5. TURBULENT SHEAR FLOWS

Jets, wakes and shear flows are most often turbulent. Three examples of turbulent shear flows are developed below: the free shear layer, the plane jet and the circular jet.

5.1 Free-shear layer

A simple turbulent shear flow is the free shear layer sketched in Figure 4-14. Assuming no pressure gradient, the differential form of the momentum equation can be simplified into :

$$V_x \times \frac{\partial V_x}{\partial x} + V_y \times \frac{\partial V_x}{\partial y} = \nu_T \times \frac{\partial^2 V_x}{\partial y^2} \qquad (4\text{-}27)$$

where ν_T is the turbulent momentum exchange coefficient. For a plane shear layer GOERTLER (1942) solved the equation of motion assuming a constant eddy viscosity ν_T across the shear layer at a given longitudinal distance x from the singularity :

$$\nu_T = \frac{1}{4 \times K^2} \times x \times V_o \qquad (4\text{-}28)$$

where K is a constant. The constant K provides some information on the expansion rate of the momentum shear layer as the rate of expansion is proportional to 1/K. K equals between 9 and 13.5 with a generally-accepted value of 11 for monophase free shear layers (RAJARATNAM 1976, SCHLICHTING 1979, SCHETZ 1993).

Let us introduce the dimensionless terms η and Λ :

$$\eta = K \times \frac{y - y_{50}}{x} \qquad (4\text{-}29)$$

$$\Lambda = \frac{\psi}{x \times \frac{V_o}{2}} \qquad (4\text{-}30)$$

where y_{50} is the location where $V_x = V_o/2$ and ψ is the stream function. Λ is simply a dimensionless stream function. Ideally y_{50} is zero, but experimental observations demonstrated that y_{50} is usually negative and increases with increasing longitudinal distance x as sketched in Figure 4-14.

Basic similarity considerations show that Λ is a function of η only, and Equation (4-27) may be rewritten as a differential function for $\Lambda(\eta)$:

$$\Lambda''' + 2 \times K \times \Lambda \times \Lambda'' = 0 \qquad (4\text{-}31)$$

where $\Lambda' = \partial\Lambda/\partial\eta$.

GOERTLER (1942) obtained the solution in the first approximation :

$$\frac{V_x}{V_o} = \frac{1}{2} \times \left(1 + \operatorname{erf}\left(K \times \frac{(y - y_{50})}{x}\right)\right) \qquad (4\text{-}32)$$

where erf is the Gaussian error function defined as :

$$\operatorname{erf}(u) = \frac{2}{\sqrt{\pi}} \times \int_0^u \exp(-t^2) \times dt \qquad (4\text{-}33)$$

Tabulated values of the Gaussian error function erf are listed in Table 4-1.