CHARACTERISTICS OF UNDULAR HYDRAULIC JUMPS: EXPERIMENTAL APPARATUS AND FLOW PATTERNS

Discussion by Iwao Ohtsu, Youichi Yasuda, and Hiroshi Gotoh

The authors' paper on the undular jump is valuable, and presents some important characteristics of the undular jump.

The discussers have experimentally investigated undular jumps in horizontal-rectangular channels with different channel widths, and would like to discuss the effects of the channel width and the Reynolds number on the characteristics of the undular jump.

The discussers' experiments were conducted using three horizontal-rectangular channels of \( W = 20, 40, \) and 80 cm, respectively.

The fundamental factors governing the undular jump are generally expressed as follows: the supercritical inflow Froude number \( F_i \); the inflow condition (the state of the boundary-layer development of the supercritical flow); the aspect ratio \( W/h_i \); and the Reynolds number \( R \) \( (R = q/\nu; \, q = \text{discharge per unit width}) \).

Recently, the discussers proposed the nondimensional factor \( L_{sw}/L_{sw} \), which represents the effect of the channel width on characteristics of the undular jump (Ohtsu et al. 1995). Here, \( L_c \) is the longitudinal length from the toe of the lateral-shock wave to the cross point of the shock wave, and \( L_{sw} \) is the longitudinal length from the toe of the shock wave to the first wave crest (Fig. 14). If the cross point of the shock wave is located upstream of the first wave crest \( (L_c/L_{sw} < 1) \), the effect of the shock wave on the characteristics of the undular jump is large. When the shock wave does not cross upstream of the first wave crest \( (L_c/L_{sw} > 1) \), the effect of the shock wave is negligibly small.

For \( R \approx 6.5 \times 10^4 \), the Reynolds number \( R \) has no effect on the characteristics of the undular jump.

For \( L_c/L_{sw} > 1, F_i \approx 1.2 \), and fully developed inflow, the flow conditions are two-dimensional, although the undulations near the wall are small and unstable (Fig. 15). In this case, the effect of the channel width is not shown. However, for \( L_c/L_{sw} < 1, F_i \approx 1.2 \), and developed inflow, the flow conditions are three-dimensional, and change with \( F_i \) and \( W/h_i \) \( (or \, y_1/W) \).

Also, the flow patterns shown in the authors' paper [Types B to E (Fig. 4)] are observed.

Regarding the hydraulic quantities of the first wave height \( h_{max} \), the wave length \( L_c \), the shock-wave length \( L_{sw} \), and the angle of the shock wave to the side wall \( \theta \), \( h_{max}/h_1, L_c/h_1, L_{sw}/h_1 \), and \( \theta \) are independent of the aspect ratio \( W/h_i \), and depend on the Froude number \( F_i \) for \( L_c/L_{sw} > 1, F_i \approx 1.2 \), and developed inflow. The following equations for \( h_{max}/h_1, L_c/h_1, L_{sw}/h_1 \), and \( \theta \) have been proposed (Ohtsu et al. 1995).

\[
\begin{align*}
    h_{max}/h_1 &= 1.51 \cdot F_i - 0.35, \\
    L_c/h_1 &= 2.75 + 0.90/(F_i - 1) \\
    L_{sw}/h_1 &= 1.30 \cdot F_i + 2.56, \\
    \theta &= 30.6 \cdot F_i^{0.65}
\end{align*}
\]

FIG. 14. Definition Sketch of Undular Jump

FIG. 15. Undular Jump: (a) Flow Condition for \( L_c/L_{sw} > 1 \), Fully Developed Inflow, \( F_i \approx 1.2 \), and \( R \approx 6.5 \times 10^4 \); (b) Photograph for \( L_c/L_{sw} > 1 \), Fully Developed Inflow, \( F_i = 1.52, w/h_1 = 9.90 \), and \( R = 8.6 \times 10^4 \)
FIG. 18. Side Views of Selected Undular Jumps

For the case of \( L_c/L_{sw} < 1 \), \( L_c/h_2 \) and \( L_{sw}/h_1 \) depend on \( F_1 \) and \( W/h_1 \) (or \( y_2/W \)). Also, \( h_{max}/h_1 \) and \( \theta \) show the same results as for \( L_c/L_{sw} > 1 \).

For \( F < 6.5 \times 10^4 \), the characteristics of the undular jump depend on the Reynolds number \( R \) (Ohtsu et al. 1995). The effect of the Reynolds number on the characteristics of the undular jump can be clarified by using test channels with different channel widths (e.g., \( W = 20, 40, \) and 80 cm).

Consequently, the characteristics of the undular jump are quite different between \( L_c/L_{sw} > 1 \) and \( L_c/L_{sw} < 1 \).

Using the nondimensional factor \( L_c/L_{sw}(L_c/L_{sw} = \sqrt{(W/h_1)(2 \cdot \tan \theta)}/(L_{sw}/h_1)) \), it is possible to clarify the effect of the channel width on the characteristics of the undular jump. For fully developed inflow, \( F \geq 1.2 \) and \( R \geq 6.5 \times 10^4 \), if the factor \( L_c/L_{sw} \) is replaced by the fundamental factors \( W/h_1 \) and \( F_1 \), the regions of \( L_c/L_{sw} > 1 \) and \( L_c/L_{sw} < 1 \) and the boundary \( L_c/L_{sw} = 1 \) are shown by Fig. 16. In addition, the authors' experiments for \( R \geq 6.5 \times 10^4 \) may be conducted within the region of \( L_c/L_{sw} < 1 \).

APPENDIX. REFERENCE


Discussion by Willi H. Hager, Member, ASCE, and Roger Reinauer

The authors have presented an interesting paper in a classic topic of open channel flow. After 130 years of its first description by Darcy and Bazin (1865), they classify the flow. Also, they are able to demonstrate that an undular surge and an undular hydraulic jump have few characteristics in common. The undular hydraulic jump is also one of those topics of which the hydraulician might think that nothing new can be added to the existing knowledge. Careful observations and extended analyses give novel insight and reveal interesting features of flow.

One fact is evident: According to Table 1, only few observations on undular jumps are available. Systematic series of experiments were conducted only by Iwasa (1955), Montes (1979), Ryabenko (1990), and Yasuda et al. (1993). The authors' new data series are thus most welcome from this point of view.

Five flow types have been introduced. The particular features of the novel classification are the effects of air entrainment and shock waves. Those two effects are normally not present in undular surges. The undular jump is thus more complex except for type A, which can occur similar to an undular surge. The discussers are not sure if the types D and E are different enough to warrant separate classification, and would like to suggest the modified type D as follows: "At higher Froude numbers, the undular jump is characterized by lateral shock waves that break at the channel axis to form a surface roller. The wave breaking may entrain air bubbles. At the upper limit the roller extends over the entire channel width and blocks the surface undulations."

The discussers have made some additional observations in a smooth horizontal channel of 500 mm width. Photographs to some specific flow patterns are provided, given that the present propositions are not sufficiently documented. The experiments relate to an approach flow depth of 110–120 mm and the flows are partially developed. Typical flow patterns include: (1) incipient shock-wave breaking (transition between types B and C); (2) incipient roller formation (transition between types C and D); (3) organized roller flow (present prop-

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osition type D); and (4) full roller flow. Increasing the Froude number over the upper limit of the fourth step of the preceding list yields the weak direct jump.

Fig. 17 shows views in the flow direction. The differences between the flow types are clearly revealed. Also, the highly spatial currents may be observed, and such flow cannot be called two-dimensional, in contrast to undular surges, that are documented for example by Treske (1994).

Side views as shown in Fig. 18 demonstrate the features of shock waves. In Fig. 18(a) the shocks are thin and continuous; in Fig. 18(b) incipient shock breaking is seen to occur, and all breaking water particles are directed downstream. If breaking is developed, the surface current is unable to direct them toward the tailwater and a central surface roller forms, instead. In Fig. 18(c) the roller extends over a comparatively small portion of the channel width, but tailwater undulations have significantly decreased already. In Fig. 18(d) the stems of shocks can still be recognized close to the channel walls, but the axial portions of shock waves are now covered with the roller. Tailwater undulations have practically disappeared.

Tailwater views also exhibit distinctly different flow patterns for the four flow types considered. In Fig. 19(a) the shock is about to form; in Fig. 19(b) the shock waves are completely present, including the system of reflections just without breaking at the first wave crest. A small central roller has formed in Fig. 19(c) and the shocks are clearly damped. If the surface roller extends nearly over the width of channel as shown in Fig. 19(d), the shockwaves disappear and the weak direct jump is developed.

The authors have suggested transition criteria from one type flow to the other (Table 2). The transitions are characterized by the approach Froude number $F_r$ and the parameter $y_r/W$. The discussers, in turn, would like to point out first that the parameter $y_r/W$ involves nothing else than the approach aspect ratio, given that $y_r/W = F_r^2(\gamma_r/W)$. Also, it can be realized from Table 2 that the transition Froude numbers depend strongly on the aspect ratio. The writers have considered the published data and some selected observations of their own, and found that the undular jump may be significantly influenced by scale effects. Any typical parameter of the undular jump, such as the crest height or wave length, can easily be shown to be functionally related to

- The approach flow depth $y_o$
- The approach aspect ratio $y_r/W$

FIG. 18. Side Views of Selected Undular Jumps

FIG. 19. Views from Tailwater