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Jean-Baptiste Bélanger: hydraulic engineer and academic

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Jean-Baptiste Bélanger (1790–1874) worked as a hydraulic engineer at the beginning of his career. He developed the backwater equation to calculate the free-surface profile of gradually varied, steady open channel flow. He also introduced the concept of critical flow and the numerical technique called the direct step method. Later, as an academic staff member at the leading French engineering schools (Ecole Centrale des Arts et Manufactures, Ecole des Ponts et Chaussées and Ecole Polytechnique), he developed a new university curriculum in mechanics and wrote several textbooks including a seminal text in hydraulic engineering. His influence on his contemporaries was considerable, and his name is written on the border of one of the four façades of the Eiffel Tower. Bélanger's leading role demonstrated the dynamism of practising engineers at the time, and his contributions paved the way to many significant works in hydraulics and fluid dynamics during the late nineteenth and early twentieth centuries.

1. INTRODUCTION

The application of the momentum principle to the hydraulic jump is commonly called the Bélanger equation in modern fluid mechanics texts. It is named after Jean-Baptiste Bélanger (1790–1874) (Figure 1) who first derived the correct relationship between the flow properties upstream and downstream of a hydraulic jump in a horizontal rectangular prismatic channel (Bélanger, 1841)

$$\frac{d_2}{d_1} = \frac{1}{2}\left(\sqrt{1 + 8\,Fr_1^2} - 1\right) \qquad (1)$$

where d is the flow depth, the subscripts 1 and 2 refer to the upstream and downstream flow conditions respectively, $Fr = V/\sqrt{g\,d}$ is the Froude number, V is the flow velocity and g is the gravity acceleration. Historical contributions on the hydraulic jump flows included the physical experiments of Bidone (1819) performed in France in 1818, the theoretical analyses of Bélanger (1828, 1841), the experiments of Darcy and Bazin (1865), the solutions of Boussinesq (1877) and the work of Bakhmeteff (1932). Recent reviews encompassed (Chanson, 2009a) and Hager (1992).

Despite this contribution, Bélanger's considerable influence on his contemporaries is too often lost. In this paper, the contribution of Bélanger to civil and mechanical engineering is reconsidered. His career included two distinct periods: the professional engineer (1816–1838) and the university academic (1838–1864) at the French leading engineering schools (Ecole Centrale des Arts et Manufactures, Ecole des Ponts et Chaussées and Ecole Polytechnique). Herein a parallel is developed between his career and his contribution to open channel hydraulics (Table 1).

Figure 1. Photograph of Bélanger (courtesy of the Bibliothèque de l'Ecole Centrale de Paris)

1.1. The life of Jean-Baptiste Bélanger

Jean-Baptiste Charles Joseph Bélanger was born in Valenciennes (Parish of St Vaast en Ville) in northern France, on 4 April 1790. He was the son of Charles Antoine Aimé Joseph Bélanger, master locksmith, and of Jeanne Françoise Joseph Fauconnier who were married on 21 April 1789 at Valenciennes (Parish of St Géry). He was baptised on his birth

Period (1)	Career (2)	Ref. (3)	Contribution in open channel hydraulics (4)	Comment (5)
1816–1838	Professional engineer (Ingénieur des Ponts)	(Bélanger, 1828)	Backwater equation, direct step method, Froude number, critical flow, gradually varied flow (GVF) against rapidly varied flow (RVF)	Incorrect treatment of the hydraulic jump
1838–1864	Academic lecturer	(Bélanger, 1841)	Hydraulic jump equation, composite channel calculations, broad-crested weir, maximum discharge	Several editions and re-editions

Table 1. Parallel between Bélanger's career and his contributions to open channel hydraulics

day, and his godfather and godmother were his uncle and aunt, respectively Jean Baptiste Laurent Honnis and Marie Heleine Fauconnier (Chanson, 2008).

He studied in Paris at the Ecole Polytechnique in the 1808 cohort (promotion 1808). He graduated together with Gustave Gaspard Coriolis (1792–1843) and Jean-Victor Poncelet (1788–1867), finishing second in his cohort. From 1812, he studied at the Ecole des Ponts et Chaussées (Tarbé de St Hardouin (1884)). Bélanger started his professional career as Ingénieur du Corps des Ponts et Chaussées (Bridges and Roads Corps of Engineers) in 1816. He was promoted to Ingénieur en Chef du Corps des Ponts (Chief Engineer) in 1843. From 1838, he became a lecturer at the Ecole Centrale des Arts et Manufactures, Ecole des Ponts et Chaussées and Ecole Polytechnique until retirement.

Bélanger married Louise Aimée Dumas (1797–1877). He retired in 1864. He died on 8 May 1874 at Neuilly-sur-Seine, and was buried in the old cemetery together with his wife (cimetière ancien, 5ème division) (Figure 2).

Figure 2. Tombstone of Bélanger at Neuilly-sur-Seine (France) (courtesy of Philippe Landru); note his wife's engraved name in foreground and the destroyed bust

2. BÉLANGER: THE ENGINEER (1816–1838)

Bélanger began his engineering career in 1816 at La Réole. From 1821, he moved to work on the Somme navigation canal and after 1826 on the Ardennes navigation canal. During these two missions he designed navigation canals, their water supplies and water resource management (Chanson, 2009b).

Later Bélanger worked with Antoine-Rémi Polonceau (1778–1847) in the Seine-et-Oise region. His contribution included the first design of the Paris–Rouen–Le Havre railway, opened in 1837 between Paris–Saint-Lazare and La Garenne-Colombes, extended to Rouen in 1843 and completed in 1847 to Le Havre. The study integrated also the design of the Paris–Dieppe railway (Polonceau and Bélanger (1837)). This project was regarded by some contemporaries as one of the greatest accomplishment of Bélanger (Quérard, 1842).

2.1. Bélanger's (1828) memoir in open channel hydraulics

For the design of navigation canals, Bélanger needed to develop a solution of gradually varied, open channel flows, and he published a preliminary report in the *Journal des Mines* in 1823, although the work lacked theoretical foundations. In 1826, he developed new ideas and submitted a revised report to the Commission des Ponts et Chaussées et des Mines in 1827. It was successfully peer-reviewed and published in 1828 (Bélanger, 1828).

Bélanger (1828) developed a new equation for the prediction of the free-surface profile in gradually varied flows: the backwater equation (Chanson, 2009b). The basic assumptions were: (*a*) steady flow, (*b*) one-dimensional flow motion, (*c*) gradual variation of the wetted surface with distance x along the channel, (*d*) friction losses that are the same as for uniform equilibrium flow (i.e. normal flow) for the same depth and discharge and (*e*) hydrostatic pressure distribution. Within the above assumptions, Bélanger (1828) derived the backwater equation from momentum considerations as

$$\sin\theta\,\partial x - \cos\theta\,\partial d - \frac{P_{\mathrm{w}}}{A}(aV + bV^2) + \frac{Q^2}{gA^3}\partial A = 0 \quad (2)$$

where θ is the angle between the bed and the horizontal, x is the longitudinal distance positive downstream, d is the flow depth measured normal to the invert, A is the cross-section area, P_w is the wetted perimeter and Q is the discharge. In Equation 2, Bélanger estimated the friction losses using the Prony formula

$$-\frac{\partial H}{\partial x} = \frac{4}{D_H}(aV + bV^2) = \frac{f}{D_H}\frac{V^2}{2g} \tag{3}$$

where H is the total head, D_H is the hydraulic diameter: $D_H = 4A/P_w$, and a and b are constant ($a = 4{\cdot}44499{\times}10^{-5}$, $b = 3{\cdot}093140 \times 10^{-4}$ in SI units with V in m/s). Equation 3 is compared with modern expressions in terms of the Darcy–Weisbach friction factor f on the right-hand side. Interestingly Bélanger (1849) was aware of the work of Henry Darcy (1803–1858) in pipe flows, but he continued to use the Prony formula for its simplicity.

Denoting S_f the friction slope: $S_f = -\partial H/\partial x$, and S_o the bed slope: $S_o = \sin\theta$, Equation 2 may be combined with the continuity equation to yield

$$\frac{\partial}{\partial x}\left(d\cos\theta + \frac{V^2}{2g}\right) = S_o - S_f \tag{4}$$

Equation 4 is identical with modern expressions of the backwater equation (Chanson, 2004; Henderson, 1966; Montes, 1998). Equation 2 was tested for a long prismatic channel (Figure 3). Figure 3 shows some measurements performed by Darcy and Bazin (1865) in a prismatic, rectangular channel down a steep slope with a downstream control gate. An undular hydraulic jump was observed at $x = 125$ m. In Figure 3, the free-surface measurements are compared with Equation 2 in which the flow resistance was calculated using Equation 3, and with Equation 4 in which the friction slope was calculated in terms of the Darcy–Weisbach friction factor. The location of the hydraulic jump was derived from the application of the momentum principle neglecting the effects of bed slope (Equation 1). The results in Figure 3 show little differences between Equation 2, Equation 4 and the experimental data, despite the crude nature of the Prony formula. Bélanger's calculations (Bélanger, 1828) give identical results to modern estimates. Note that Bélanger performed his calculations manually, justifying the usage of Prony's simplified formula at the time (Brown, 2002). Let us remember that the modern slide rule was introduced in 1859, 31 years later, by the French artillery officer Amédée Mannheim (1831–1906).

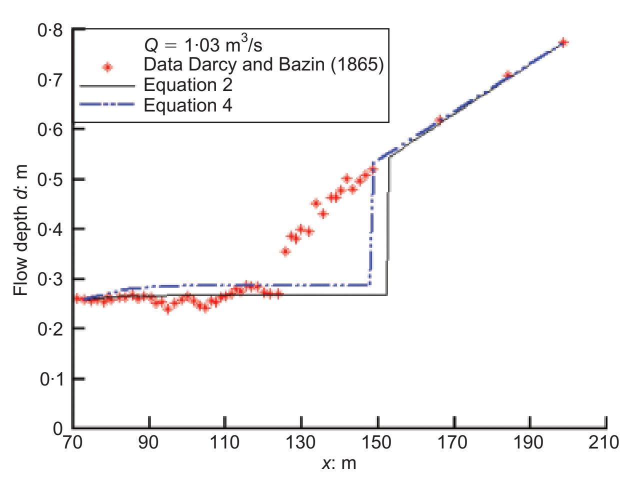

Figure 3. Free-surface profile $d(x)$ in a long prismatic channel: comparison between backwater calculations and experimental data by Darcy and Bazin (1865). Flow conditions: experimental channel along the Canal de Bourgogne, $Q = 1{\cdot}03$ m^3/s, $\theta = 0{\cdot}281°$, $B = 1{\cdot}99$ m, planed boards, series 89; Backwater calculations include Equations 2 and 4

Bélanger integrated the backwater equation by selecting known flow depths and calculating manually the distance in between. This numerical technique is called the step method distance calculated from depth (Chanson, 2004; Henderson, 1966) or the direct step method. In the same 1828 memoir, Bélanger introduced the concept of critical flow as one of two singularities of the backwater equation. He also recognised the importance of the ratio $V/\sqrt{gd}$ now called the Reech–Froude number after the works of Ferdinand Reech (1852) and William Froude (1872), respectively 24 and 44 years later. Further, Bélanger understood the concepts of gradually varied flows (GVF) and rapidly varied flows (RVF). Simply, his 1828 treatise (Bélanger, 1828) marked a seminal advance in open channel hydraulics.

3. BÉLANGER: A LEADING UNIVERSITY ACADEMIC (1838–1864)

In 1838, Bélanger became a lecturer at Ecole Centrale des Arts et Manufactures and he lectured there until 1864 (Figure 4). He lectured also at Ecole des Ponts et Chaussées from 1841 to 1855, and at Ecole Polytechnique from 1851 to 1860 (Table 2).

From 1851, as a full professor at Ecole Polytechnique, Bélanger developed a new university curriculum in mechanics (cours de Mécanique) in response to a restructure of the engineering programme at Ecole Polytechnique (Chatzis, 1995). Linking kinematics and dynamics, he argued that the mechanics is based upon three principles: inertia, action–reaction and constant ratio force to acceleration at any point. Among the innovations, he considered statics as a limited case of dynamics

Figure 4. Photograph of Bélanger (second row, third from the left with white hair) among his colleagues of Ecole Centrale des Arts et Manufactures (courtesy of the Bibliothèque de l'Ecole Centrale de Paris)

Engineering school	Ecole Centrale des Arts et Manufactures	Ecole Polytechnique	Ecole des Ponts et Chaussées
Start	1838: Chair in geometry (analyse géometrique) and mechanics (mécanique générale) and Director of Studies in replacement of Jean-Baptiste Liouville	1851: Chair in mechanics	1841: Chair in applied mechanics (mécanique appliquée) Lectures in hydraulic engineering
End	1864 (or 1866)	1861	1855 (or 1853)

Table 2. Lecturing responsibilities of Bélanger

which was most innovative in France at the time. His basic ideas were first developed in his textbook (Bélanger, 1847), although criticised by Adhémar Jean Claude Barré de Saint-Venant (1797–1886) and Ferdinand Reech (1805–1880) without much success (Chatzis, 1995). Barré de Saint-Venant believed that kinematics could be taught without the concept of force. Instead Reech argued that Bélanger's definition of force was erroneous. Both arguments received little support and Bélanger's curriculum in mechanics was widely accepted in all French engineering schools.

Bélanger wrote further textbooks, including a series of treatises in mechanics and applied mechanics (Bélanger, 1864a, 1864b, 1866). His ideas influenced strongly the teaching of statics and dynamics, and mechanics in France and Europe during the nineteenth century as well as into the twentieth century. For example, the author was taught engineering mechanics and fluid dynamics, in line with Bélanger's curriculum. The contribution of Bélanger in mechanics and applied mechanics was highlighted in his necrology: 'M. Bellanger a puissamment contribué à répandre en France les connaissances de la mécanique appliquée' ('M. Bellanger [note the typographic mistake] contributed strongly to diffuse in France the fundamental knowledge of applied mechanics') (Figuier, 1875).

3.1. Bélanger's contribution in hydraulics

Twenty-one years after this original essay, Bélanger (1841) expanded his hydraulics memoir into a series of lecture notes for Ecole des Ponts et Chaussées. The lecture notes were comparable to a modern textbook and they were used at Ecole des Ponts et Chaussées and Ecole Centrale des Arts et Manufactures, and were available at the Ecole Polytechnique et Ecole des Mines de Paris. The comprehensive treatise in hydraulic engineering was re-edited at least five times with relatively small to moderate differences (Table 3). The textbook has had a profound influence in leading hydraulic engineering scholars including Boussinesq (1877), Forchheimer (1914), Bakhmeteff (1932), Jaeger (1956) and Montes (1998).

Despite an incorrect treatment of the hydraulic jump in his 1828 memoir, Bélanger corrected his theory in 1838 (Bélanger, 1841). He solved the momentum equation for a hydraulic jump in a flat channel. For a rectangular channel and neglecting the friction force, he obtained

$$\frac{d_2}{d_1} = -\frac{1}{2} + \sqrt{\frac{1}{4} + 2\alpha' Fr_1^2} \quad (5)$$

where α' is a velocity correction coefficient. Equation 5 is equivalent to Equation 1 for $\alpha' = 1$. This reasoning became commonly accepted thereafter (Bélanger, 1849; Bresse, 1860). For a hydraulic jump in a horizontal, rectangular, prismatic channel, Bélanger (1849) calculated the loss in kinetic energy head as

$$\frac{V_1^2}{2g} - \frac{V_2^2}{2g} = \frac{(d_1 + d_2)^2}{4 d_1 d_2}(d_2 - d_1) \quad (6)$$

that may be rewritten in terms of the head loss in the hydraulic jump

$$\Delta H = \frac{(d_2 - d_1)^3}{4 d_1 d_2} \quad (7)$$

Equation 7 is a well-known result for a hydraulic jump in a horizontal rectangular channel (Chanson, 2004; Henderson, 1966).

In the same treatise, Bélanger (1841, 1849) presented explicitly a number of basic features of open channel flows. He developed an expression of the uniform equilibrium flow depth (normal depth) that was derived from energy considerations. He further developed the calculations of the normal depth for a composite channel (Figure 5), showing accurately that the total

Date (1)	Title (2)	University year (3)	Engineering school (4)
1841	Notes sur l'Hydraulique	1841–42	Ecole des Ponts et Chaussées
1845	Notes sur l'Hydraulique	1845–46	Ecole des Ponts et Chaussées
1846	Résumé du cours d'hydraulique	1846–47	Ecole Centrale des Arts et Manufactures
1849	Notes sur le Cours d'Hydraulique	1849–50	Ecole des Ponts et Chaussées
1850	Notes sur le cours d'hydraulique	1850–51	Ecole Centrale des Arts et Manufactures
1850	Notes sur l'Hydraulique		

Table 3. Editions of Bélanger's lecture notes in hydraulic engineering

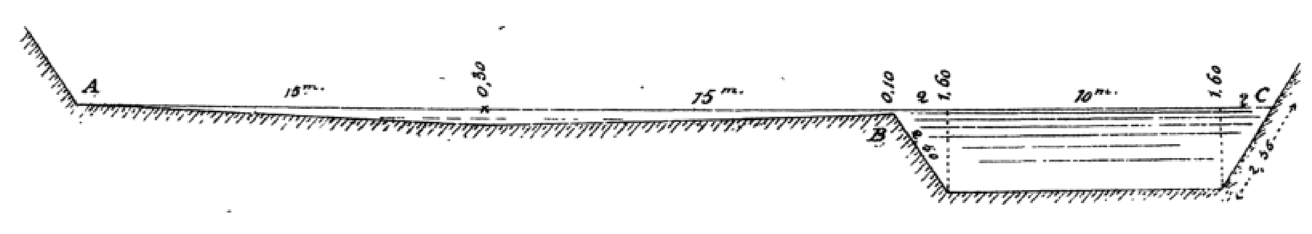

Figure 5. Composite open channel cross-section after J. B. Bélanger's sketch

discharge is the sum of the flow rates in the main channel and in the floodplain, and that the friction slope is identical for both channel sections, but with different friction coefficients (e.g. Bélanger, 1849, pp. 69–70).

Bélanger (1841, 1849) showed that, in a rectangular channel, the discharge per unit width is maximum at critical flow conditions for a given specific energy E. This result is sometimes called Bélanger's principle (Jaeger, 1956). Bélanger derived the expression of the critical depth d_c

$$d_c = \frac{2}{3}E \tag{8}$$

Note that Bélanger used the notion of specific energy more than 70 years before Bakhmeteff (1912, 1932) who is often credited for the introduction of this concept (Henderson, 1966).

In the same section, Bélanger analysed the flow over a broad-crested weir (Figure 6). His treatment yielded further the classical expression of the flow rate Q

$$\frac{Q}{B} = \sqrt{g}\left(\frac{2}{3}H\right)^{3/2} \tag{9}$$

for a rectangular channel of width B, where H is the total head above the crest invert. All these results are common knowledge today (Chanson, 2004; Henderson, 1966), but were new and important developments in the early 1840s. It is worth noting that Bélanger (1841, 1849) used both momentum and energy considerations in a somewhat inconsistent manner. Such inconsistencies were discussed by Yen (2002) and Chanson (1999, 2004) in a broader context.

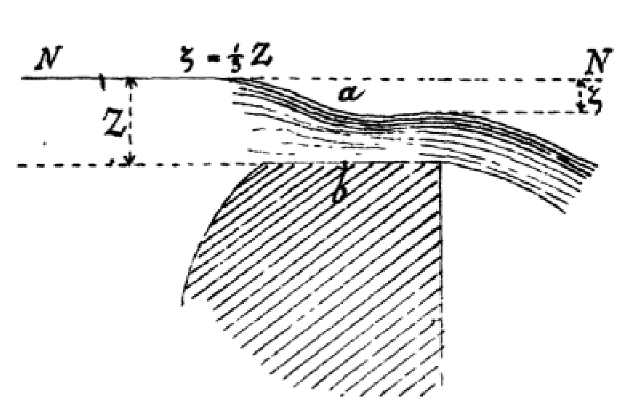

Figure 6. Overflow above a broad-crested weir after J. B. Bélanger's sketch

4. DISCUSSION

Between 1816 and 1864, the work of Bélanger demonstrated the dynamism of practising engineers in France. His own contributions were remarkable and influenced many leading hydraulic engineers including H. Darcy, E. Mach, J. Boussinesq, P. Forchheimer, B. Bakhmeteff, and most likely R. Manning, R. Freeman, L. Prandtl, and R. von Mises. Figure 7 shows the lifespan of Bélanger and that of other leading hydraulicians and fluid dynamicists during the nineteenth and early twentieth centuries.

Henry Darcy (1805–1858) had a high respect for Bélanger's hydraulic engineering notes. For example, he cited the work of Bélanger (1828) in his own publications (Darcy and Bazin, 1865). He also praised Bélanger for his support and inputs into his research on flow resistance in pipes and channels (Darcy, 1858). Bélanger, and Poncelet, attended in person several experiments conducted by H. Darcy. In Germany, Franz Reuleaux (1829–1905) respected highly Bélanger's (Bélanger, 1847) text: for example, it is cited several times in his hydromachinery textbook (Reuleaux, 1877). In Austria, Ernst Mach (1838–1916) listed Bélanger's treatise (Bélanger, 1847) among a few basic references in Mechanics, together with the works of Coriolis and Poncelet (Mach, 1883). In France, Joseph Boussinesq (1842–1929) included ample references of Bélanger's works in his papers (e.g. Boussinesq, 1877).

More specifically in hydraulic engineering, Bélanger's works have been commonly cited in classical textbooks during the nineteenth, twentieth and twenty-first centuries, including Bakhmeteff (1932), Bresse (1860), Forchheimer (1914) and

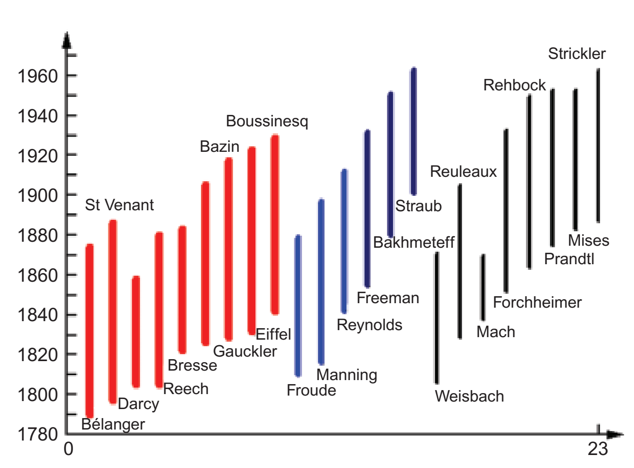

Figure 7. Lifespan of leading hydraulic engineers including J. B. Bélanger (far left). Red = French; Blue = English/American; Black = German speakers

Jaeger (1956), among other books. Today, the modern open channel hydraulic texts continue to acknowledge his seminal contributions for example Montes (1998) and Chanson (1999, 2004).

4.1. Names engraved on the Eiffel Tower

Bélanger lectured for more than 26 years the best engineering students in France. One of his students, at Ecole Centrale, was Gustave Eiffel (1832–1923) who built the Eiffel Tower and engraved his name (Figure 8) around the first floor together with the names of 71 other scientists. Altogether, 14 hydraulic engineers and scholars had their name engraved: J. B. Bélanger (1789–1874), Eugène Belgrand (1810–1878), Jean-Charles de Borda (1733–1799), Jacques Antoine Charles Bresse (1822–1883) who was Professor at Ecole des Ponts et Chaussées as the successor of J. B. Bélanger, Augustin Louis de Cauchy (1789–1857), Gustave Gaspard Coriolis (1792–1843), Jean Baptiste Joseph Fourier (1768–1830), Joseph-Louis Lagrange (1736–1813), Pierre-Simon Laplace (1746–1827), Gaspard Monge (1746–1818), Claude Louis Marie Henri Navier (1785–1835), Siméon Denis Poisson (1781–1840), Jean-Victor Poncelet (1788–1867) and Gaspard Clair François Marie Riche de Prony (1755–1839).

Of these 14 hydraulic engineers, academics and scholars, half played an active role during the French Revolution and the Napoléonic Era. Most were active lecturers (80%) in the leading French engineering schools emphasising the influence of engineering lecturers on the nineteenth century French engineering society. Notably these leading hydraulic scholars were educated mostly (57%) at Ecole des Ponts et Chaussées and at Ecole Polytechnique. In his selection, Gustave Eiffel acknowledged the leading scientists in the field of hydrodynamics and fluid mechanics. Yet one notes the absence of Henry Darcy (1805–1858). This might be a consequence of Darcy's provincial career in Dijon, his short career lifespan (Figure 7) and a lack of teaching involvement. It might also illustrate that some of Darcy's findings were not widely used until the twentieth century (e.g. Darcy friction factor). The omission of the names of Antoine Chézy (1717–1798) and Joseph Valentin Boussinesq (1842–1929) is noticeable but more understandable. Chézy was less famous and Boussinesq was still a relatively young researcher in the 1880s.

Figure 8. Inscription Bélanger on the Eiffel Tower (Tour Eiffel) between Lagrange and Cuvier, with Poncelet and Bresse on the left. Photograph taken on 25 July 2008

5. CONCLUSION

Bélanger (1790–1874) is a forgotten champion of hydraulic engineering and fluid mechanics, but his name engraved on the Eiffel Tower is a reminder of his praiseworthy contributions (Figure 8). His life spanned over six major political regimes in France. For nearly 45 years, J. B. Bélanger generated a major stimulus in the development of rationale mechanics, fluid dynamics and open channel hydraulics. His works influenced leading fluid mechanics researchers ranging from H. Darcy and J. Boussinesq, to E. Mach, P. Forchheimer, B. Bakhmeteff and many more.

The unusual feature of Bélanger's contribution was his successful involvement in both professional engineering (1816–1838) and academic teaching (1838–1864), and his abilities to develop new fundamental textbook materials. As an academic, he was highly respected by his former students and peers in France and overseas. Another aspect was the longevity of his career.

Considering his contribution to hydraulics and fluid mechanics, the backwater equation should be renamed the 'Bélanger equation' (Chanson, 2009b), the maximum discharge equation for a given specific energy should be called the 'Bélanger principle' (Jaeger, 1956) and the application of the momentum principle to the hydraulic jump should be renamed the 'Bélanger method'.

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