CHARACTERISTICS OF UNDULAR HYDRAULIC JUMPS: EXPERIMENTAL APPARATUS AND FLOW PATTERNS[a]

Discussion by Iwao Ohtsu,[3] Youichi Yasuda,[4] and Hiroshi Gotoh[5]

The authors' paper on the undular jump is valuable, and presents some important characteristics of the undular jump.

The discussers have experimentally investigated undular jumps in horizontal-rectangular channels with different channel widths, and would like to discuss the effects of the channel width and the Reynolds number on the characteristics of the undular jump.

The discussers' experiments were conducted using three horizontal-rectangular channels of W = 20, 40, and 80 cm, respectively.

The fundamental factors governing the undular jump are generally expressed as follows: the supercritical inflow Froude number F_1; the inflow condition (the state of the boundary-layer development of the supercritical flow); the aspect ratio W/h_1; and the Reynolds number R ($\mathsf{R} = q/\nu$; q = discharge per unit width).

Recently, the discussers proposed the nondimensional factor L_c/L_{sw}, which represents the effect of the channel width on characteristics of the undular jump (Ohtsu et al. 1995). Here, L_c is the longitudinal length from the toe of the lateral-shock wave to the cross point of the shock wave, and L_{sw} is the longitudinal length from the toe of the shock wave to the first wave crest (Fig. 14). If the cross point of the shock wave is located upstream of the first wave crest ($L_c/L_{sw} < 1$), the effect of the shock wave on the characteristics of the undular jump is large. When the shock wave does not cross upstream of the first wave crest ($L_c/L_{sw} > 1$), the effect of the shock wave is negligibly small.

For $\mathsf{R} \geqq 6.5 \times 10^4$, the Reynolds number R has no effect on the characteristics of the undular jump.

For $L_c/L_{sw} > 1$, $\mathsf{F}_1 \geqq 1.2$, and fully developed inflow, the flow conditions are two-dimensional, although the undulations near the wall are small and unstable (Fig. 15). In this case, the effect of the channel width is not shown. However, for $L_c/L_{sw} < 1$, $\mathsf{F}_1 \geqq 1.2$, and developed inflow, the flow conditions are three-dimensional, and change with F_1 and W/h_1 (or y_c/W). Also, the flow patterns shown in the authors' paper [Types B to E (Fig. 4)] are observed.

Regarding the hydraulic quantities of the first wave height $h_{\max}$, the wave length L_w, the shock-wave length L_{sw}, and the angle of the shock wave to the side wall θ, $h_{\max}/h_1$, L_w/h_2, L_{sw}/h_1, and θ are independent of the aspect ratio W/h_1, and depend on the Froude number F_1 for $L_c/L_{sw} > 1$, $\mathsf{F}_1 \geqq 1.2$, and developed inflow. The following equations for $h_{\max}/h_1$, L_w/h_2, L_{sw}/h_1, and θ have been proposed (Ohtsu et al. 1995).

$$h_{\max}/h_1 = 1.51 \cdot \mathsf{F}_1 - 0.35, \quad L_w/h_2 = 2.75 + 0.90/(\mathsf{F}_1 - 1) \tag{15}$$

$$L_{sw}/h_1 = 1.30 \cdot \mathsf{F}_1 + 2.56, \quad \theta = 30.6 \cdot \mathsf{F}_1^{0.65} \tag{16}$$

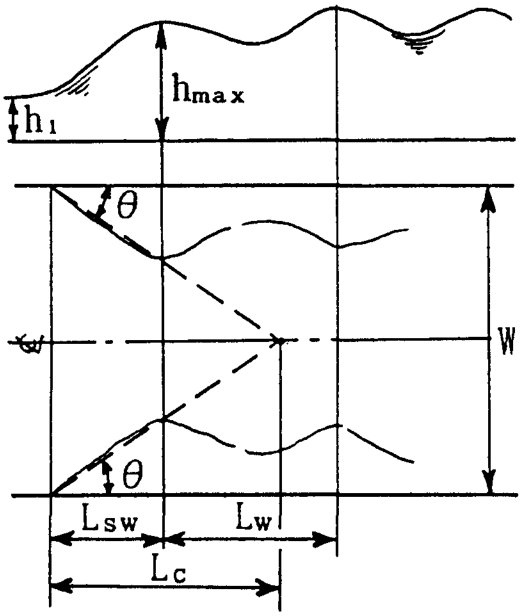

FIG. 14. Definition Sketch of Undular Jump

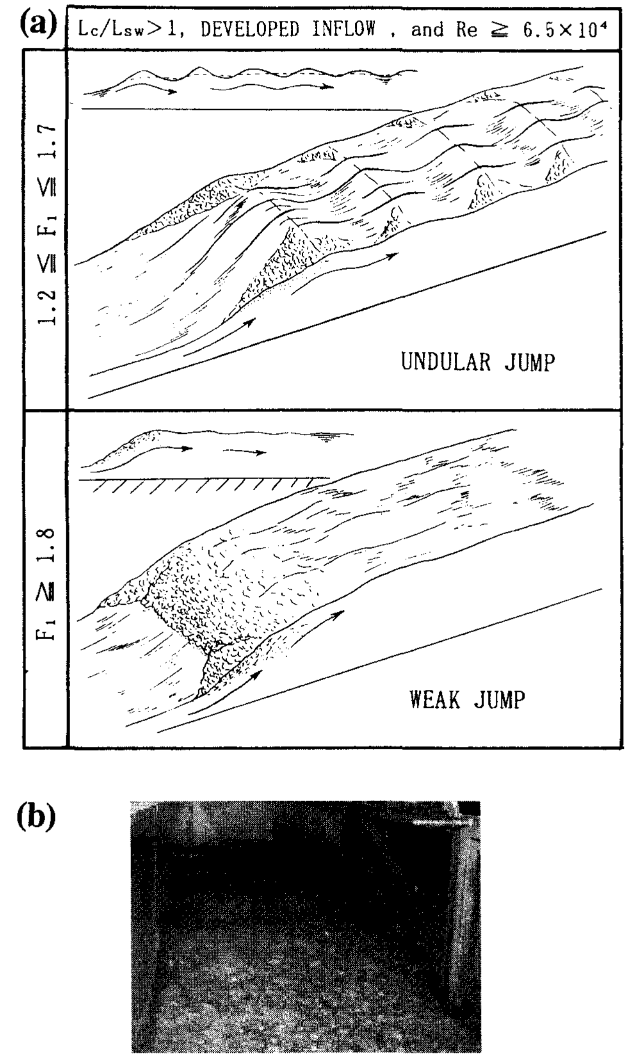

FIG. 15. Undular Jump: (a) Flow Condition for $L_c/L_{sw} > 1$, Fully Developed Inflow, $\mathsf{F}_1 \geqq 1.2$, and $\mathsf{R} \geqq 6.5 \times 10^{-4}$; (b) Photograph for $L_c/L_{sw} > 1$, Fully Developed Inflow, $\mathsf{F}_1 = 1.52$, $w/h_1 = 9.90$, and $\mathsf{R} = 8.6 \times 10^{-4}$

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[a] February 1995, Vol. 121, No. 2, by H. Chanson and J. S. Montes (Paper 7859).

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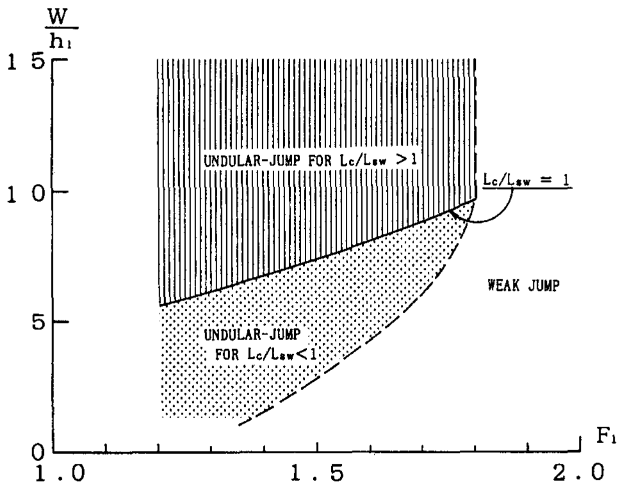

FIG. 16. Regions of Undular-Jump Formations for $L_c/L_{sw} > 1$ and $L_c/L_{sw} < 1$

For the case of $L_c/L_{sw} < 1$, L_w/h_2 and L_{sw}/h_1 depend on F_1 and W/h_1 (or y_c/W). Also, h_{max}/h_1 and θ show the same results as for $L_c/L_{sw} > 1$.

For $\mathsf{R} < 6.5 \times 10^4$, the characteristics of the undular jump depend on the Reynolds number R (Ohtsu et al. 1995). The effect of the Reynolds number on the characteristics of the undular jump can be clarified by using test channels with different channel widths (e.g., $W = 20$, 40, and 80 cm).

Consequently, the characteristics of the undular jump are quite different between $L_c/L_{sw} > 1$ and $L_c/L_{sw} < 1$.

Using the nondimensional factor $L_c/L_{sw}\{L_c/L_{sw} = [(W/h_1)/(2 \cdot \tan\theta)]/(L_{sw}/h_1)\}$, it is possible to clarify the effect of the channel width on the characteristics of the undular jump. For fully developed inflow, $\mathsf{F} \geqq 1.2$, and $\mathsf{R} \geqq 6.5 \times 10^4$, if the factor L_c/L_{sw} is replaced by the fundamental factors W/h_1 and F_1, the regions of $L_c/L_{sw} > 1$ and $L_c/L_{sw} < 1$, and the boundary $L_c/L_{sw} = 1$ are shown by Fig. 16. In addition, the authors' experiments for $\mathsf{R} \geqq 6.5 \times 10^4$ may be conducted within the region of $L_c/L_{sw} < 1$.

APPENDIX. REFERENCE

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Discussion by Willi H. Hager,[6] Member, ASCE, and Roger Reinauer[7]

The authors have presented an interesting paper in a classic topic of open channel flow. After 130 years of its first description by Darcy and Bazin (1865), they classify the flow. Also, they are able to demonstrate that an undular surge and an undular hydraulic jump have few characteristics in common. The undular hydraulic jump is also one of those topics of which the hydraulician might think that nothing new can be added to the existing knowledge. Careful observations and extended analyses give novel insight and reveal interesting features of flow.

One fact is evident: According to Table 1, only few observations on undular jumps are available. Systematic series of experiments were conducted only by Iwasa (1955), Montes (1979), Ryabenko (1990), and Yasuda et al. (1993). The authors' new data series are thus most welcome from this point of view.

Five flow types have been introduced. The particular features of the novel classification are the effects of air entrainment and shock waves. Those two effects are normally not present in undular surges. The undular jump is thus more complex except for type A, which can occur similar to an undular surge. The discussers are not sure if the types D and E are different enough to warrant separate classification, and would like to suggest the modified type D as follows: "At higher Froude numbers, the undular jump is characterized by lateral shock waves that break at the channel axis to form a surface roller. The wave breaking may entrain air bubbles. At the upper limit the roller extends over the entire channel width and blocks the surface undulations."

The discussers have made some additional observations in a smooth horizontal channel of 500 mm width. Photographs to some specific flow patterns are provided, given that the present propositions are not sufficiently documented. The experiments relate to an approach flow depth of 110–120 mm and the flows are partially developed. Typical flow patterns include: (1) incipient shock-wave breaking (transition between types B and C); (2) incipient roller formation (transition between types C and D); (3) organized roller flow (present prop-

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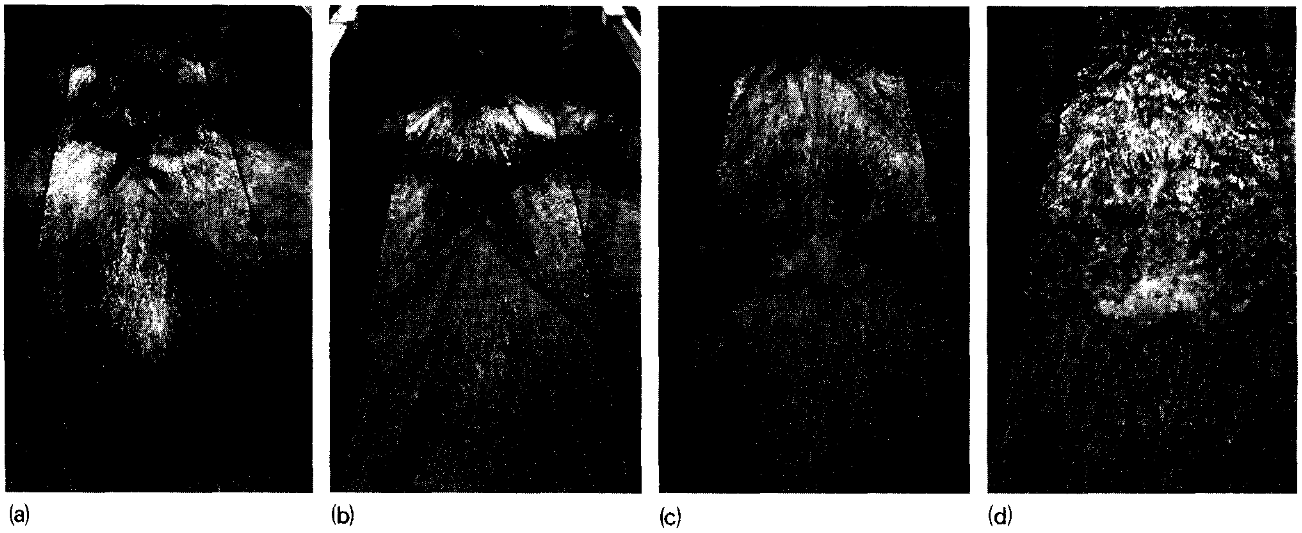

FIG. 17. Undular Jumps in Flow Direction

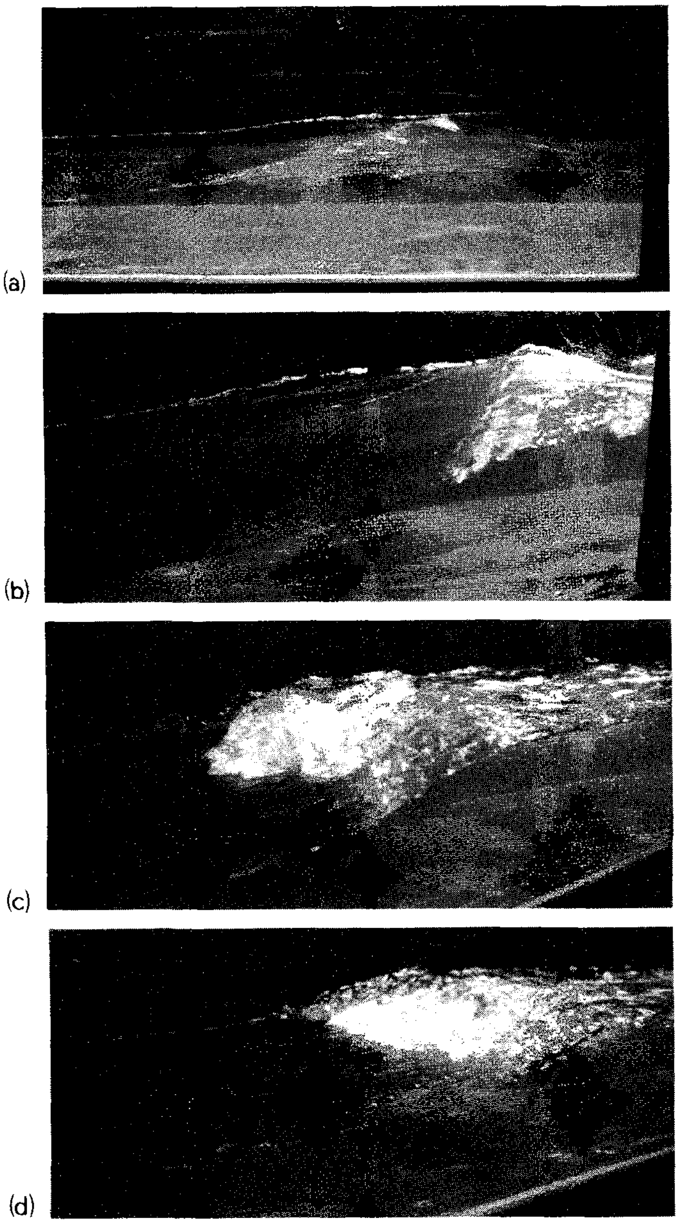

FIG. 18. Side Views of Selected Undular Jumps

osition type D); and (4) full roller flow. Increasing the Froude number over the upper limit of the fourth step of the preceding list yields the weak direct jump.

Fig. 17 shows views in the flow direction. The differences between the flow types are clearly revealed. Also, the highly spatial currents may be observed, and such flow cannot be called two-dimensional, in contrast to undular surges, that are documented for example by Treske (1994).

Side views as shown in Fig. 18 demonstrate the features of shock waves. In Fig. 18(a) the shocks are thin and continuous; in Fig. 18(b) incipient shock breaking is seen to occur, and all breaking water particles are directed downstream. If breaking is developed, the surface current is unable to direct them toward the tailwater and a central surface roller forms, instead. In Fig. 18(c) the roller extends over a comparatively small portion of the channel width, but tailwater undulations have significantly decreased already. In Fig. 18(d) the stems of shocks can still be recognized close to the channel walls, but the axial portions of shock waves are now covered with the roller. Tailwater undulations have practically disappeared.

Tailwater views also exhibit distinctly different flow patterns for the four flow types considered. In Fig. 19(a) the shock is about to form; in Fig. 19(b) the shock waves are completely present, including the system of reflections just without breaking at the first wave crest. A small central roller has formed in Fig. 19(c) and the shocks are clearly damped. If the surface roller extends nearly over the width of channel as shown in Fig. 19(d), the shockwaves disappear and the weak direct jump is developed.

The authors have suggested transition criteria from one type flow to the other (Table 2). The transitions are characterized by the approach Froude number F_o and the parameter y_c/W. The discussers, in turn, would like to point out first that the parameter y_c/W involves nothing else than the approach aspect ratio, given that $y_c/W = \mathsf{F}_o^{2/3}(y_o/W)$. Also, it can be realized from Table 2 that the transition Froude numbers depend strongly on the aspect ratio. The writers have considered the published data and some selected observations of their own, and found that the undular jump may be significantly influenced by scale effects. Any typical parameter of the undular jump, such as the crest height or wave length, can easily be shown to be functionally related to

- The approach flow depth y_o
- The approach aspect ratio y_o/W

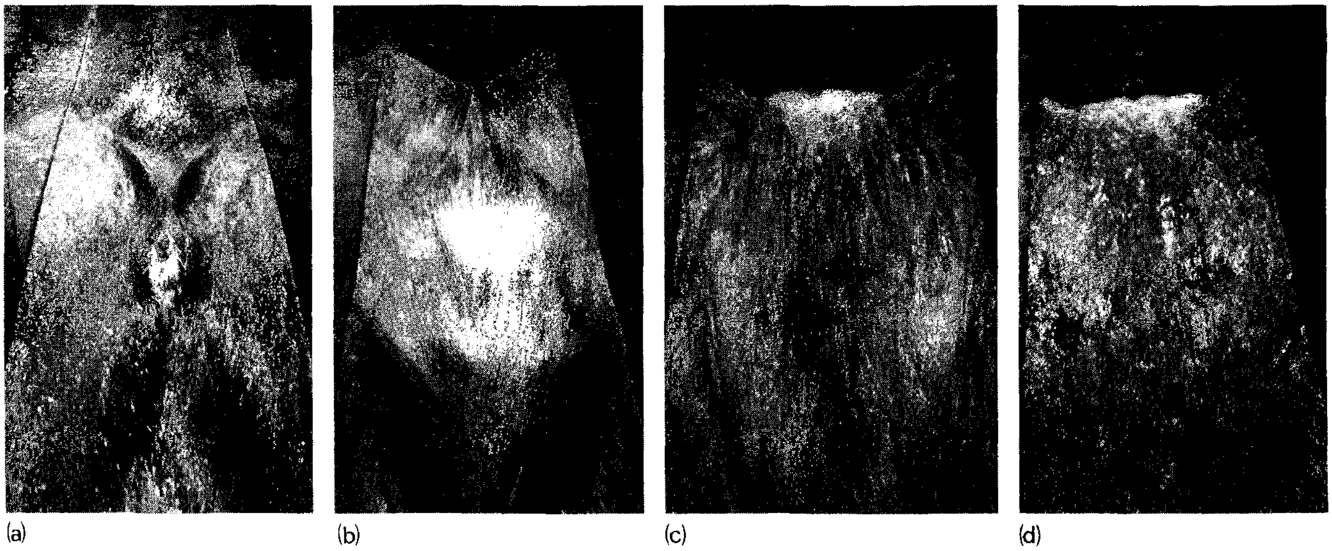

FIG. 19. Views from Tailwater

TABLE 4. Modified Characteristic Froude Numbers (without Scale Effects)

Froude number (1)	Value (2)
F^A	1.20
F^B	1.28
F^C	1.36
F^D	1.60

- The approach Froude number F_o
- The approach Reynolds number R_o
- The approach Weber number W_o
- The approach turbulence number, T_o

From observations to be published separately, the effects of R_o and W_o can be demonstrated to be small in water provided $y_o > 50-70$ mm. The effects of y_o/W and T_o could not be evaluated in detail, but they are small for channels with minimum width $W = 200$ mm. With these preliminary findings and the numbers presented in Table 2, limit Froude numbers practically free of scale effects may be suggested as shown in Table 4. The discussers are interested in the authors' opinion on these suggestions.

APPENDIX. REFERENCE

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Closure by H. Chanson[8] and J. S. Montes[9]

The writers thank the discussers for their interest on the topic and for some new photographic information. They wish to highlight some important points and indicate that additional photographic evidence on their work can be found in Chanson (1993, 1995a,b).

First the writers wish to emphasize that, for each experiment, the first wave crest was located at least 10 m downstream of the channel intake. The upstream flow was uniform equilibrium: i.e., the upstream flow was always fully developed in terms of both the bottom and sidewall boundary layers, and the upstream flow was neither accelerated nor decelerated. Neither Ohtsu et al. nor Hager and Reinauer presented new experimental evidence with these same flow conditions. The experiments of Ohtsu et al. were performed with partially developed inflow and fully developed inflows with gradually varied flow conditions while Hager and Reinauer performed experiments with partially developed inflow conditions. The writers showed clearly the significance of the inflow conditions on the undular jump flow characteristics (Chanson and Montes 1995). Therefore, the discussions cannot be compared exactly with the writers' work.

In their experiments, the writers observed always that the lateral shock waves intersect first next to the first wave crest and on the channel centerline. Some different flow patterns observed by Ohtsu et al. might result from different upstream flow conditions.

Second, an aspect ratio can be defined in term of the upstream depth or the critical depth. A fundamental characteristic of undular hydraulic jump is the transition from supercritical to subcritical flow and the occurrence of critical flow conditions. In the writers' opinion, it is more relevant to define the aspect ratio in term of the critical flow depth. At the location where $h = y_c$, a blockage effect induced by the sidewalls exists and it is best described by the ratio y_c/W.

Additional experiments were performed with identical upstream flow conditions (i.e., q_W, d_1, W) but with different sidewall roughness (Chanson 1995a). The results provide valuable information on the effects of sidewall roughness on the flow properties independently of the other parameters (i.e., Froude number and aspect ratio). First the flow patterns differ from the smooth-wall channel experiments as defined by Chanson and Montes (1995): e.g., the undular jump type A is never observed. Although the pressure distributions are little (if not) affected by the change of sidewall roughness, some substantial changes in the velocity distributions are observed (Chanson 1995a). Such results confirm the relevance of the dimensionless sidewall roughness as a significant parameter. Undular hydraulic jump flows are indeed three-dimensional.

The writers refute the suggestion of Hager and Reinauer that the effect of the aspect ratio is small. The writers performed very careful experiments that showed explicitly the effects of the aspect ratio y_c/W on undular flow with identical upstream Froude numbers (and uniform equilibrium upstream flow conditions). Further the study of Ryabenko (1990) showed exactly the same effects (e.g., Figure 5 of Chanson and Montes 1995) with a 1-m wide channel.

Although some early experimental studies included undular jump cases (e.g., Darcy and Bazin 1865), few researchers highlighted the specific nature of undular hydraulic jump. The writers believe that the first significant study of undular jump flow can be attributed to Fawer (1937). Undular jump flows should be called Fawer's jump in homage to Fawer's work in a similar manner as the Italians named the hydraulic jump after Bidone: i.e., "the jump of Bidone" ("il salto di Bidone").

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