My first observation on Fig. 1 is that empirical relationships such as (3) perform better than the Rankine model in the transition between rotational and irrotational domains, that is, $0.5 < \bar{r} < 2$. Nevertheless, the experimental results also confirm the suitability of the Rankine model with constant circulation, $\Gamma = 2\pi r v_\theta$, when $2 < \bar{r} < 14$.

The discussion focuses on whether or not the radial velocity depends on the eddy viscosity, as stated in the authors' conclusion. In previous experiments with a mechanically driven vortex, Julien (1986) measured the eddy viscosity from sediment diffusion under conditions where the radial velocity was nonexistent. A relationship between eddy viscosity and radial velocity must therefore be viewed with suspicion. In the paper, the authors link radial velocity and eddy viscosity through a simplified form of the equation of motion in (18). I question the relevance of this approach at least in the range $2 < \bar{r} < 14$, because the Rankine velocity profile with constant circulation, rv_θ = constant, demonstrates that both sides of (18) are identically zero. Instead of an analysis of second-order momentum terms, may I suggest a first-order continuity analysis that seems more promising. To examine the fundamental nature of this problem, consider a circular cylinder of radius r and forebay depth d. The tangential velocity is v_θ and the radial velocity is $-v_r$ toward the center of the cylinder. At $\bar{r} > 2$, the constant circulation is given by $\Gamma = 2\pi r v_\theta$, and the steady intake discharge at the bottom end of the cylinder calculated by continuity is $Q = 2\pi r d v_r$. It is simple to combine the expressions for Γ and Q to obtain

$$\Gamma = \frac{v_\theta}{v_r}\frac{Q}{d} \tag{25}$$

It is interesting to note that (25) is solely a result of continuity, while the authors found $\Gamma = 1.94Q/d$ after complex consideration of eddy viscosity in (18)–(21). Instead of Fig. 4, it should be rewarding to plot v_θ versus v_r for the experiments; the graphical slope of the expected straight line can be used in (25). I suspect that the very high values of v_r in the authors' experiments can be attributed to the presence of the pier in the close proximity of the vortex, as shown in Fig. 7(a), which forces streamline deviation toward the air core.

As a last point of discussion, the authors suggest that the eddy viscosity should be linearly proportional to circulation in (21), based on dimensional similarity. This seems plausible but it is not convincing because the kinematic viscosity could also be used for the same reason. For instance, equation (19) in Julien (1986) showed that the similarity parameter for the distribution of silts in a vortex is of the form $\Gamma^2/\varepsilon\nu$. Two experimental points were available from the same study, namely ε = 1.67 and 9.4 cm^2/s, respectively at Γ = 410 and 1,230 cm^2/s, while $Q = 0$. I attempted to check these against (20) and (21) with limited success. Comparisons with (20) are impossible because $Q = 0$ in the experiment, whereas k_2 is about an order of magnitude smaller than when calculated using (21). It is impossible to ascertain the nonlinearity of ε versus Γ with only two points. With their experimental data, the authors could plot ε versus Γ to demonstrate the linearity of (21).

With three-dimensional velocity measurements, the authors definitely contribute to fundamental understanding of the mechanics of air-core vortices. A summary table including key experimental measurements would also be useful for further reference.

JET FLOW ON STEPPED SPILLWAYS[a]

Discussion by Hubert Chanson[3]

INTRODUCTION

The authors presented an interesting commentary on the nappe-flow regime above stepped spillways. The writer would like to add some information on energy dissipation calculations of nappe flow and discuss the comparison between the rate of energy dissipation with nappe flow and skimming flow. It will be shown that, in fact, more flow energy is dissipated with a skimming flow regime.

ENERGY DISSIPATION IN NAPPE-FLOW REGIME

In a nappe-flow regime, the total head loss on the spillway ΔE equals the difference between the maximum head available E_0 and the residual head at the spillway toe. For an ungated spillway (Fig. 1), the writer (Chanson 1993) showed that it yields

$$\frac{\Delta E}{E_0} = 1 - \left(\frac{0.54 \left(\frac{y_c}{h}\right)^{0.275} + \frac{3.43}{2}\left(\frac{y_c}{h}\right)^{0.55}}{\frac{3}{2} + \frac{H}{y_c}} \right) \tag{6}$$

In Fig. 4, (6) is compared with experimental data (Moore 1943; Rand 1955; Horner 1969; Stephenson 1979). The results indicate a reasonable agreement. For professional engineers, (6) is simpler than (2) and it does not require an empirical estimate of the rate of dissipation at each step.

COMPARISON OF ENERGY DISSIPATION BETWEEN NAPPE AND SKIMMING FLOWS

Several researchers (Ellis 1989; Peyras et al. 1991), including the authors, suggested that there is much higher energy dissipation in nappe flows than in skimming-flow situations. But, in a recent paper (Chanson 1994), the writer showed that, for long stepped channels where uniform flow conditions are reached, higher energy dissipation takes place in a skimming flow regime.

Such a result is illustrated on Fig. 5, where the energy dissipation with nappe flow (Horner 1969) is compared with energy dissipation of skimming-flow data. Fig. 5 shows consistently that the nappe-flow data indicate a lesser energy dissipation than skimming-flow data. Note that, although Fig. 5 suggests that the difference is small, it is more appropriate to consider the residual energy [i.e., $(1 - \Delta E/E_0)$]. Horner's (1969) data show that the residual energy with nappe flows is 50–100% larger than for the skimming-flow data (Fig. 5).

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[a] February, 1994, Vol. 120, No. 2, by M. R. Chamani and N. Rajaratnam (Technical note 5344).

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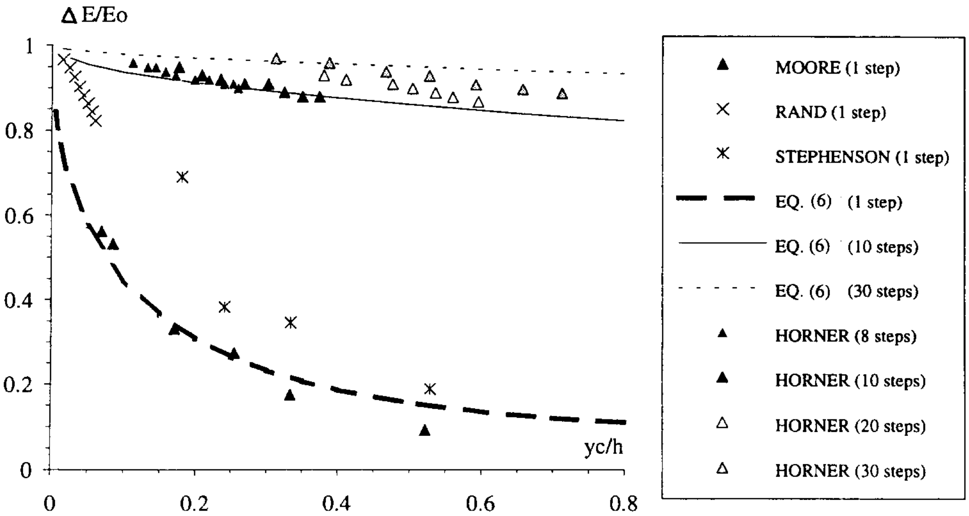

FIG. 4. Energy Dissipation in Nappe Flow Regime—Comparison between Eq. (6) and Data

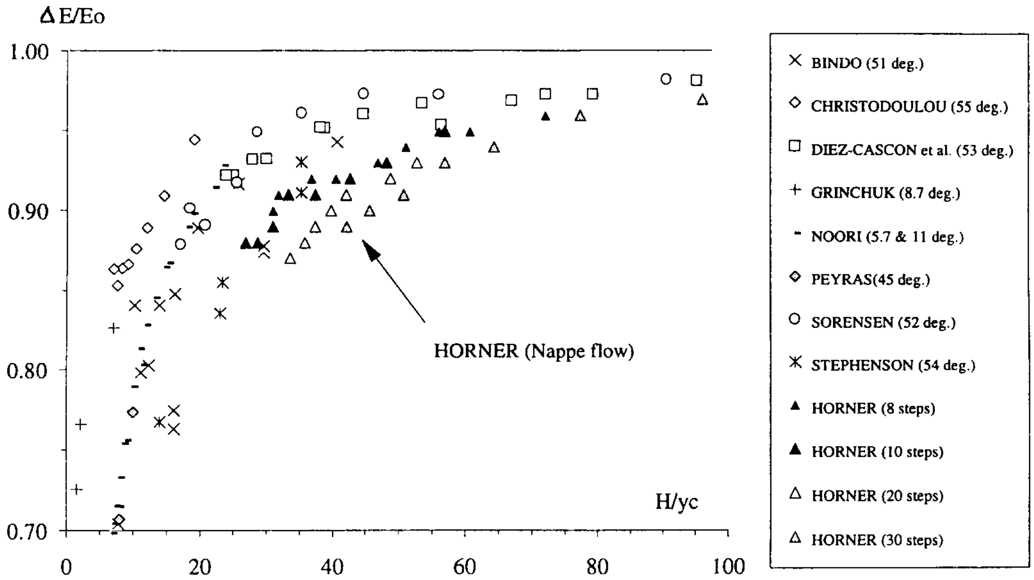

FIG. 5. Comparison of Energy Dissipation in Nappe-Flow Regime (Horner 1969) and Skimming-Flow Regime

For long chutes where uniform flow conditions are reached, higher energy dissipation takes place in a skimming-flow regime. But, for short channels, nappe flows would dissipate more kinetic energy than skimming flows. In a nappe-flow regime, energy dissipation takes place at each step. It is believed that nappe-flow situations can dissipate higher energy than skimming-flow regime on short chutes. It must be noted, however, that for a given discharge, a nappe-flow regime requires flatter slope and larger steps than a skimming-flow regime. In some cases, such requirements might increase the cost of the structure or are not possible.

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Discussion by Jorge Matos[4] and António Quintela[5]

After presenting an innovative method to estimate the energy loss on stepped spillways for jet (nappe) flow, the authors have concluded that the average energy loss per step for skimming-flow regime is expected to be less than that for jet flow. The writers considered it of interest to also compare the total energy loss downstream of an N-step spillway, for the same values of spillway total height H, critical depth y_c (or water discharge per unit width q_w), and spillway slope (h/l). The results seem to show that the total energy loss for skimming flow is less than that for jet flow. On the basis of the authors' work, a formula is presented to verify if the uniform jet flow is established upstream of the Nth step.

UNIFORM FLOW

The uniform jet flow is reached when $(E_N - E_{N-1})$ approaches zero. From (1), the following equation is obtained:

$$\frac{E_N - E_{N-1}}{E_N} = \frac{(1-\alpha)^{N-1}\left[(1-\alpha) - 1.5\alpha\left(\frac{y_c}{h}\right)\right]}{(1-\alpha)^N\left[1 + 1.5\left(\frac{y_c}{h}\right)\right] + \sum_{i=1}^{N-1}(1-\alpha)^i} \tag{7}$$

Fig. 6 is a result of (7), where α is obtained from (3)–(5), for two spillways with the same slope ($h/l = 0.842$): one for $N = 10$ and $20 \leq Nh/y_c \leq 100$ and the other for $N = 30$ and $40 \leq Nh/y_c \leq 100$. Fig. 6 shows that the uniform flow is practically reached upstream of the Nth step (toe of the spillway) for most of the Nh/y_c values within the mentioned range, both for $N = 100$ and $N = 30$. This conclusion was verified for the other values of h/l analyzed by the authors and is in agreement with the observations of Essery and Horner (1978).

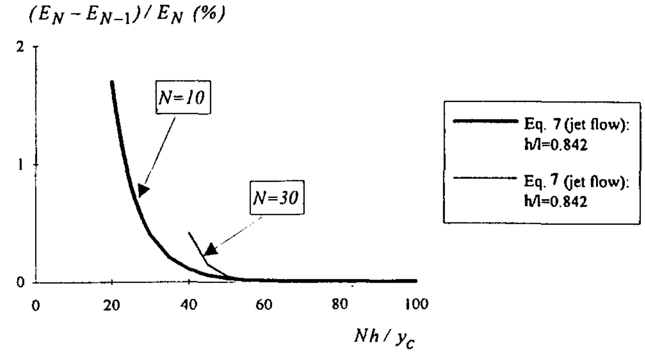

FIG. 6. Specific Energy Difference near Toe of *N*-Step Spillways

ENERGY DISSIPATION

Assuming that uniform flow conditions in the skimming-flow regime are reached upstream of the spillway toe, the friction slope equals the spillway slope, and, as indicated in Stephenson (1991) and Chanson (1994), $\Delta E/E_0$ can be expressed as a function of H/y_c. Fig. 7 shows values of $\Delta E/E_0$ obtained from skimming-flow experiments in which different measurement techniques were used. The $\Delta E/E_0$ values for the second series of the model tests of Diez-Cascon et al. (1991) are recalculated by the writers using the momentum equation to estimate the equivalent water depth upstream of the hydraulic jump (at the toe). This indirect or nonintrusive method has already been referred by Stephenson (1991) and Diez-Cascon et al. (1991), and applied by Tozzi (1992). The recalculated $\Delta E/E_0$ values plotted in Fig. 7 are considerably lower than those obtained from Table V in Diez-Cascon et al. (1991) as well as those plotted in Fig. 2 of Chanson (1994). In fact, ΔE can be significantly overestimated if it is calculated on the basis of the aerated-flow depth instead of the equivalent water depth. On the other hand, the $\Delta E/E_0$ values obtained from the experiments of Sorensen (1985) are considerably higher than those obtained for the other studies. However, it appears that these values might be overestimated. The writers verified that the flow depths on the horizontal slope downstream of the toe in Sorensen's experiments (calculated from available values of discharge and toe velocity) are generally slightly higher than the normal water depths on the steps at which air entrainment commences. This observation leads to the conclusion that perhaps air was not significantly released from the flow. Fig. 7 also contains two

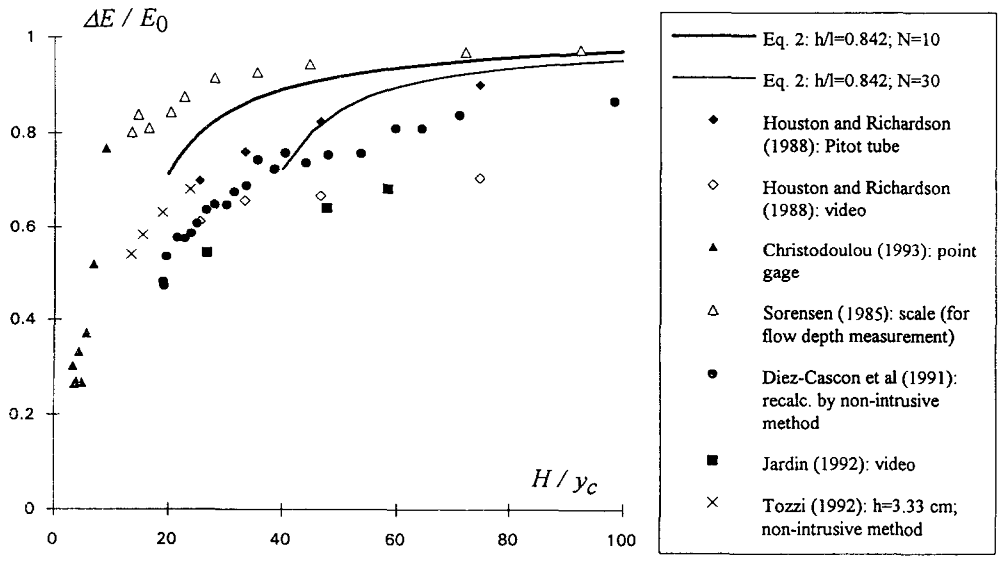

FIG. 7. Energy Loss Ratio in Stepped Spillways

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graphs for jet flow resulting from (2), one for $h/l = 0.842$ ($\theta = 40°$) and $N = 10$, and the other for the same h/l and $N = 30$.

Fig. 7 shows that (1) for the same value of H/y_c (or spillway height and water discharge per unit width) and spillway slope, the total energy loss in a jet flow is greater in the case that $N = 10$ than for $N = 30$. The difference in $\Delta E/E_0$ reduces as H/y_c increases. This tendency was verified for the other values of h/l analyzed by the authors; and (2) the total energy losses obtained from the experiments of Diez-Cascon et al. (1991), Tozzi (1992), Jardin (1992), and Houston and Richardson (1988) are in general lower than those estimated for jet flow.

Taking into account that for the same slope and H/y_c the total energy loss in jet flow decreases when the number of steps increases (this means that the flow regime tends to a skimming flow), it seems reasonable to accept that the total energy loss in jet flow is greater than that observed for skimming flow and that the difference in $\Delta E/E_0$ reduces when H/y_c becomes large. This is to be expected once a quasi-uniform flow is attained over a considerable extension of the spillway; in fact, for the same spillway slope the friction slope is the same for a uniform jet flow and for a uniform skimming flow.

Although the results obtained using the nonintrusive method seems to be relatively accurate, a more developed research taking into account the effects of air entrainment is considered important to obtain a precise estimate of $\Delta E/E_0$.

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Discussion by P. Veerabhadra Rao,[6] Member, ASCE, and P. L. N. Rao[7]

The authors are to be commended for a unique presentation of the results pertaining to the ratio of energy lost on the spillway and the proportion of energy lost per step as a function of y_c/h for different values of h/l and N in the jet-flow regime. The interesting aspects of the results are (1) small values of y_c/h result in large values of α; and (2) universalization of α versus y_c/h curves including the observation of the α value being independent of h/l for $y_c/h \geq 0.25$. While discussing the jet flow over gabion weirs, Peyras et al. (1992) classified nappe flows into isolated nappe flows (at small flow rates, i.e., $y_c/h < 0.5$) and partial nappe flows (at higher discharges, i.e., $0.5 \leq y_c/h \leq 0.8$). However, one of the present authors (Rajaratnam 1990) suggested a simple nappe-flow regime (i.e., $y_c/h \leq 0.8$). The present writers wish to bring the attention of authors to the 3D flow over the steps of Chew Valley spillway (*Hydraulic* 1978) shown in Fig. 8, which is very pertinent to the present results and explains the dissipation phenomenon in some detail during a nappe-flow regime and at low y_c/h values. The following are our further observations and comments.

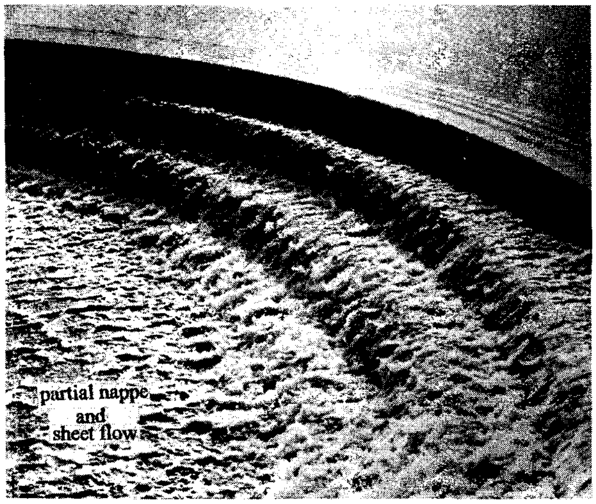

FIG. 8. Flow over Steps (Chew Valley Spillway) (Reproduced with Permission of Controller of Her Majesty's Stationery Office)

For low discharges, the maximum energy loss occurs as the friction factor tends to be very high. This was reported by earlier investigators including one of the present authors. Apart from 2D experimental observations and explanations advanced by different investigators, Fig. 8 clearly depicts a 3D dissipation process when $y_c/h << 1$ for jet flow over steps including the formation of a partial hydraulic jump. The actual process of energy dissipation appears to be more involved than the simple explanation from 2D experiments.

According to Chanson (1994), the onset of skimming flow is when $y_c/h = 1.057 - 0.465\,(h/l)$. For jet flow $y_c/h \leq 0.8$. Substitution of this value in the above relation gives $h/l \geq 0.43$. This relationship is a condition for jet or nappe flow to continue. From Fig. 3 it is clear that for $y_c/h < 0.25$, the values of α are very high and independent of h/l. Does this result mean that nappe flow tends to sheet flow at all h/l ratios, possibly without the jet separating from the surface to effectively dissipate maximum energy? Analysis of the authors' results and (3) also indicates that the effect of N appears negligible on the relation of $\Delta E/E_0$ at $y_c/h = 0.2$. On the other hand, the effect of N is prominent of $\Delta E/E_0$ at high y_c/h ratios ($0.4 \leq y_c/h \leq 0.8$). The relations are very interesting but need clarifications with respect to the experimental observations.

The authors stated "that for skimming flow, which occurs for y_c/h larger than 0.8, an analysis of the observation of Sorensen as well as Fig. 3 indicate that the average energy loss per step would be less than that of jet flow." This is believed to be true (Chanson 1994) for short channels. On the other hand, a skimming-flow regime enables higher energy dissipation than a nappe-flow regime for long chutes. The observations by the authors may be explained with special

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reference to flow mechanisms as to why such energy dissipation characteristics are possible.

The authors have presented an equation [(3)] for α as a function of y_c/h for different values of h/l and another equation [(2)] for energy loss. These equations appear to have a close relation to the equation presented by Chanson (1994) for energy loss in jet-flow regime on stepped spillways. Dissipation of more energy in a nappe-flow situation, as noted by the authors, however, depends on the long or short channels.

The analysis of the results of Horner for stepped drops with steps having a reverse slope also supported the idea of more loss for multiple steps than for a single step, as presented by the authors. Spillways with wedge-shaped block (with adverse slope) technology were used in Russia since 1976 (Baker 1994). In this case, the optimum conditions for maximum energy dissipation occur at a different h/l value. Fig. 3 indicates that the energy loss increases as h/l decreases for a constant y_c/h ratio. Indeed, the unified results presented in Fig. 3 are very interesting. Recently Christodoulou (1993) also arrived at different universalized relationship that is, a $\Delta H/H_0$ versus y_c/hN curve for nappe and skimming-flow regimes (where H = local energy head, above chosen step; H_0 = upstream head, referred to step elevation; and $\Delta H = H_0 - H$). It may be noted that the ranges of y_c/h in these two studies are different. In spite of this, general nature of presentation in both studies are identical and are very good for universal representation of data sets.

Recently, it has been reported by McCorquodale and Mohamed (1994) that the length of roller of an adverse jump is less than that of an equivalent horizontal jump. It has further been observed that the adverse sloping jump has a lower energy dissipation than the equivalent horizontal jump. We ask that the authors comment on the energy-dissipation characteristics with respect to jet flow and jump phenomenon with respect to adverse slopes with specific reference to stepped spillways.

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Discussion by Sandip P. Tatewar[8] and Ramesh N. Ingle[9]

The authors are to be commended for developing a method to estimate the energy loss on stepped spillways for the jet-flow regime, as the procedure for accurate estimation of this energy loss is not available in literature. The authors' method is based on a regression analysis of the experimental data of Horner (1969), and hence predicts the energy loss accurately when applied to Horner's experimental observations. It is necessary to determine the accuracy of (2)–(5) for prediction of energy loss for the experimental data of others or for prototype observations. Using Rand's (1955) expression for the depth at the toe of the nappe of a straight-drop spillway, the energy loss over a stepped spillway for jet flow can be expressed as

$$\frac{\Delta E}{E_0} = 1 - \frac{(y_1/h) + 0.5D(h/y_1)^2}{N + 1.5(y_c/h)} \tag{8}$$

where

$$y_1/h = 0.54D^{0.425} \tag{9}$$

and D is drop number

$$D = q^2/gh^3 \tag{10}$$

The writers predicted the relative energy loss from (8) for all the values shown in Fig. 2. Comparison of relative energy loss predicted by (8), (2) and (3), and Horner's experimental observations is shown in Fig. 9. Compared to experimental observations, the average error in prediction from (2) and (3) is 0.85% with standard deviation of 0.64, while in prediction from (8), it is 2.89% with standard deviation of 1.63. Since prediction by (8) is based on Rand's (1955) expressions, which

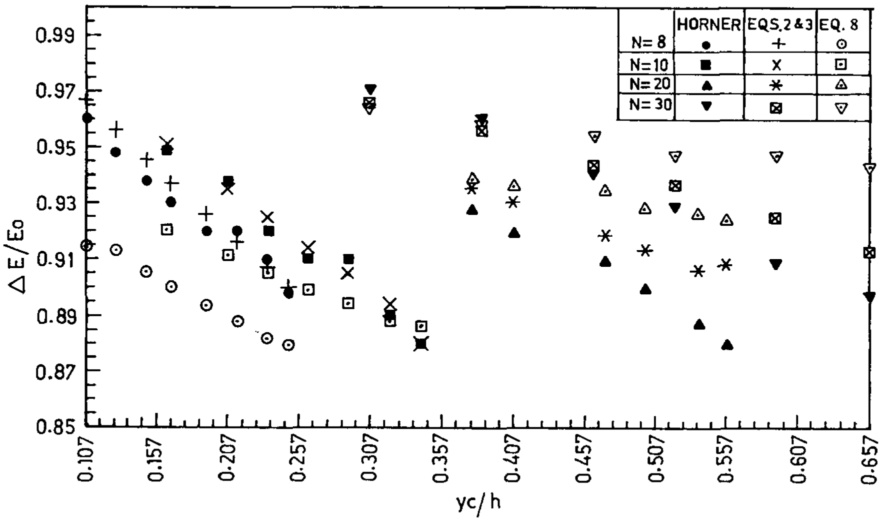

FIG. 9. Comparison of Eq. (8) with Eqs. (2) and (3) and Horner's Experimental Data

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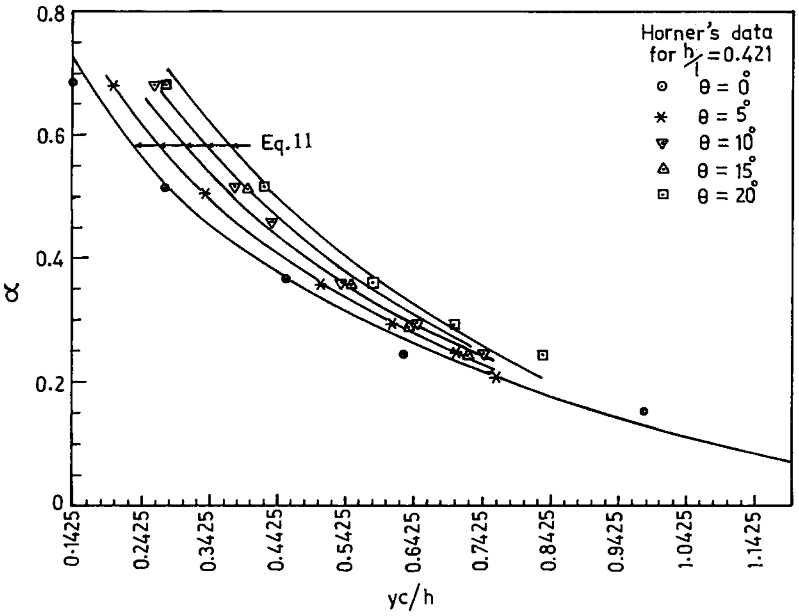

FIG. 10. Variation of α with y_c/h for Different Values of θ

are derived from his own experimental data and those of Moore (1943) and Bakhmeteff and Feodoroff, it cannot be as accurate as prediction from (2) and (3) for Horner's experimental data. However, it can be considered more general and comparatively convenient for application.

It is expected that the reverse slope of steps would increase the relative energy loss on stepped spillway in a nappe-flow regime. To study the effect of the angle of inclination of reverse slope (θ), the writers carried out a regression analysis, following the procedure suggested by the authors for determining α. For constant value of $h/l = 0.421$, the variation of α with θ in degrees can be described by

$$\alpha = A - B \log(y_c/h) \tag{11}$$

where the coefficients A and B are described by the following equations:

$$A = 0.12712 - 0.000280\theta \tag{12}$$

$$B = 0.70794 + 0.017240\theta \tag{13}$$

where θ is in degrees.

The variation of α with y_c/h for different values of θ is shown in Fig. 10. The continuous lines represent (11), and the points represent the experimental data. For the same value of y_c/h, α increases with θ as expected. For y_c/h more than 0.8, α becomes independent of θ, probably because the flow regime changes to skimming flow.

Closure by M. R. Chamani,[10] and N. Rajaratnam,[11] Member, ASCE

The writers appreciate the interesting comments from all the discussers. The comments and questions of the discussers have enhanced the value of this technical note. The comparison of energy loss between jet and skimming flows presented by Matos and Quintela, and Chanson as well as the photograph provided by Rao and Rao are interesting and useful.

The main thrust of this note was to develop a method to estimate the energy loss on stepped spillways for the jet-flow regime. Since it was difficult to accomplish this from a physical understanding of the flow, we introduced the concept of relative energy loss per step α, which was evaluated from the extensive experimental observations of Horner.

In the discussions, three questions were raised: (1) Does the prediction of relative energy loss from the simple equation of Rand differ considerably from that predicted by the writers' method? (2) Is the relative energy loss in the jet-flow regime greater than that in the skimming-flow regime? and (3) Why does the adverse slope of the steps increase the relative energy loss? We attempt to address these issues in the following paragraphs.

The first equation of Tatewar and Ingle, (8), is essentially the same as (6), in Chanson's discussion, and is based mainly on Rand's work. Eq. (7), in Matos and Quintela, is also based on the assumption that the head loss in the developed or uniform flow region at any step in the jet-flow regime equals the step height. Hence, it was concluded that for the last step, the flow is similar to that at a single drop. Then Rand's equation an be used to estimate the depth of flow at the base of a stepped spillway. In Rand's study, the approaching flow was subcritical and the nappe was aerated. For stepped spillways, after the first step, the flow is likely to be supercritical on the other steps. Second, stepped spillways are not aerated. Our observations in the laboratory indicate that the pocket behind the falling jet is generally filled with water and this would enhance the energy dissipation. It may also be pointed out that if the uniform-flow assumption is extended to the last step, the total head loss would be equal to the spillway height. Then, the relative energy loss can be written as

$$\frac{\Delta E}{E_0} = \frac{H}{H + 1.5y_c} = \frac{N}{N + 1.5\left(\frac{y_c}{h}\right)} \tag{14}$$

To compare the different equations for relative energy loss, the experimental results of Horner for the 30-step model for

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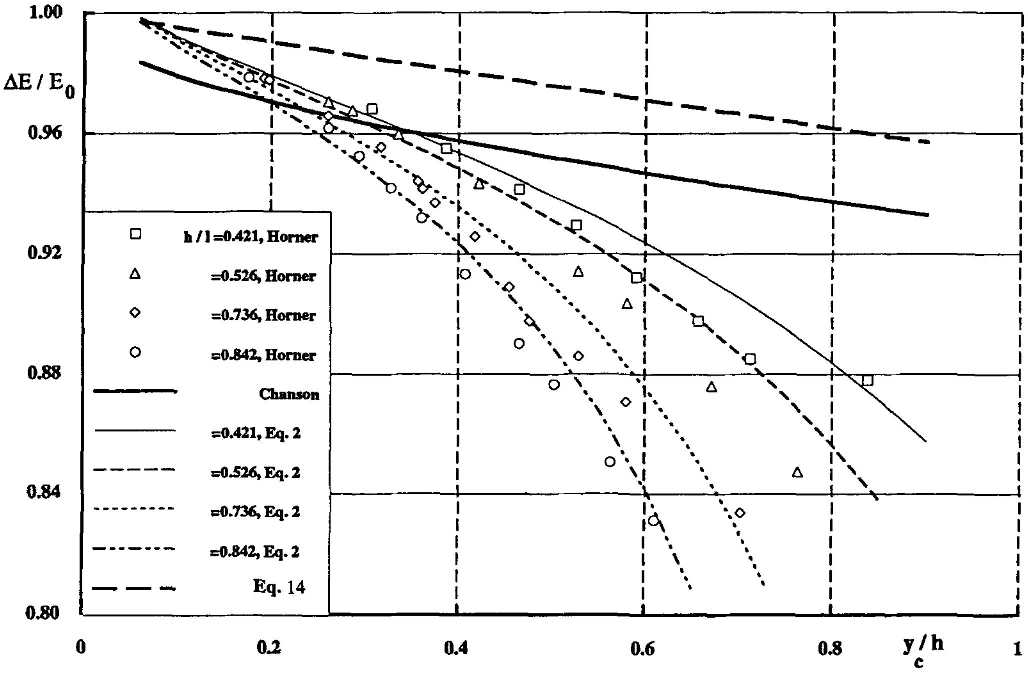

FIG. 11. Variation of Relative Energy Loss with h_c/h for 30-Step Model

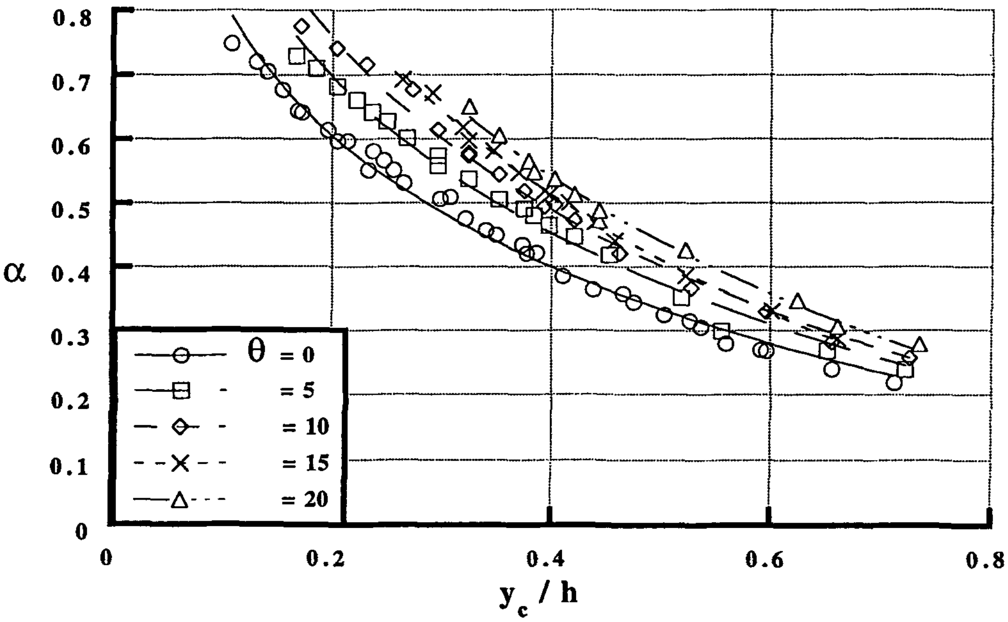

FIG. 12. Variation of α with y_c/h for h/l = 0.421 for Inclined Steps

different values of h/l are shown in Fig. 11 along with the plots of (6), Chanson, and (2) and (14). Fig. 11 shows that (2) shows better agreement with the experimental observations of Horner than (6) or (14). Further, this agreement improves as the slope of the stepped spillway increases.

Now addressing the second question, we refer to the discussion supporting the statement that the relative energy loss $\Delta E/E_0$ in the jet-flow regime is greater than that observed in the skimming flow. Matos and Quintela plotted the experimental data for both regimes and pointed out that the difference becomes negligible for larger values of H/y_c. By plotting different sets of data, Chanson showed that the jet-flow regime dissipates less energy than skimming flow. Rao and Rao, and Chanson commented that for a short channel, $\Delta E/E_0$ for jet flow is larger than that for skimming flow, whereas the reverse is true for long stepped chutes.

The writers only mentioned that α is greater for jet-flow regime. The lack of a precise solution for estimating $\Delta E/E_0$ in skimming flow makes it difficult to make a proper comparison. The method proposed by Rajaratnam (1990), which was used later by Chanson (1993) in a different form, is based on the idea that uniform-flow condition is reached on stepped spillways. The estimated energy loss depends on evaluating the friction factor f, which appears to vary over a wide range. This makes it difficult to calculate $\Delta E/E_0$ precisely. The scatter in the data shown in Fig. 7 demonstrates this point. The flow characteristics of these two regimes are different and they occur under different conditions.

As to the third question, we simply pointed out that the provision of an adverse slope helps to form at least a partial jump that makes some contribution to the dissipation of energy. Tatewar and Ingle showed that variation of α with y_c/h

for different values of the adverse slope for one value of h/l. We present herein Fig. 12 (from the thesis of the first writer), to show the variation of α with y_c/h for $h/l = 0.421$ in more detailed form. Similar results for other values of h/l are available in the thesis of the first writer. The variation of α with y_c/h for adverse slopes, where θ is the angle of the step, can be described by

$$\alpha = c + d \log(y_c/h) \tag{15}$$

wherein the coefficients c and d are described by the following equations:

$$c = 0.476 + 0.431 \cos(\theta) + [-2.043 + 1.583 \cos(\theta)]\log(h/L) \tag{16a}$$

$$d = -0.795 - 0.223(\theta) + [-0.187 + 1.657(\theta)]\log(h/L) \tag{16b}$$

INCIPIENT MOTION OF SAND-GRAVEL SEDIMENT MIXTURES[a]

Discussion by Roger Bettess[2]

The discusser found the paper interesting, and the author should be congratulated on his contribution to the subject.

In treating problems of initiation of motion, there is always a difficulty in defining what is meant by initiation of motion. The author has chosen to define initiation as the point at which the nondimensional sediment transport rate achieves a fixed value. To carry out his procedure, the author based his technique on the work of Parker et al. (1982) and used the nondimensional sediment transport rate W_i^* defined by (3).

In many sediment transport problems the method by which the sediment transport rate is nondimensionalized does not have a significant impact on the results of a study. In this case the writer suspects that the equation used could have a significant impact on the calculated results. It is of interest, for example, that (3) does not directly involve the grain size D_i. This may perhaps explain the insensitivity to grain size that is demonstrated in Fig. 5. The writer would be interested to know whether similar results would have been obtained if an alternative form of nondimensional sediment transport, for example, that proposed by Ackers and White (1973), had been used.

An alternative definition of initiation of motion can be based on the observed movement of grain sizes. Yalin (1977) has done much significant work on indicating how sediment size should be taken into account in such observations.

It would be interesting to know how the author's approach relates to that of Yalin and whether the two are reconcilable.

APPENDIX. REFERENCES

Ackers, P., and White, W. R. (1973). "Sediment transport: new approach and analysis." *J. Hydr. Div.*, 99(11), 2041–2060.

Yalin, M. S. (1977). *Mechanics of sediment transport*, 2nd ed., Pergamon Press, Elmsford, N.Y.

Closure by Roger A. Kuhnle[3]

I thank Roger Bettess for his discussion and interest in this work.

The definition and measurement of the initiation of motion for sediments in alluvial streams has been a problem for many years. Since some movement of the sediment is necessary for determining the initial motion of the bed, a precisely defined criteria is needed to define initial motion. For unisize sediments Neil and Yalin (1969) defined an initial motion criterion

$$\frac{nD^3}{u_*} = \text{constant} \tag{7}$$

where n = number of grains displaced per unit area per unit time; D = grain diameter; and u_* = shear velocity. From practical concerns Neil and Yalin suggested a value of 10^{-6} for the constant in (7) for the beginning of bed movement. Yalin (1977) introduced another criterion for the beginning of bed movement that was similar to that proposed by Neil and Yalin (1969)

$$\frac{nD^3}{\sqrt{(s-1)gD}} = \text{constant} \tag{8}$$

where s = ratio of the density of the sediment to that of the water; and g = acceleration of gravity.

Neither of these initial motion criteria can be directly applied to mixed-size sediments (Wilcock 1988) because of the difficulty of defining a common time scale for the individual grain sizes in a bed with mixed sizes. Wilcock (1988) proposed using either (7) or (8) for comparisons between different mixed-sediment beds and using the following criterion to scale the observations for the different sediment sizes in each mixed bed

$$\frac{m_i D_i^3}{f_{mi}} = \text{constant} \tag{9}$$

where m_i = number of grains of ith size fraction displaced per unit bed area (= nt); D_i = grain diameter of the ith size fraction; f_{mi} = proportion of the ith size fraction present on the bed surface; and t = time. In practice, however, the large areas, long time periods, or large number of grains that must be used to scale observations of the different grain sizes make (9) very difficult to implement for most mixed-size sediments [see Wilcock (1988)].

The method chosen (Parker et al. 1982) to calculate the reference (or critical) shear stress from transport rates measured from a series of flows over a given sediment bed, does not measure the beginning of movement of the sediment directly, but the scaling problems are less severe than for other

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[a] December, 1993, Vol. 119, No. 12, by Roger A. Kuhnle (Paper 5392).

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