$$\eta = \frac{y}{x} \times \sqrt{Re_x}$$

$$\Lambda = -\frac{\psi}{\frac{\mu}{\rho} \times \sqrt{Re_x}}$$

in which Λ is a dimensionless stream function that is a function of the dimensionless co-ordinate η. Equation (3-11) may be rewritten into a differential equation in terms of the dimensionless stream function Λ:

$$\frac{\partial^3 \Lambda}{\partial \eta^3} + \frac{\Lambda}{2} \times \frac{\partial^2 \Lambda}{\partial \eta^2} = 0 \qquad (3\text{-}12)$$

Equation (3-12) is the *Blasius equation*. The boundary conditions are : $\Lambda = 0$ and $\partial\Lambda/\partial\eta = 0$ at the plate ($\eta = 0$), and $\partial\Lambda/\partial\eta = 1$ for $\eta = +\infty$. If Λ is only a function of η, the differentiation of Λ with respect of η becomes the first derivative (i.e. $\partial\Lambda/\partial\eta = \Lambda'$) and so on for the second and third differentiation (i.e. $\partial^2\Lambda/\partial\eta^2 = \Lambda''$ & $\partial^3\Lambda/\partial\eta^3 = \Lambda'''$).

The Blasius equation may be expressed as :

$$\Lambda''' + \frac{\Lambda}{2} \times \Lambda'' = 0 \qquad (3\text{-}12b)$$

Equation (3-12b) may be integrated as a power series. The solution is shown in Figure 3-2 and tabulated in Table 3-1.

Remarks

1- The Blasius equation was derived by BLASIUS (1907,1908) as part of his doctoral thesis at the University of Göttingen under the supervision of Ludwig PRANDTL (1875-1953). The problem was considered the first historical application of the boundary layer theory of L. PRANDTL (LIGGETT 1994, MEIER 2004)).
2- Paul Richard Heinrich BLASIUS (1883-1970) was a German fluid mechanician. Commonly called Heinrich BLASIUS, he lectured at the University of Hamburg from 1912 to 1950. His career in Germany spaned over the period of two World Wars, and his greatest contributions included the Blasius equation and the Blasius friction factor formula for smooth turbulent pipe flows (HAGER 2003).
3- The dimensionless stream function Λ is assumed to be a power series :

$$\Lambda = a_o + a_1 \times \eta + \frac{a_2}{2!} \times \eta^2 + \frac{a_3}{3!} \times \eta^3 + ...$$

where 3! is the factorial of 3 : $3! = 3\times2\times1$.
4- The Blasius equation was extended (e.g. ROUSE 1959, pp. 325-328). Some applications may include boundary layer flows with a temperature-dependant property (e.g. FANG et al. 2006).
5- Practically, the solution of the Blasius equation yields $\Lambda' \approx 1$ and $V_x/V_o \approx 1$ for $\eta > 6$.