$$\frac{l_m}{\delta} = 0.085 \times \tanh\left(\frac{K}{0.085} \times \frac{y}{\delta}\right) \qquad 0 \le y \le \delta$$

where tanh is the hyperbolic tangent function, K is the von Karman constant and δ is the boundary layer thickness (SCHLICHTING and GERSTEN 2000, p. 557). The momentum exchange coefficient, or "eddy viscosity", ν_T derives then from :

$$\nu_T = l_m^2 \times \frac{\partial v_x}{\partial y}$$

Power law velocity distribution in smooth boundary layers

For a smooth turbulent flow, the velocity distribution in the entire boundary layer may be approximated by a power law :

$$\frac{v_x}{V_o} = \left(\frac{y}{\delta}\right)^{1/N} \qquad 0 \le \frac{y}{\delta} \le 1 \quad (4\text{-}21)$$

Equation (4-21) is valid close to and away from the boundary without distinction between the inner and outer regions. It is also valid for turbulent boundary layer flows along rough walls. BARENBLATT (1994,1996) showed that the velocity power law (Eq. (4-21)) can be derived theoretically from some basic self-similarity. A self-similar process is one whose spatial distribution of properties at various times can be obtained from one another by a similarity transformation. Self-similarity is a powerful tool in turbulence flow research. Although more simple than the law of the wall, the power law velocity distribution gives results very close to the log-law (GEORGE 2006).

For a power-law velocity distribution :

$$\frac{v_x}{V_o} = \left(\frac{y}{\delta}\right)^{1/N} \qquad (4\text{-}21)$$

the characteristic parameters of the boundary layers can be expressed analytically. The displacement thickness and the momentum thickness become :

$$\frac{\delta_1}{\delta} = \frac{1}{1 + N}$$

$$\frac{\delta_2}{\delta} = \frac{N}{(1 + N) \times (2 + N)}$$

where N is the inverse of the velocity exponent. The shape factor equals :

$$\frac{\delta_1}{\delta_2} = \frac{N + 2}{N}$$

For two-dimensional turbulent boundary layers, SCHLICHTING (1979) indicated that separation occurs for $\delta_1/\delta_2 > 1.8$ to 2.4. Such a condition implies separation for $N < 1.4$ to 2.5.

Discussion

NIKURADSE (1932) performed detailed velocity measurements in fully-developed circular pipe flows. A re-analysis of his data set showed that the data were self-similar and followed closely a power law :

$$\frac{v_x}{V_*} \propto \left(\frac{\rho \times V_* \times y}{\mu}\right)^{\frac{3}{2 \times \mathrm{Ln}(Re)}}$$