The free surfaces are located by trial and error to satisfy the condition that equipotential lines and streamlines are equally spaced.

Application No. 1

Considering an uniform flow past a cambered Joukowski airfoil with zero angle of attack (Fig. 3-2), draw the flow net. Deduce the pressure field on the airfoil surface and total lift force if the uniform velocity away from the airfoil is 8 m/s and the foil chord is 0.5 m. *Assume a wind flow at standard conditions.*

Solution

(a) First the obvious streamlines are drawn. These are the streamlines away from the airfoil and the boundary (perimeter) of the airfoil. Another streamline is that through the stagnation points (i.e. $\psi = 0$) (Fig. 3-2).

(b) Then the remaining portions of streamlines are sketched with smooth curves.

(c) Equipotential lines are drawn to be normal to all streamlines and fixed boundaries, and forming squares with the streamlines. This is a trial-and-error method.

Four basic steps of the flow net construction are illustrated in Figure 3-2.

At completion of the flow net, the velocity next to the airfoil is deduced from the continuity equation. In each stream tube, the flow rate per unit width δq is constant :

$$\delta q = V_o \times \delta n_o = V \times \delta n \qquad \text{Continuity equation} \quad (3\text{-}9)$$

where V_o is the uniform velocity away from the airfoil, δn_o is the distance between two adjacent streamlines in the uniform flow, V is the velocity in a flow net element, and δn is the distance between two adjacent streamlines (Fig. 2-2). In this application, $V_o = 8$ m/s and $\delta n_o = 0.032$ m for $\delta\psi = 0.25$. Note that δn_o is scaled from the flow net knowing the airfoil chord length (i.e. 0.5 m).

The pressure at the boundary is calculated from the Bernoulli equation. In steady flow, it yields:

$$P_d = P_{d_o} + \rho \times \frac{V_o^2}{2} \times \left(1 - \left(\frac{V}{V_o}\right)^2\right) \qquad (2\text{-}23b)$$

where P_d is the dynamic pressure (Chapter 2, paragraph 4.3)

The lift force is calculated by integrating the pressure force component perpendicular to the approach flow direction along the airfoil perimeter :

$$\text{Lift} = - \int_0^{2\times\pi} P \times r \times \sin\theta \times d\theta \qquad (3\text{-}10)$$

where r and θ are the radial coordinates of the foil surface. The integration of Equation (3-10) gives: Lift ~ +5 N/m, where the lift is positive upwards.

Notes

1- The selection of $\delta\psi = 0.25$ was made arbitrarily to obtain several flow net elements next to the airfoil. The accuracy of the calculations increases with increasingly smaller flow net elements (i.e. $\delta\psi \to 0$).

2- The lift force is the pressure force component in the direction perpendicular to the flow direction.