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AIR BUBBLE DIFFUSION

IN SUPERCRITICAL OPEN CHANNEL FLOWS

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ABSTRACT

Air bubble entrainment in open channels is called 'white waters'. It is observed in supercritical turbulent flows. The air bubble diffusion process is analysed both analytically and experimentally in uniform equilibrium flows. The solution of the diffusion equation is compared with model and prototype data. The results indicate that the turbulent diffusivity is of the same order of magnitude as the momentum transfer coefficient (i.e. eddy viscosity.). However D_t/ν_T is larger on models than on prototype, suggesting that model investigations might not reproduce accurately the air bubble diffusion process.

INTRODUCTION

In supercritical open channel flows, free-surface aeration is frequently observed. It is also called 'self-aeration'. Air entrainment occurs when the turbulence acting next to the free-surface is large enough to overcome both the surface tension and buoyancy effects.

The process of self-aeration in chutes and storm waterways was initially studied because of the effects of entrained air on the thickness of the flow and the possible reduction in cavitation damage. Recently self-aeration on chutes has been recognised also for its contribution to the air-water transfer of atmospheric gases such as oxygen and nitrogen.

After a brief bibliography, the paper presents a new analysis of the air bubble diffusion. An analytical solution of the diffusion equation in uniform equilibrium flow is deduced. The result are compared with existing and new experimental data. The results are analysed also in term of the air bubble diffusion coefficient.

Bibliography

Self-aerated open channel flows have been studied only recently. Although some researchers observed 'white waters' and discussed possible effects, no experimental investigation was conducted successfully before the first quarter of the 20-th century. EHRENBERGER (1926) presented probably the first set of conclusive data. Another milestone was the experimental work lead by L.G. STRAUB (e.g. STRAUB and ANDERSON 1958).

The early models of air bubble diffusion derived from sediment-laden flow studies. Two original models were developed by STRAUB and ANDERSON (1958) and WOOD (1984). STRAUB and ANDERSON described the structure of self-aerated open channel flows as consisting of the inner region consisting of air bubbles distributed through the water flow by turbulent transport fluctuations and the outer flow region with a heterogeneous mixture of water droplets ejected from the flowing liquid stream. In the writer's opinion, the model of STRAUB and ANDERSON does not reflect the physical nature of the air-water flow. Measured air concentration and velocity distributions show clearly that the air-water flow behaves as a homogeneous mixture between 0 and Y_{90} as shown by WOOD (1985) and CHANSON (1993).

WOOD (1984) developed the conservation equation for the air-water mixture density in the equilibrium region. His model provides a very good fit with the experimental data for mean air contents between 10% and 75% (WOOD 1984,1985). It fits well also air concentration distributions in the gradually-varied flow region. But the model is based upon empirical constants derived from the concept of "diffusivity of the average density" and the need to estimate a "fall velocity of water".

AIR BUBBLE DIFFUSION

In a spillway chute, the upstream flow region is non-aerated, followed by a gradually-varied flow region and eventually an uniform equilibrium flow region (fig. 1). In the uniform equilibrium flow region, the flow properties (including the air concentration distribution) are independent of the distance x along the channel. The turbulence diffusion normal to the bottom counterbalances exactly the buoyancy effect. For a small control volume, the continuity equation for air in the air-water flow yields :

$$0 = \frac{d}{dy}\left(D_t * \frac{d\,C}{dy}\right) - \cos\alpha * \frac{d}{dy}(u_r * C) \tag{1}$$

where C is the air content, y the distance normal to the channel, D_t the diffusion coefficient, u_r the bubble rise velocity (in air-water mixture) and α the slope. The bubble rise velocity u_r in a fluid of density $\rho_w*(1 - C)$ can be related to the rise velocity in hydrostatic pressure gradient as : $u_r^2 = (u_r)_{Hyd}^2 *(1 - C)$ (see Appendix). Assuming a homogenous turbulence, a solution of equation (1) is then :

$$C = 1 - \tanh^2\left(K' - \frac{y'}{2 * D'}\right) \tag{2}$$

where $D' = D_t/((u_r)_{Hyd}*\cos\alpha*Y_{90})$, $y' = y/Y_{90}$, Y_{90} the distance where C = 0.9, α is the channel slope and tanh is the hyperbolic tangent function. K' and D' are integration constants satisfying :

$$C(y' = 1) = 0.9$$

and $$C_{mean} = \int_0^1 C * dy'$$

K' and D' are dimensionless functions of the mean air concentration only (table 1). Full details of the integration are reported in CHANSON (1995b).

Equation (2) is compared firstly with uniform equilibrium flow data (STRAUB and ANDERSON 1958) (fig. 2). Figure 3 compares equation (2) with experimental data obtained in a 4.0-degree slope chute (CHANSON 1995b). The data were recorded in the gradually-varied flow region. WOOD (1985) showed that the longitudinal variation of air concentration is gradual. Hence equation (2) can be used with local rather than equilibrium value. On figures 2 and 3, the agreement between equation (2) and data is good in both equilibrium and gradually-varied flows.

DIFFUSION COEFFICIENTS

In open channel flows, the relationship between the air bubble diffusion coefficient and the momentum transfer coefficient (ν_T) is :

$$\frac{D_t}{\nu_T} = \frac{2}{K} * D^* \tag{3}$$

where K is the Von Karman constant (K = 0.40) and $D^* = D_t/(V_* * Y)$ where V_* is the shear velocity and Y is the characteristic flow depth. D* is a classical dimensionless expression of the diffusion coefficient in sediment-laden flows and for the vertical dispersion of matters (e.g. dye, salt). In non-aerated flows, Y is the flow depth while Y_{90} is the characteristic depth in self-aerated flows.

The ratio D_t/ν_T describes the combined effects of : 1- the difference in the diffusion of a discrete particle (e.g. air bubble, sediment) and the diffusion of a small coherent fluid structure, and 2- the influence of the particles on the turbulence field (e.g. turbulence damping or drag reduction). Values of D_t/ν_T for several self-aerated flow experiments (uniform equilibrium flow conditions) are summarised in table 2 and on figure 4. They are compared with sediment diffusion coefficients and diffusion coefficients of matter in open channels.

First note that D_t is of the same order of magnitude as the eddy viscosity.

In equilibrium self-aerated flows, equation (3) can be rewritten as :

$$\frac{D_t}{\nu_T} = \frac{2}{K} * \frac{u_r * \cos\alpha}{V_*} * D' \tag{3b}$$

where D' is a function of the mean air content (table 1). Equation (3b) implies that D_t/ν_T depends not only upon $u_r*\cos\alpha/V_*$ (in a similar form as for sediment-laden flows) but also upon the mean air content.

On figure 4, the reader shall note that, on large prototypes, the ratio of the turbulent diffusivity over the eddy viscosity is less than unity while it is larger than one on models. Such a result (fig. 4) suggests that scale-model studies of self-aerated flows might not describe accurately the air bubble diffusion process in uniform equilibrium self-aerated flows. The trend shown on figure 4 should however be confirmed with additional uniform equilibrium field data.

Table 1 - Relationship between C_{mean}, D' and K

C_{mean} (1)	D' (2)	K' (3)
0.01	0.007312	68.70445
0.05	0.036562	14.0029
0.10	0.073124	7.16516
0.15	0.109704	4.88517
0.20	0.146489	3.74068
0.30	0.223191	2.567688
0.40	0.3111	1.93465
0.50	0.423441	1.508251
0.60	0.587217	1.178924
0.70	0.878462	0.896627

C_{mean} : mean air concentration defined in term of Y_{90}

DISCUSSION

1- The air concentration distribution (eq. (2)) has been obtained assuming : a homogeneous turbulence and a constant bubble rise velocity $(u_r)_{Hyd}$ (from 0 to Y_{90}). The latter is an approximation. The author (CHANSON 1995a) showed that the air bubble size varies across the flow from micro-sizes next to the bottom up to large air packets in the upper flow region. For such a wide range of bubble sizes, the rise velocity $(u_r)_{Hyd}$ is not a constant (e.g. COMOLET 1979).

For sediment-laden flows, several studies (e.g. COLEMAN 1970, GRAF 1971) indicated that the sediment diffusion coefficient D_t is not a constant across the flow. However model experiments (COLEMAN 1970)

and river data (ANDERSON 1942) showed that D_t is constant in the outer flow region (i.e. typically y/Y > 0.15).
2- The relationship between the dimensionless diffusivity D' and the mean air concentration is unique and independent of the discharge and bottom friction.
3- Note that equation (2) does not describe the air bubble diffusion next to the wall (i.e. within the air concentration boundary layer). Next to the wall, the interactions between the air bubbles and the turbulent shear layers result in a different profile and induce some drag reduction. A detailed review can be found in CHANSON (1994).

Table 2 - Ratio D_t/ν_T in open channels

Reference	D_t/ν_T	Comments
(1)	(2)	(3)
Self-aerated flows		
STRAUB and ANDERSON (1958)	0.88 to 3.62 ([a])	Model data.
AIVAZYAN (1986)	1.85 to 4.1 ([a])	Model data.
	0.6 to 2.6 ([a])	Prototype data.
Sediment-laden flows		
LANE and KALINSKE (1941)	0.335	
ANDERSON (1942)	0.4 to 1.5 ([b])	Enoree river, USA.
COLEMAN (1970)	0.25 to 2 ([b])	Model data.
Matter diffusion in open channel		
FISCHER et al. (1978)	0.335	Vertical mixing.

Notes :
([a]) Calculations performed assuming $(u_r)_{Hyd}$ = 0.4 m/s
([b]) Sediment diffusion coefficient in outer flow region.

CONCLUSION

The air bubble diffusion in self-aerated supercritical flows is analysed both analytically and experimentally. A new solution of the diffusion equation (eq. (2)) compares favourably with experimental data in both the equilibrium flow region and gradually-flow region (fig. 2 and 3).

The solution of the advective diffusion equation implies turbulent diffusivities of the same order of magnitude as the momentum transfer coefficient (table 2). The ratio D_t/ν_T is however larger with model experiments than with prototype data. If the trend is confirmed, it would suggest that model experiments overestimate the air bubble diffusion coefficient.

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APPENDIX - RISE BUBBLE VELOCITY IN NON-HYDROSTATIC PRESSURE GRADIENT

The buoyant force on a submerged body (e.g. an air bubble in a fluid) is the difference between the vertical components of the pressure force on its underside and on its upper side. For a single bubble rising at a constant velocity in a quiescent surrounding fluid, the drag force counterbalances the resultant of the weight force and the buoyant force. Neglecting the weight of an air bubble, the rise velocity squared is proportional to the pressure gradient :

$$u_r^2 \sim -\frac{dP}{dy} \qquad \text{(A-1)}$$

In an air-water flow (fig. 1), the local pressure and the pressure gradient at any position y are :

$$P(y) = \int_y^{+oo} \rho_w * (1 - C) * g * \cos\alpha * d\xi \qquad \text{(A-2)}$$

$$\frac{dP}{dy}(y) = \rho_w * (1 - C) * g * \cos\alpha \qquad \text{(A-3)}$$

Considering the bubble rise velocity in a hydrostatic pressure gradient (i.e. $dP/dy = \rho_w*g*\cos\alpha$), the expression of the bubble rise velocity in a fluid of density $\rho_w*(1-C)$ becomes :

$$u_r^2 = [(u_r)_{Hyd}]^2 * (1 - C) \qquad \text{(A-4)}$$

At the limit the rise velocity is zero in air (i.e. C = 1).

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Fig. 1 - Air entrainment on an open channel chute

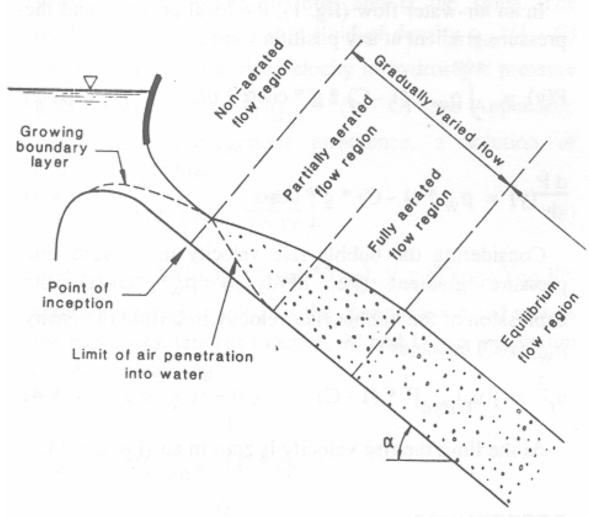

Fig. 2 - Air concentration distributions in uniform equilibrium flows - STRAUB and ANDERSON (1958)

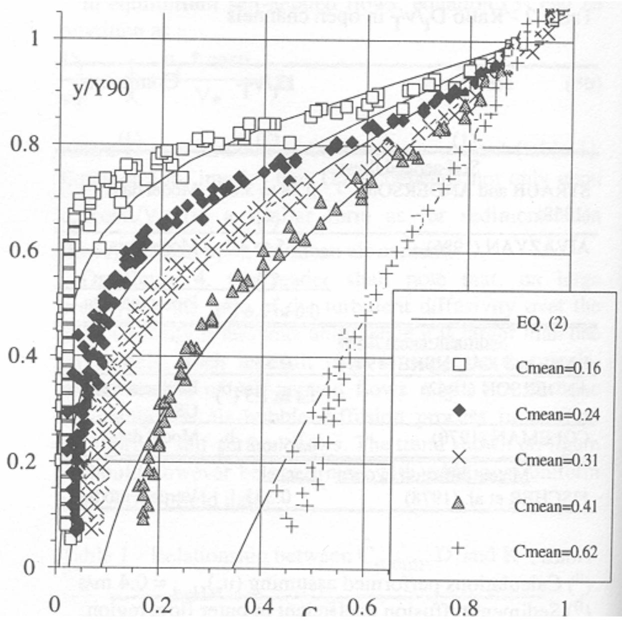

Fig. 3 - Air concentration distributions in gradually-varied self-aerated flows - CHANSON (1995b)

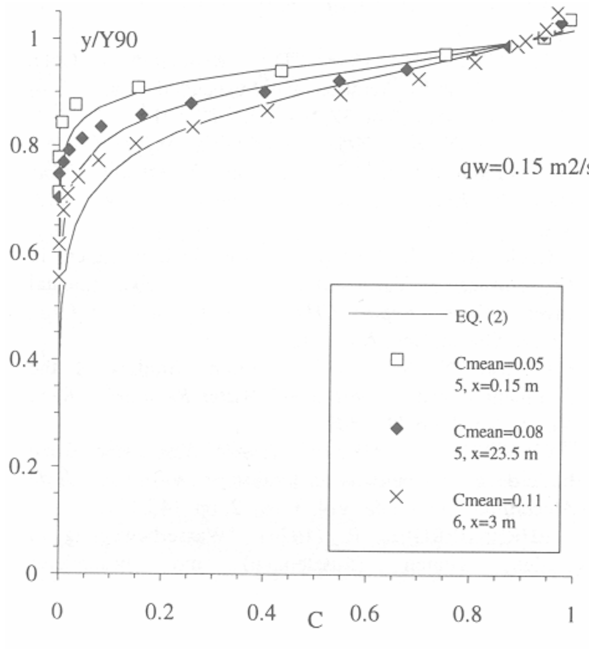

Fig. 4 - Ratio D_t/ν_T in uniform equilibrium self-aerated flow and in sediment-laden flows