough to model the prototype flow conditions?

Step 3. Check if the flow resistance scaling is achievable in the model.

Is it possible to achieve the required f_m or $(n_{Manning})_m$ in the model?

Step 4. Check the model Reynolds number Re_m for the smallest test flow rate.

For $(Q_p)_{min}$, what are the flow conditions in the model: e.g. laminar or turbulent, smooth-turbulent or fully-rough-turbulent? If the prototype flow is turbulent, model flow conditions must be turbulent (i.e. typically $Re_m > 5000$).

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Step 5. Choose the convenient scale.

When a simple physical model is not feasible, more advanced modelling techniques can be used: e.g. a two-dimensional model (e.g. spillway flow), a distorted scale model (e.g. river flow).

14.6.3 Distorted scale models

For a distorted scale model, the engineer must select two (or three) geometric scales. The model design procedure is again an iterative process:

Step 1. Select the smallest horizontal scale ratio X_r to fit within the constraints of the laboratory.

Step 2. Determine the possible range of vertical scale Z_r such as:
+ the smallest scale $(Z_r)_1$ is that which gives the limit of the discharge scaling ratio, based upon the maximum model discharge $(Q_m)_{max}$,
+ the largest scale $(Z_r)_2$ is that which gives the feasible flow resistance coefficient (i.e. feasible f_m or $(n_{Manning})_m$), and
+ check the distortion ratio X_r/Z_r.
(X_r/Z_r should be less than 5 to 10.)

Step 3. Check the model Reynolds number Re_m for the smallest test flow rate. This might provide a new largest vertical scale ratio $(Z_r)_3$.
+ check the distortion ratio X_r/Z_r.

Step 4. Select a vertical scale ratio which satisfies: $(Z_r)_1 < Z_r < \text{Min}[(Z_r)_2, (Z_r)_3]$. If this condition cannot be satisfied, a smaller horizontal scale ratio must be chosen.
+ check the distortion ratio X_r/Z_r.
In practice it is recommended that X_r/Z_r should be less than 5 to 10.

Step 5. Choose the convenient scales (X_r, Z_r).

14.7 Summary

Physical hydraulic modelling is a design technique used by engineers to optimize the structure design, to ensure the safe operation of the structure and/or to facilitate the decision-making process.

In practice, most hydraulic models are scaled with either a Froude or a Reynolds similitude: i.e. the selected dimensionless number is the same in the model and in the prototype (i.e. full-scale).

The most common fluids are air and water. Free-surface flow modelling is most often performed with the same fluid (e.g. water) in full-scale and the model. Fully-enclosed flow modelling might be performed with water in the prototype and air in the model. The selection of fluid in the model and the prototype fixes the density scale ratio ρ_r.

Table 14.1 summarizes the scaling ratios for the Froude and Reynolds similitudes.

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Table 14.1 Scaling ratios for Froude and Reynolds similitudes (undistorted model)

Parameter	Unit	Scale ratio with		
		Froude law[a]	Froude law[a] (distorted model)	Reynolds law
(1)	(2)	(3)	(4)	(5)
Geometric properties				
Length	m	L_r	X_r, Z_r	L_r
Area	m^2	L_r^2	–	L_r^2
Kinematic properties				
Velocity	m/s	$\sqrt{L_r}$	$\sqrt{Z_r}$	$1/L_r \times \mu_r/\rho_r$
Discharge per unit width	m^2/s	$L_r^{3/2}$	$\sqrt{Z_r}X_r$	μ_r/ρ_r
Discharge	m^3/s	$L_r^{5/2}$	$Z_r^{3/2}X_r$	$L_r\mu_r/\rho_r$
Time	s	$\sqrt{L_r}$	$X_r/\sqrt{Z_r}$	$L_r^2\rho_r/\mu_r$
Dynamic properties				
Force	N	$\rho_r L_r^3$	–	μ_r^2/ρ_r
Pressure	Pa	$\rho_r L_r$	$\rho_r Z_r$	$\mu_r^2/\rho_r \times 1/L_r^2$
Density	kg/m^3	ρ_r	ρ_r	ρ_r
Dynamic viscosity	Pa s	$L_r^{3/2}\sqrt{\rho_r}$	–	μ_r
Surface tension	N/m	L_r^2	–	$\mu_r^2/\rho_r \times 1/L_r$

Note: [a] assuming identical gravity acceleration in model and prototype.

14.8 Exercises

Numerical solutions to some of these exercises are available from the Web at www.arnoldpublishers.com/support/chanson

A butterfly valve is to be tested in a laboratory to determine the discharge coefficient for various openings of the disc. The prototype size will be 2.2 m in diameter and it will be manufactured from cast steel with machined inside surfaces (roughness height estimated to be about 0.5 mm). The maximum discharge to be controlled by the valve is 15 m^3/s. The laboratory model is a 5:1 scale model.

(a) What surface condition is required in the model? What model discharge is required to achieve complete similarity with the prototype, if water is used in both? (b) Can these conditions be achieved? (c) If the maximum flow available for model tests is 200 L/s, could you accurately predict prototype discharge coefficients from the results of the model tests?

Summary sheet:

(a) $(k_s)_m$ =	Q_m =
(b) Yes/No	Reasons:
(c) Re_p =	Re_m
Discussion:	

The inlet of a Francis turbine is to be tested in a laboratory to determine the performances for various discharges. The prototype size of the radial flow rotor will be:

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inlet diameter = 0.6 m, width = 0.08 m, inlet crossflow area = $\pi \times$ diameter $\times$ width. It will be manufactured from cast steel with machined inside surfaces (roughness height estimated to be about 0.3 mm). The maximum discharge to be turbined (by the Francis wheel) is 1.4 m^3/s. The laboratory model is a 5:1 scale model.

(a) What surface condition is required in the model? What model discharge is required to achieve complete similarity with the prototype, if water is used in both?
(b) Can these conditions be achieved? (Compute the minimum required model total head and flow rate. Compare these with the pump performances of a typical hydraulic laboratory: $H \sim 10$ m, $Q \sim 100$ L/s.)
(c) If the maximum flow available for model tests is 150 L/s, would you be able to predict accurately prototype performances from the results of the model tests? (Justify your answer.)

Summary sheet:

(a) $(k_s)_m =$	$Q_m =$
(b) Yes/No	Reasons:
(c) $Re_p =$	Re_m
Discussion:	

An overflow spillway is to be designed with an uncontrolled crest followed by a stepped chute and a hydraulic jump dissipator. The maximum spillway capacity will be 4300 m^3/s. The width of the crest, chute and dissipation basin will be 55 m. A 50:1 scale model of the spillway is to be built. Discharges ranging between the maximum flow rate and 10% of the maximum flow rate are to be reproduced in the model.

(a) Determine the maximum model discharge required. (b) Determine the minimum prototype discharge for which negligible scale effects occur in the model. (Comment on your result.) (c) What will be the scale for the force ratio?

Laboratory tests indicate that operation of the basin may result in unsteady wave propagation downstream of the stilling basin with a model wave amplitude of about 0.05 m and model wave period of 47 seconds. Calculate: (d) the prototype wave amplitude and (e) the prototype wave period.

Summary sheet:

(a) Maximum $Q_m =$	
(b) Minimum $Q_p =$	Why?
(c) $\text{Force}_r =$	
(d) $A_p =$	
(e) $t_p =$	

A 35.5:1 scale model of a concrete overfall spillway and stilling basin is to be built. The prototype discharge will be 200 m^3/s and the spillway crest length is 62 m. (a) Determine the maximum model discharge required and the minimum prototype discharge for

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which negligible scale effects occur in the model. (b) In tests involving baffle blocks for stabilizing the hydraulic jump in the stilling basin, the force measured on each block was 9.3 N. What is the corresponding prototype force? (c) The channel downstream of the stilling basin is to be lined with rip-rap (angular blocks of rock) approximately 650 mm in size. The velocity measured near the rip-rap is as low as 0.2 m/s. Check whether the model Reynolds number is large enough for the drag coefficient of the model rocks to be the same as in the prototype. What will be the scale for the force ratio?

Summary sheet:

(a) Maximum Q_m =	
Minimum Q_p =	Why?
(b) Force_p =	
(c) Re_m =	Comment:
Force ratio =	

A sluice gate will be built across a 25 m wide rectangular channel. The maximum prototype discharge will be 275 m^3/s and the channel bed will be horizontal and concrete-lined (i.e. smooth). A 35:1 scale model of the gate is to be built for laboratory tests.

(a) What similitude should be used? Calculate: (b) model width and (c) maximum model flow rate.

For one particular gate opening and flow rate, the *laboratory* flow conditions are: upstream flow depth of 0.2856 m, downstream flow depth of 0.0233 m. (d) Compute the model discharge. State the basic principle(s) involved. (e) Compute the model force acting on the sluice gate. State the basic principle(s) involved. (f) What will be the corresponding prototype discharge and force on the gate? (g) What will be the scale for the force ratio?

Gate operation may result in unsteady flow situations. If a prototype gate operation has the following characteristics: gate opening duration = 15 minutes, initial discharge = 180 m^3/s, new discharge = 275 m^3/s, calculate: (h) gate opening duration and (i) discharges to be used in the model tests.

Summary sheet:

(a) Similitude:	Why?
(b) B_m =	
(c) $(Q_m)_{max}$ =	
(d) Q_m =	Principle(s):
(e) F_m =	Principle(s):
(f) Q_p =	
Force_p =	
(g) Force_r =	
(h) t_m =	
(i) Q_m = (before)	Q_m = (after)

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A hydraulic jump stilling basin, equipped with baffle blocks, is to be tested in a laboratory to determine the dissipation characteristics for various flow rates. The maximum prototype discharge will be 220 m^3/s and the rectangular channel will be 10 m wide. (Assume the channel bed to be horizontal and concrete-lined, i.e. smooth.) A 40:1 scale model of the stilling basin is to be built. Discharges ranging between the maximum flow rate and 10% of the maximum flow rate are to be reproduced in the model.

(a) What similitude should be used? (Justify your selection.) (b) Determine the maximum model discharge required. (c) Determine the minimum prototype discharge for which negligible scale effects occur in the model. (Comment on your result.)

For one particular inflow condition, the *laboratory* flow conditions are: upstream flow depth of 0.019 m, upstream flow velocity of 2.38 m/s, downstream flow depth of 0.122 m. (d) Compute the model force exerted on the baffle blocks. (State the basic principle(s) involved.) (e) What is the direction of force in (d): i.e. upstream or downstream? (f) What will be the corresponding prototype force acting on the blocks? (g) Compute the prototype head loss.

Operation of the basin may result in unsteady wave propagation downstream of the stilling basin. (h) What will be the scale for the time ratio?

Tests will be made on a model sea wall of 1/18 prototype size. (a) If the prototype wave climate is: wave period = 12 seconds, wave length = 20 m, wave amplitude = 2.1 m, what wave period, wave length and wave amplitude should be used in the model tests? (b) If the maximum force exerted by a wave on the model sea wall is 95 N, what corresponding force will be exerted on the prototype?

Summary sheet:

(a) t_m =	L_m =
A_m =	
(b) $Force_p$ =	

A fixed bed model is to be built of a certain section of a river. The maximum full-scale discharge is 2750 m^3/s, the average width of the river is 220 m and the bed slope is 0.16 m per kilometre. The Gauckler–Manning coefficient for the prototype is estimated at 0.035 $s/m^{1/3}$. Laboratory facilities limit the scale ratio to 200:1 and maximum model discharge is 45 L/s. Note that the smoothest model surface feasible has a Gauckler–Manning coefficient of about 0.014 $s/m^{1/3}$. Discharges ranging between the maximum flow rate and 15% of the maximum flow rate are to be reproduced in the model.

Determine the acceptable maximum and minimum values of the vertical scale ratio Z_r. Select a suitable scale for practical use, and calculate the corresponding model values of the Gauckler–Manning coefficient, maximum discharge and normal depth (at maximum discharge). (It can be assumed that the river channel is wide (i.e. $D_H \sim 4d$) in both the model and prototype for all flows. Assume that uniform equilibrium flow is achieved in the model and prototype.)

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Summary sheet:

Minimum Z_r =	Why?
Maximum Z_r =	Why?
Alternative Max. Z_r =	Why?
Allowable range for Z_r =	
Your choice for Z_r =	Why?
Corresponding values of Q_r =	$(n_{\text{Manning}})_r$ =
$(n_{\text{Manning}})_r$ =	Maximum Q_m =
d_m =	at maximum flow rate

An artificial concrete channel model is to be built. Laboratory facilities limit the scale ratio to 50:1 and the maximum model discharge is 50 L/s. The maximum full-scale discharge is 150 m^3/s, the cross-section of the channel is approximately rectangular (50 m bottom width) and the bed slope is 0.14 m per kilometre. (Note: The roughness height of the prototype is estimated as 3 mm while the smoothest model surface feasible has a Darcy friction factor of about $f = 0.03$.) Discharges ranging between the maximum flow rate and 10% of the maximum flow rate are to be reproduced in the model.

For an undistorted model: (a) what would be the model discharge at maximum full-scale discharge? (b) what would be the Darcy coefficient of the model flow? (c) what would be the Darcy coefficient of the prototype channel? (d) comment and discuss your findings. (Assume normal flow conditions.)

A distorted model is to be built. (e) Determine the acceptable maximum and minimum values of the vertical scale ratio Z_r. (f) Select a suitable scale for practical use. Calculate the corresponding model values of: (g) Darcy coefficient, (h) maximum discharge and (i) normal depth (at maximum discharge).

A fixed bed model is to be made of a river with a surface width of 80 m. The Gauckler–Manning coefficient for the river is estimated at 0.026 $s/m^{1/3}$. Scale ratios of $X_r = 150$ and $Z_r = 25$ have been selected. (a) Find the required model values of the Gauckler–Manning coefficient corresponding to prototype depths of water of 2.0 and 5.0 m, if the cross-sectional shape is assumed to be rectangular. (b) What material would you recommend to use in the laboratory model for a prototype depth of 2.0 m?

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