(b) Using the momentum integral equation, calculate the boundary shear stress at x = 0.1 m.

(c) Based upon the momentum integral equation, integrate numerically the boundary shear stress to estimate the friction force per unit width on the 0.2 m long plate.

(d) Compare your results with the theoretical calculations (Blasius equation) and with the approximate solution assuming a quadratic velocity distribution. *Check that the flow laminar before conducting the comparison.*

The fluid is a bentonite suspension (density: 1115 kg/m^3, viscosity: 0.19 Pa.s, mass concentration: 17%).

Solution

(a) The free-stream velocity is about 11.6 cm/s.

(b)

	Momentum integral equation	Blasius solution
τ_o (x=0.1 m) (Pa) =	0.41	0.6
F_{shear}/B (N/m) =	0.083 (?)	0.17

Note: The application of the momentum integral equation betwen x = 0 and x = 0.05 m not impossible, but this region is affected by large shear stress because of the flow singularity.

EXERCISE NO. 5

Assuming that the velocity profile in a laminar boundary layer satisfies a polynomial of fourth degree, apply the momentum integral equation.

(a) Derive the expression of the velocity profile. *Write carefully the boundary conditions.*

(b) Derive mathematically the expression of the boundary layer thickness, bed shear stress and total shear force.

EXERCISE NO. 6

Assuming that the velocity profile in a laminar boundary layer satisfies a polynomial of third degree, apply the momentum integral equation.

(a) Derive the expression of the velocity profile. *Write carefully the boundary conditions.*

(b) Derive mathematically the expression of the boundary layer thickness, displacement thickness and momentum thickness.

(c) Derive the expressions of the bed shear stress and total shear force.

(d) Compare your results with the Blasius solution.

Solution

(a) Let us assume that the velocity profile above a flat plate may be expressed as :

$$\frac{V_x}{V_o} = a_0 + a_1 \times \frac{y}{\delta} + a_2 \times \left(\frac{y}{\delta}\right)^2 + a_3 \times \left(\frac{y}{\delta}\right)^3$$

where a_0, a_1, a_2 and a_3 are undetermined coefficients, The coefficients are determined from the boundary conditions : $v_x(y=0) = 0$, $v_x(y=+\infty) = V_o$, $(\partial v_x/\partial y) = 0$ for $y = \delta$ and $(\partial^2 v_x/\partial y^2) = 0$ for $y = \delta$ The velocity distribution is found to be:

$$\frac{v_x}{V_o} = 3 \times \frac{y}{\delta} - 3 \times \left(\frac{y}{\delta}\right)^2 + \left(\frac{y}{\delta}\right)^3$$

(b) The von Karman momentum integral equation for a flat plate becomes :

$$V_o^2 \times \frac{\partial}{\partial x}(\delta_2) = \frac{\tau_o}{\rho}$$

For a velocity profile satisfying a polynomial of third degree, the momentum thickness equals :

$$\delta_2 = \int_0^{+\infty} \frac{v_x}{V_o} \times \left(1 - \frac{v_x}{V_o}\right) \times dy = \frac{3}{28} \times \delta$$

The bed shear stress is defined as:

$$\tau_o = \mu \times \left(\frac{\partial v_x}{\partial y}\right)_{y=0} = \frac{3 \times \mu \times V_o}{\delta}$$

The momentum integral equation yields :

$$\frac{3}{28} \times V_o^2 \times \frac{\partial \delta}{\partial x} = \frac{\mu}{\rho} \times \frac{3 \times V_o}{\delta}$$

The integration gives the expression of the boundary layer growth:

$$\delta = \sqrt{14} \times \frac{x}{\sqrt{Re_x}}$$

The displacement thickness and momentum thickness equal:

$$\delta_1 = \frac{\delta}{4} = \sqrt{\frac{7}{2}} \times \frac{x}{\sqrt{Re_x}}$$

$$\delta_2 = \frac{3}{28} \times \delta = \sqrt{\frac{9}{56}} \times \frac{x}{\sqrt{Re_x}}$$

(c) The boundary shear stress is deduced from the momentum integral equation:

$$\frac{\tau_o}{\frac{1}{2} \times \rho \times V_o^2} = 2 \times \frac{\partial}{\partial x}(\delta_2) = \sqrt{\frac{9}{56}} \times \frac{1}{\sqrt{Re_x}}$$

The dimensionless boundary shear force per unit width equals:

$$\frac{\int_{x=0}^{L} \tau_o \times dx}{\frac{1}{2} \times \rho \times V_o^2 \times L} = \sqrt{\frac{9}{14}} \times \frac{1}{\sqrt{Re_L}}$$

(d) The results are compare with the Blasius analytical solution below:

Boundary layer parameter	Approximate solution	Theoretical solution
Velocity distribution:	$\frac{V_x}{V_o} = 3 \times \frac{y}{\delta} - 3 \times \left(\frac{y}{\delta}\right)^2 + \left(\frac{y}{\delta}\right)^3$	Blasius equation
$\delta =$	$3.74 \times \frac{x}{\sqrt{Re_x}}$	$4.91 \times \frac{x}{\sqrt{Re_x}}$
$\delta_1 =$	$1.87 \times \frac{x}{\sqrt{Re_x}}$	$1.72 \times \frac{x}{\sqrt{Re_x}}$
$\delta_2 =$	$0.40 \times \frac{x}{\sqrt{Re_x}}$	$0.664 \times \frac{x}{\sqrt{Re_x}}$
$\frac{\tau_o}{\frac{1}{2} \times \rho \times V_o^2}$	$\frac{0.40}{\sqrt{Re_x}}$	$\frac{0.664}{\sqrt{Re_x}}$
$\frac{\int_{x=0}^{L} \tau_o \times dx}{\frac{1}{2} \times \rho \times V_o^2 \times L}$	$\frac{0.80}{\sqrt{Re_L}}$	$\frac{1.328}{\sqrt{Re_L}}$

EXERCISE NO. 7

Let us consider a laminar wake behind a 0.5 m long plate.

(a) Calculate the total drag force (per unit width) on the plate.

(b) Estimate at what distance, downstream of the plate, the velocity profile will recovers (within 2% of the free-stream velocity) ?

The free-stream velocity is 0.35 m/s and the fluid is a viscous SAE40 oil (density: 871 kg/m^3, viscosity: 0.6 Pa.s).

Solution

(a) Drag per unit width = 4.4 N/m

(b) x/L = 350 (x = 175 m !!!)

EXERCISE NO. 8

Some fluid is injected in a vast container where the surrounding fluid is at rest. The nozzle height is 0.15 mm and the injected velocity is 1 cm/s.

(b) Calculate the maximum jet velocity at distances of 1.5 mm, 2 cm and 18 cm from the nozzle.

(b) At a distance of 18 cm from the nozzle, estimate the volume discharge of entrained fluid (per unit width).

Assume a two-dimensional jet.

The fluid is blood (density: 1050 kg/m^3, viscosity: 4 E-3 Pa.s).