5. Applications

5.3 Application No. 2

An alluvial stream in which the flow depth is 1.35-m has a longitudinal bed slope of $\sin\theta = 0.0002$. Characteristics of the sediment mixture are : $d_{50} = 1.5$ mm, $d_{90} = 2.5$ mm.

Predict the sediment transport capacity (taking into account the bed form).

Solution

The calculations proceed in four steps.

Step [1] :

Assuming a wide channel and uniform equilibrium flow, the bed shear stress equals :

$$\tau_o = \rho * g * \frac{D_H}{4} * \sin\theta \approx \rho * g * d * \sin\theta = 2.64 \text{ Pa} \qquad \text{Uniform equilibrium flow [M]}$$

Assuming a flat bed and $k_s = d_s = d_{50}$ (i.e. $k_s/D_H \approx 5.1\text{E-4}$), the mean velocity is :

$$V = \sqrt{\frac{8 * g}{f}} * \sqrt{\frac{D_H}{4} * \sin\theta} \qquad \text{Uniform equilibrium flow and flat bed}$$

The (iterative) calculations give : V = 1.12 m/s and f = 0.017.

The Shield parameter equals :

$$\tau_* = \frac{\tau_o}{\rho * (\mathbf{s} - 1) * g * d_{50}} = 0.11$$

and the particle Reynolds number ($d_{50}*V_*/\nu$) equals 77. For these values, the Shields diagram (fig. II-3-4) predicts sediment motion.

Step [2] :

Using the preliminary results, let us use the design chart of ENGELUND and HANSEN (1967) (fig. II-7-7). The known parameters are :

$$(V*d)/\sqrt{g*(\mathbf{s}-1)*d_{50}^3} = 6.5\text{E+3}$$

the bed slope : $\sin\theta = 2\text{E-4}$

the dimensionless flow depth : $d/d_{50} = 900$

Figure II-7-7 indicates that the bed forms are dunes. Note that figure II-7-7 suggests : $q_s \sim 0.05*\sqrt{(\mathbf{s}-1)*g*d_s^3}$

Step [3] :

The complete flow calculations are developed. The total bed shear stress must be satisfy the momentum equation :

$$\tau_o = \tau_o' + \tau_o'' \approx \rho * g * d * \sin\theta = 2.64 \text{ Pa} \qquad \text{[M]}$$

where τ_o' is the skin friction shear stress and τ_o'' is the form-related shear stress. The skin friction shear stress equals :

$$\tau_o' = \frac{f}{8} * \rho * V^2$$

assuming $k_s = 2*d_{90}$ and the bed form shear stress is calculated as :

$$\tau_o'' = \frac{1}{2} * \rho * V^2 * \frac{h^2}{l * d}$$

where the dune dimensions may be calculated using the correlations of van RIJN (1984c) :

$$\frac{h}{d} = 0.11*\left(\frac{d_{50}}{d}\right)^{0.3}*\left(1 - \exp\left(-0.5*\left(\frac{\tau_o}{(\tau_o)_c} -1\right)\right)\right)*\left(25 - \left(\frac{\tau_o}{(\tau_o)_c} -1\right)\right)$$

$$\frac{l}{d} = 7.33$$

The calculations are iterative :

1- assume V,

2- compute h, l, τ_o' and τ_o''

3- check the momentum equation by comparing ($\tau_o' + \tau_o''$) and τ_o (=2.64 Pa)

4- if ($\tau_o' + \tau_o''$) > τ_o, select a smaller flow velocity V; if ($\tau_o' + \tau_o''$) < τ_o, select a larger flow velocity

5- repeat the calculations until ($\tau_o' + \tau_o''$) = τ_o with an acceptable accuracy.

Complete calculations give : V = 0.65 m/s, τ_o'= 1.05 Pa, τ_o'' = 1.6 Pa, h = 0.32 m, l = 9.9 m.

Step [4] :

The sediment transport capacity is the sum of the bed-load transport rate and suspension transport rate.

The bed-load transport rate (see chap. II-5) is calculated using flow properties based upon the effective shear stress τ_o' : i.e., V_*', τ_*', $(\tau_*)_c'$, $(C_s)_{bl}$, $(d_s)_{bl}$, $(V_s)_{bl}$.

Complete calculations confirm bed-load motion and the bed-load transport rate equals 5E-6 m^2/s.

The suspension transport rate (see chap. II-6) must be computed using the overall bed shear stress τ_o. For the present application, the ratio V_*/w_o equals 0.2, where $V_* = \sqrt{\tau_o/\rho}$ = 0.051 m/s.

The flow condition are near the onset of suspension and the suspension transport rate may be assumed negligible.

Overall the sediment transport capacity of the alluvial stream with dune bed forms is : q_s = 5E-6 m^2/s (i.e. 0.013 kg/s/m).