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Discussion

Critical flow in rockbed streams with estimated values for Manning's *n*—Comment

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Tinkler (1997) presented interesting experimental observations. I would like to comment some aspects of the paper regarding 'critical flow' (or more accurately near-critical flow), 'critical slope' calculations and boundary shear stress distributions in undular flows. I hope that the information will assist in refining Tinkler's work.

1. Is undular flow equivalent to critical flow?

In his paper, Tinkler (1997) based his calculations of the coefficient of mean friction on the assumption that the flow is uniform equilibrium and critical, i.e., the mean bed slope equals the critical slope. I do not believe that all the observations of undular flows (reported by Tinkler) can be identified as critical flows and critical slope conditions.

At critical flow conditions [1], the relationship between specific energy and flow depth (e.g., Henderson, 1966 pp. 31–34) implies an infinitely-large change of flow depth for a very-small change of energy. A small change of flow energy can be caused by bottom or sidewall irregularity, by turbulence generated in the boundary layers, or by an upstream disturbance. The 'unstable' nature of near-critical flows is favourable to the development of large free-surface undulations. Free-surface undulations (i.e., undular flows) may occur in numerous circumstances for which the flow is *not* critical (i.e., $F \neq 1$) but near-critical. Near-critical flows (i.e., undular flows) may be observed for $0.3 \leq F \leq 3$ (Chanson, 1995, 1996). For example, the wave train of undular jumps (Fig. 1) is observed in the subcritical section of the jump, critical flow taking place upstream of the first wave crest (Chanson and Montes, 1995a,b,Montes and Chanson, 1998).

The presence of free-surface undulations (i.e., standing waves) reveals near-critical flow conditions but not always critical flows.

2. In conditions of uniform equilibrium flow

The momentum equation along a streamline states the exact balance between the shear forces and the gravity component (e.g., Henderson, 1966; pp. 90–91; Streeter and Wylie, 1981; pp. 227–228). For a wide channel, the momentum equation yields:

$$F = \sqrt{\frac{8}{f} \times s} \qquad (1)$$

where f is the Darcy friction factor, F is the Froude number (Tinkler's Eq. (1)) and s is the bed slope.

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[1] Flow conditions in which the specific energy is minimum are called 'critical flow conditions'.

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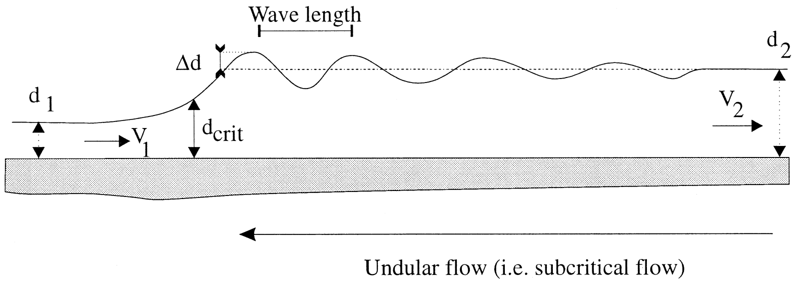

Fig. 1. Sketch of an undular hydraulic jump.

A related form is the empirical correlation called the Gauckler–Manning equation [2]:

$$F = \frac{1}{n} \times \sqrt{\frac{s}{g} \times d^{1/3}} \quad \text{wide channel} \tag{2}$$

where g is the gravity constant and n is the Gauckler–Manning coefficient.

Using the expression of the critical slope s_{crit} (defined as $F = 1$):

$$s_{crit} = \frac{f}{8} \quad \text{wide channel} \tag{3a}$$

$$s_{crit} = n^2 \times g \times d^{-1/3} \quad \text{wide channel} \tag{3b}$$

the bed slope in uniform equilibrium flow may be rewritten:

$$s = s_{crit} \times F^2 \quad \text{wide channel} \tag{4}$$

Eq. (4) is general. In the particular case of undular flows (i.e., near critical flows), it illustrates that Tinkler's Eqs. (3a), (3b) and (4) are inaccurate by a factor F^2. For example, for an undular hydraulic jump with inflow Froude number $F_1 = 1.5$, the average Froude number of the undular section is about $F = 0.7$, $s/s_{crit} = 0.47$, and Tinkler's Eqs. (3a), (3b) and (4) would overestimate the Gauckler–Manning coefficient n by one third.

3. Bed-shear stress in undular flow

The approach of Trinkler implies that the boundary shear stress is basically uniformly distributed in undular flows. This is not accurate and I recently performed new experiments that demonstrate the point.

A 20-m long fixed-bed channel (0.25-m wide rectangular cross-section) was used to investigate the boundary shear stress in undular jump flows. The bed shear stress was measured with a Prandtl–Pitot tube used as a Preston tube. The Pitot tube was calibrated in situ in uniform equilibrium flows (see Chanson, 1997).

Typical results are presented in Fig. 2, where the dimensionless shear stress $\tau_o/(0.5 \times \rho \times V_{crit}^2)$ is plotted as a function of x/d_{crit}, where τ_o is the boundary shear stress, ρ is the water density, d_{crit} and V_{crit} are, respectively, the critical depth and velocity, and x is the longitudinal distance from the channel intake. The data (Fig. 2) show large fluctuations of boundary shear stress in the longitudinal direction x and cross-wise direction z ($z = 0$ at sidewall). The bed shear is minimum at wave crests and maximum at wave troughs.

Further small variations of sidewall roughness (e.g., in a flood plain) affect substantially the boundary shear stress distributions. I performed two experiments with identical inflow conditions ($d_{crit} = 0.100$ m, $F_1 = 1.3$, $s = 0.67\%$). In one experiment (Ref. WZ3_1), the channel bed and sidewalls were glass panels. In the other experiment (Ref. WZ3_2), the channel bed was glass and the sidewalls were embossed aluminium panels (0.3-mm deep, stucco pattern, very-smooth finish [3]). The results are presented

[2] Eq. (2) was first proposed by the Frenchman Gauckler (1867) based upon the re-analysis of experimental data obtained by Darcy and Bazin (1865). It was later presented by the Irishman Manning (1890).

[3] Surface inspection by electronic microscopy of the aluminium sheets (Alcan™ Ref. KS05-10-1200-240-STU) confirmed this point.

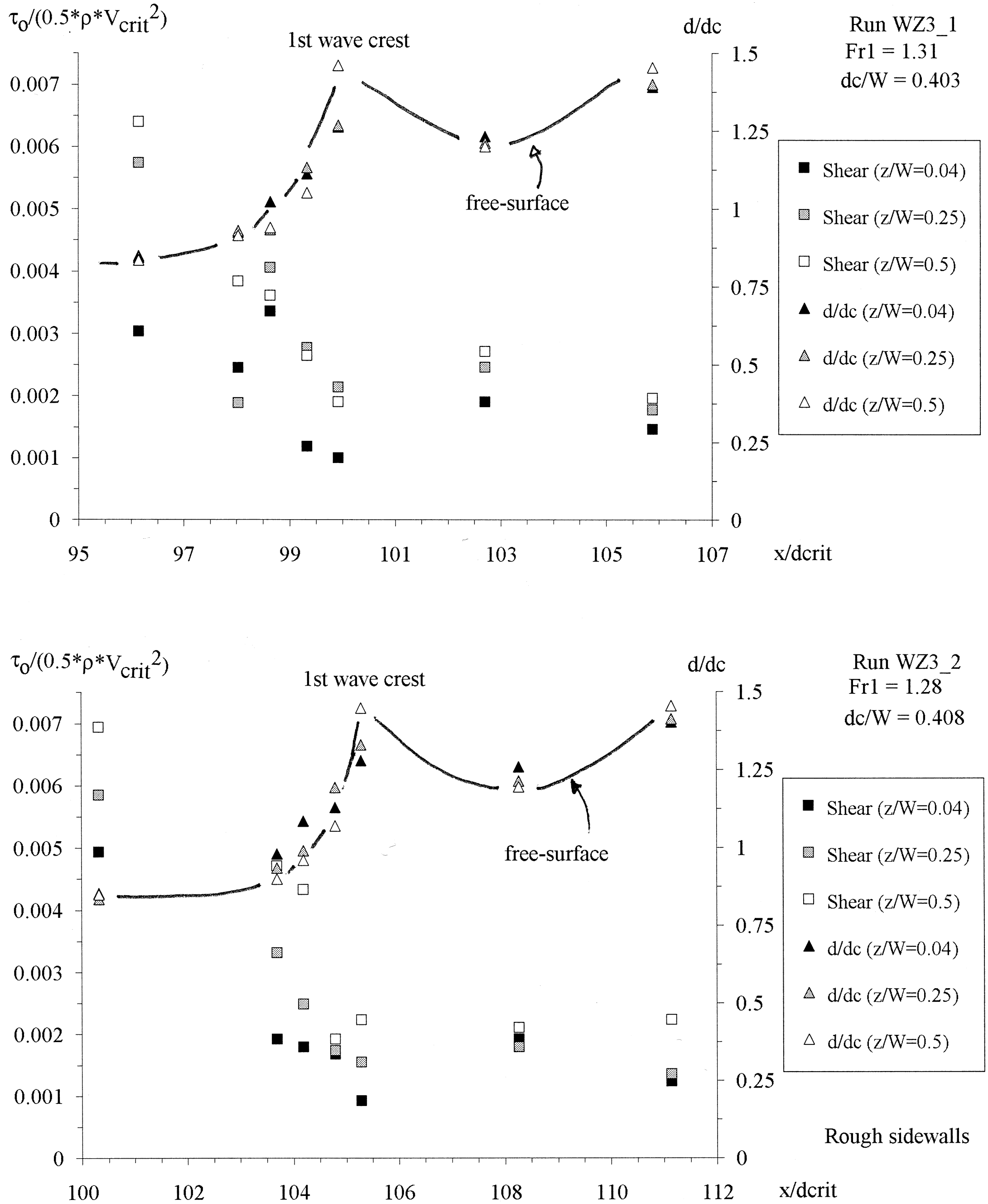

Fig. 2. Bed shear stress along an undular hydraulic jump. (Top) Flow conditions: $F_1 = 1.3$, $d_{crit} = 0.100$ m, $W = 0.25$ m (Run WZ3_1) (Smooth sidewalls). (Bottom) Flow conditions: $F_1 = 1.3$, $d_{crit} = 0.100$ m, $W = 0.248$ m (Run WZ3_2) (Rough sidewalls).

in Fig. 2 (top: Exp. Ref. WZ3_1, bottom: Exp. Ref. WZ3_2). They show different distributions of boundary shear stress near the first wave crest between experiments WZ3_1 and WZ3_2. The changes may be accounted for the difference in sidewall friction.

It is worth noting that undular hydraulic jumps are characterised by the development of lateral shock waves intersecting on the first wave crest. Montes (1986) suggested that the shock waves are connected with the existence of sidewall boundary layers and that the boundary layers would force the apparition of critical flow conditions sooner near the wall. Altogether, the sidewall shock waves are dominant features of undular jumps (Chanson and Montes, 1995a,b,Montes and Chanson, 1998) and they result from the interactions between the sidewall boundary layers and the bed boundary layer.

In natural channels, the photographs of undular flows (presented by Tinkler) show identical flow patterns as those observed in laboratory and it is believed that the distributions of boundary shear stress would also be three-dimensional. As a result, the approximation of 'mean friction loss coefficient' is not an accurate representation of the boundary shear stress. It should not be used in movable boundary channel because it will predict very poorly sediment motion in undular flows (i.e., near-critical flows).

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