### 4.5 Doublet in uniform flow (2)

We consider the air flow ( $\mathrm{V}_{\mathrm{O}}=9 \mathrm{~m} / \mathrm{s}$, standard conditions) past a suspension bridge cable ( $\emptyset=20$ mm ).
(a) Select the strength of doublet needed to portray the uniform flow of ideal fluid around the cylindrical cable.
(b) In real fluid flow, calculate the hydrodynamic frequency of the vortex shedding.

## Solution

(a) A doublet and uniform flow is analog to the flow past a cylinder of radius :

$$
R=\sqrt{\frac{-\mu}{V_{0}}}
$$

where $\mu$ is the strength of the doublet. Hence :

$$
\mu=-V_{0} \times R^{2}=9 \mathrm{E}-4 \mathrm{~m}^{3} / \mathrm{s}
$$

(b) The Reynolds number of the flow is 1.1 E-4. For that range of Reynolds number, the vortex shedding behind the cable is characterised by a well-defined Karman street of vortex. The hydrodynamic frequency satisfies :

$$
\mathrm{St}=\frac{\omega_{\text {shedding }} \times 2 \times \mathrm{R}}{\mathrm{~V}_{\mathrm{O}}} \sim 0.2
$$

It yields : $\omega_{\text {shedding }}=90 \mathrm{~Hz}$. If the hydrodynamic frequency happens to coincide with the natural frequency of the structure, the effects may be devastating : e.g., Tacoma Narrows bridge failure on 7 November 1940.

### 4.6 Flow pattern (2)

In two-dimensional flow we now consider a source, a sink and an uniform stream.
For the pattern resulting from the combinations of a source (located at (-L, 0)) and sink (located at $(+\mathrm{L}, 0)$ ) of equal strength Q in uniform flow (velocity $+\mathrm{V}_{\mathrm{O}}$ parallel to the x -axis) :
(a) Sketch streamlines and equipotential lines;
(b) Give the velocity potential and the stream function.

This flow pattern is called the flow past a Rankine body. W.J.M. RANKINE (1820-1872) was a Scottish engineer and physicist who developed the theory of sources and sinks. The shape of the body may be altered by varying the distance between source and $\operatorname{sink}$ (i.e. $2 \times \mathrm{L}$ ) or by varying the strength of the source and sink. Other shapes may be obtained by the introduction of additional sources and sinks and RANKINE developed ship contours in this way.
(c) What is the profile of the Rankine body (i.e. find the streamline that defines the shape of the body)?
(d) What is the length and height of the body ?
(e) Explain how the flow past a cylinder can be regarded as a Rankine body. Give the radius of the cylinder as a function of the Rankine body parameter.

