Technical Note

Energy Dissipation down a Stepped Spillway with Nonuniform Step Heights

Stefan Felder[1] and Hubert Chanson[2]

Abstract: Although most stepped spillway–design guidelines were developed for uniform step heights, a nonuniform stepped design might be a practical alternative in some cases. A physical study was conducted in a moderate slope-stepped chute (1V:2H) and five stepped configurations were tested for $0.7 < d_c/h < 1.9$. Detailed air-water flow measurements were performed for each configuration and the results were compared in terms of flow patterns, energy dissipation, and flow resistance. The basic findings showed minor differences between all configurations and indicated that the rate of energy dissipation was about the same for uniform and nonuniform stepped configurations. But the observations suggested that the nonuniform stepped configurations might induce some flow instabilities for smaller flow rates. **DOI: 10.1061/(ASCE)HY.1943-7900.0000455.**

CE Database subject headings: Spillways; Energy dissipation; Air entrainment; Measurement.

Author keywords: Stepped spillways; Nonuniform step heights; Energy dissipation; Air entrainment; Physical measurements.

Introduction

The design of stepped spillways has been known for at least 3,500 years, but at the beginning of the 20th century, breakthroughs in the design of hydraulic jump-stilling basins led to the disuse of stepped spillways (Chanson 2001). With the development of new, more efficient construction techniques [(e.g., roller-compacted concrete (RCC)], the design of stepped spillways regained interest in the 1980s (Hansen and Reinhardt 1991; Chanson 2001). This was associated with a substantial amount of physical modeling research (Sorensen 1985; Chamani and Rajaratnam 1999; Carosi and Chanson 2008).

Most experiments were conducted on stepped spillways with uniform flat steps to quantify the energy dissipation and to provide some design guidelines (Matos 2000; Chanson 1995, 2001). But some prototype spillways are equipped with nonuniform step heights (Malmsburry 1870, Upper Coliban 1903) (Chanson 2001), and their long operation indicates that the design is sound. However, some flow instabilities and shock waves might occur for the nonuniform step heights, as reported by Toombes and Chanson (2008) in the nappe flow regime and by Thorwarth and Köngeter (2006) for pooled stepped spillways. The only experimental test of nonuniform step heights was conducted by Stephenson (1988) on a model with a slope of 45°. In a test with occasional large drops, Stephenson observed an increase in energy dissipation of 10%.

In the present study, the effects of nonuniform step heights on the air-water flow properties down a stepped chute are tested systematically for a wide range of discharges. It is the aim of this work to assess the effects of occasional large steps and alternate large and small steps on the rate of energy dissipation and flow aeration using a large-size facility with moderate slope (1V:2H).

Experimental Facility and Instrumentation

The experimental study was conducted at the University of Queensland. The facility consisted of a large intake structure supplying a constant discharge through a sidewall convergent with a 4.8:1 contraction ratio to the test section. The test section was composed of a 1-m-wide and 0.6-m-long broad-crested weir with an upstream rounded corner followed by a 1-m-wide and 1-m-high stepped spillway section with a slope of 26.6° (1V:2H). The discharge was measured with a pointer gauge from the upstream head above the weir crest using a discharge calibration function (Gonzalez and Chanson 2007). The air-water flow properties were recorded with a double-tip conductivity probe (Ø = 0.25 mm, $\Delta x = 7.2$ mm). The probe was supported by a trolley system in the longitudinal direction and the positioning of the probe sensor normal to the pseudo-bottom was controlled with a Mitutoyo digital ruler mounted on a fine-adjustment screw-drive mechanism. The error in the translation of the probe in the direction normal to the flow was less than 0.5 mm. The accuracy on the longitudinal probe position was less than 0.5 cm and the transverse direction less than 0.1 mm (Carosi and Chanson 2008). For all experiments, the probe sensors were sampled for 45 s at 20 kHz.

The experiments were conducted for a wide range of discharges between 0.02 and 0.237 m^3/s corresponding to Reynolds numbers between 8.2×10^4 and 9.4×10^5. All experiments were conducted with a slope of 26.6°, but different configurations of step heights were investigated (Table 1). In some experiments, the stepped chute was equipped with uniform steps of 5 and 10 cm heights. In addition, several nonuniform stepped configurations with combinations of 5- and 10-cm-high steps were investigated. Table 1 summarizes the conducted experiments and all the channel configurations are sketched in Fig. 1.

[1]School of Civil Engineering, Univ. of Queensland, Brisbane QLD 4072, Australia.

[2]School of Civil Engineering, Univ. of Queensland, Brisbane QLD 4072, Australia (corresponding author). E-mail: h.chanson@uq.edu.au

Note. This manuscript was submitted on December 16, 2010; approved on May 10, 2011; published online on May 12, 2011. Discussion period open until April 1, 2012; separate discussions must be submitted for individual papers. This technical note is part of the ***Journal of Hydraulic Engineering***, Vol. 137, No. 11, November 1, 2011. ©ASCE, ISSN 0733-9429/2011/11-1543–1548/$25.00.

Table 1. Summary of Experimental Configurations with Uniform and Nonuniform Step Heights (Present Study), Including the Flow Conditions of Flow Regime Changes for Different Channel Configurations

Configuration (1)	Steps (2)	Characteristic (3)	q_w (m^2/s) (4)	d_c/h (5)	d_c/h NA-TRA (6)	d_c/h TRA-SK (7)	Comment (8)
10 cm	10 steps with $h = 0.10$ m	Uniform step heights	0.057–0.237	0.7–1.85	0.6	0.93	Calculation of d_c/h with $h = 0.1$ m
5 cm	20 steps with $h = 0.05$ m	Uniform step heights	0.021–0.218	0.7–3.5	0.53	1.07	Calculation of d_c/h with $h = 0.05$ m
A	10 steps with $h = 0.05$ m, 5 steps with $h = 0.10$ m	Regular alternation of one 10-cm step followed by two 5-cm steps	0.057–0.237	0.7–1.85	0.53	1.0	Calculation of d_c/h with $h = 0.1$ m
B	9 steps with $h = 0.10$ m, 2 steps with $h = 0.05$ m	Two 5-cm steps between step edges 7 and 8	0.057–0.237	0.7–1.85	0.6	1.0	Calculation of d_c/h with $h = 0.1$ m
C	18 steps with $h = 0.05$ m, 1 step with $h = 0.10$ m	10-cm step between step edges 13 and 15	0.021–0.21	0.7–3.4	0.53	1.73 (1.07)	Calculation of d_c/h with $h = 0.05$ m

Note: NA-TRA, change from nappe to transition flow; TRA-SK, change from transition to skimming flow.

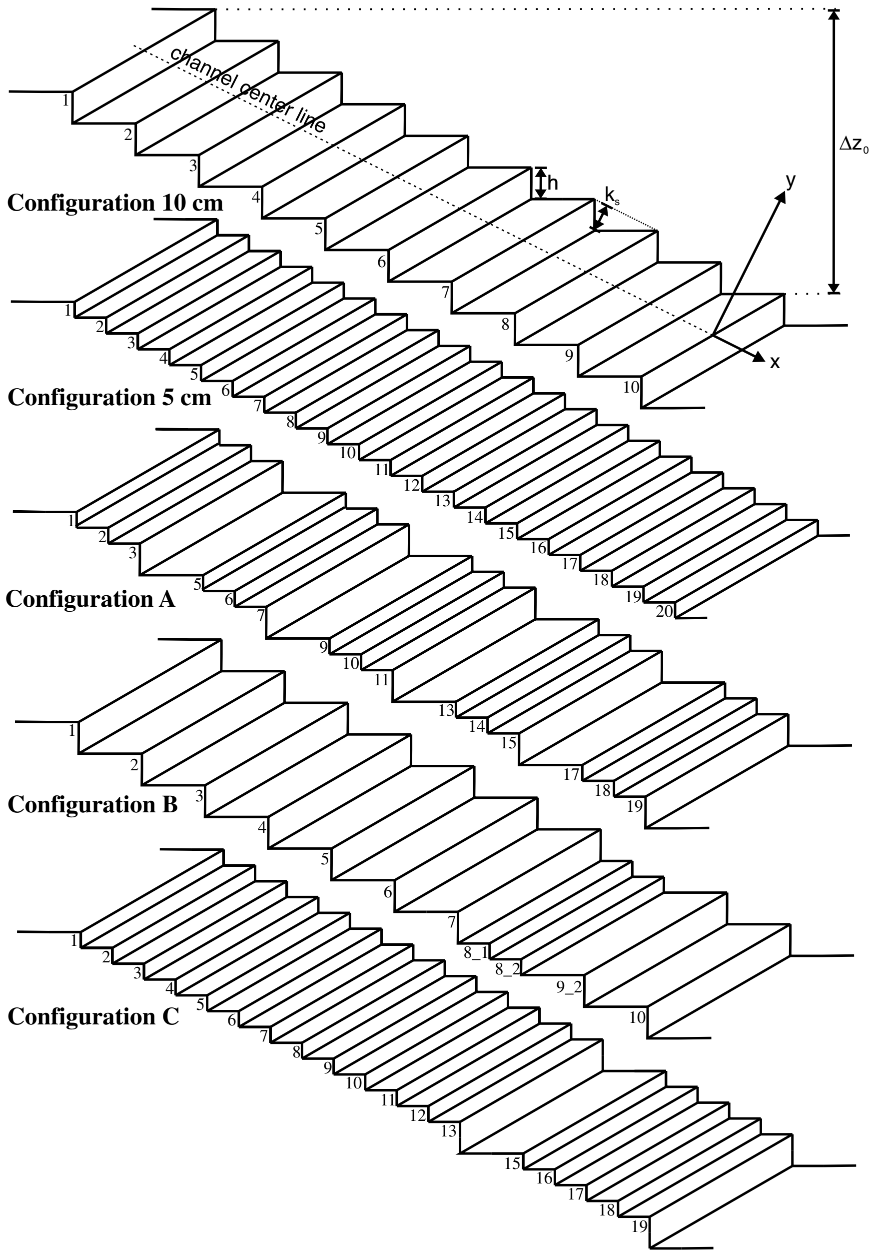

Fig. 1. Sketches of the experimental setup for all step-edge configurations and definition of step numbering, $\theta = 26.6°$

Basic Flow Patterns

On uniform stepped configurations, a nappe flow regime was observed for the smallest flow rates. For some intermediate flows, a transition flow was seen with some strong spray, splashing, and flow instabilities. For the largest flow rates, the waters skimmed over the pseudobottom formed by the stepped edges. For all stepped spillway configurations, the flow patterns were observed for the full set of discharges. Some photos for typical flows over nonuniform steps are shown in Fig. 2. The observations of flow regime changes are listed in Table 1 (columns 6 and 7). Note that for configuration C, the observation of flow-regime changes was not definite. At the upstream end of the chute, the 5-cm-high steps resulted in identical changes of flow regimes, as observed for the stepped channel with uniform step heights of 5 cm, i.e., a change from nappe to transition flow for $d_c/h = 0.53$ and from transition to skimming for $d_c/h = 1.07$. At the same time, the large drop of 10 cm at the lower end of the chute leads to different flow patterns downstream of the drop, i.e., a transition flow regime exists still for $d_c/h < 1.73$ before the flow regime changes to a skimming flow regime, whereas the change from nappe to transition flow is identical to the uniform step heights [Fig. 2(d)]. In the following sections, the present study is focused on the flow properties in transition and skimming flows.

The locations of the inception point of free-surface aeration were recorded for all configurations. The data compared favorably with the empirical correlation of Chanson (1995):

$$\frac{L_I}{k_s} = 9.719 \times (\sin\theta)^{0.0796} \times \mathsf{F}^{*0.713} \tag{1}$$

and a simple linear correlation of Carosi and Chanson (2008):

$$\frac{L_I}{k_s} = 1.05 + 5.11 \times \mathsf{F}^* \qquad 0.45 < d_c/h < 1.6 \tag{2}$$

in which L_I = longitudinal distance from the first step edge to the inception point location; k_s = step cavity height ($k_s = h \times \cos\theta$); and

$$\mathsf{F}^* = \frac{q_w}{\sqrt{g \times \sin\theta \times k_s^3}} \tag{3}$$

The results (not shown) indicated that the nonuniform stepped configurations did not have an impact on the location of the inception point of air entrainment. However, the large drop in configuration C was at the downstream end and a positioning further upstream might have yielded some different air-entrainment inception locations for smaller flow rates.

Downstream of the inception point of free-surface aeration, the flow was highly aerated (Fig. 2). The flow was rapidly varied

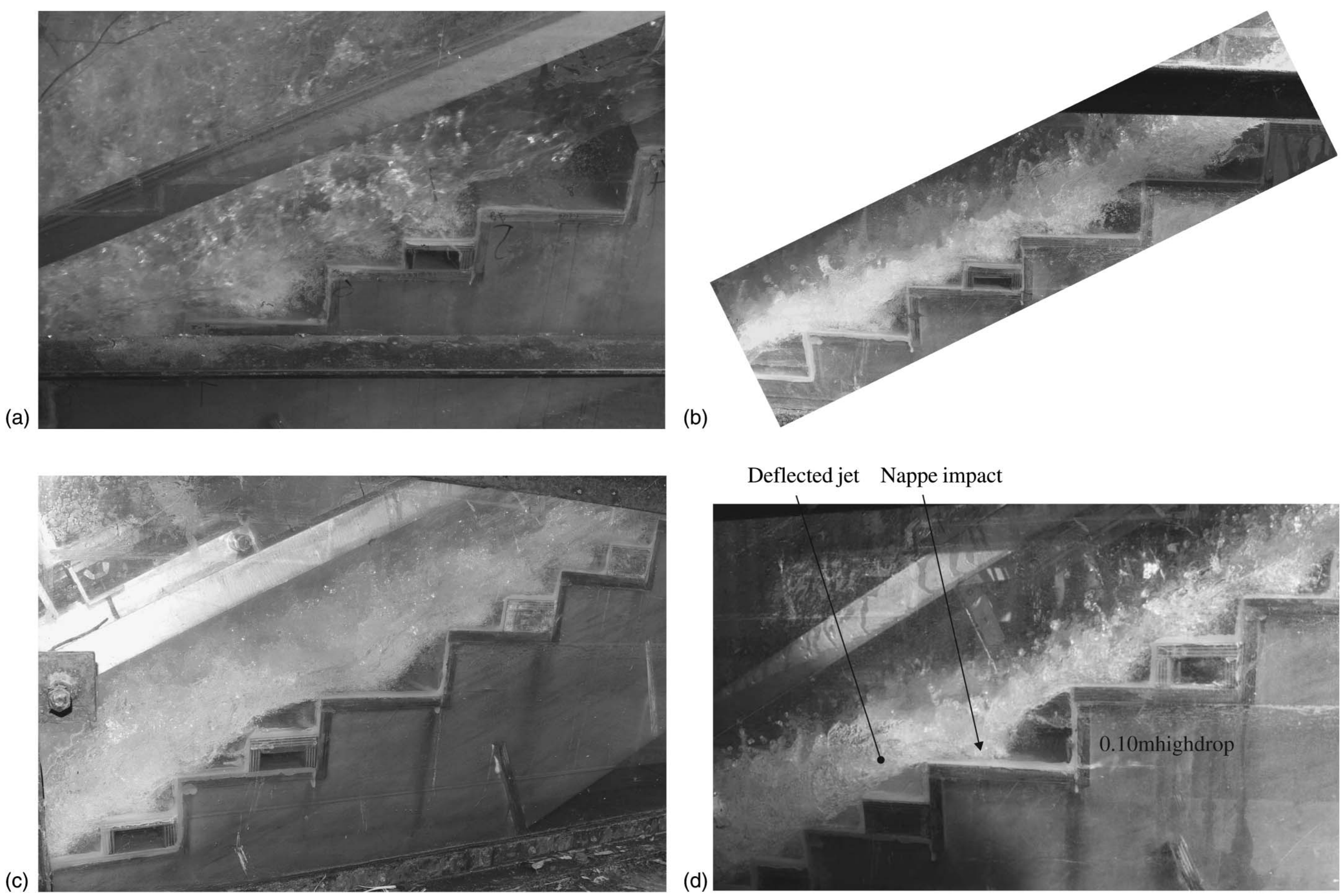

Fig. 2. Flow patterns from top right to bottom left over nonuniform stepped configurations ($\theta = 26.6°$): (a) skimming flows and cavity recirculation for configuration A, $d_c/h = 1.27$, $\mathsf{R} = 5.4 \times 10^5$, $q_w = 0.137\ \mathrm{m^2/s}$, and $h = 0.1 + 0.05$ m; (b) transition flow in large cavities and skimming flows in small cavities for stepped spillway in configuration B, $d_c/h = 0.85$, $\mathsf{R} = 3.1 \times 10^5$, $q_w = 0.078\ \mathrm{m^2/s}$, and $h = 0.1 + 2$ steps 0.05 m; (c) skimming flows and cavity recirculation in configuration C, $d_c/h = 1.7$, $\mathsf{R} = 3.1 \times 10^5$, $q_w = 0.078\ \mathrm{m^2/s}$, and $h = 0.05 + 1$ step 0.1 m; (d) skimming flow and deflected jets in configuration C, $d_c/h = 1.15$, $\mathsf{R} = 1.7 \times 10^5$, $q_w = 0.043\ \mathrm{m^2/s}$, and $h = 0.05 + 1$ step 0.1 m (images by authors)

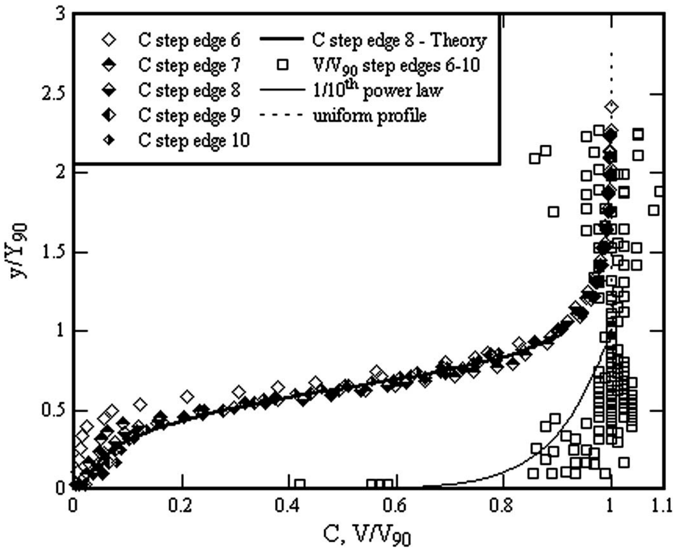

Fig. 3. Dimensionless distributions of air concentration and velocities downstream of the inception of free-surface aeration; uniform step-height configuration $h = 0.10$ m, $q_w = 0.122$ m^2/s, and $d_c/h = 1/15$

immediately downstream of the inception point, and it became gradually varied two to three step cavities downstream. True uniform equilibrium flow conditions were not achieved because the chute was relatively short. The air-water flow properties were recorded at each step edge for all configurations (Table 1). The air concentration profiles presented some self-similarity following

$$C = 1 - \tanh^2\left[K' - \frac{y/Y_{90}}{2 \times D_o} + \frac{(y/Y_{90} - 1/3)^3}{3 \times D_o}\right] \quad (4)$$

in which C = air concentration; y = distance normal to the pseudobottom; Y_{90} = characteristic depth where $C = 0.9$; and K' and D_o = functions of the depth-averaged air concentration only (Chanson and Toombes 2002). Eq. (4) is compared with some data for a discharge (Fig. 3).

Energy Dissipation and Aeration

The rate of energy dissipation, the flow resistance, and some basic depth-averaged flow properties were calculated in both transition and skimming flow regimes for all configurations. For the design of stepped spillways, some basic parameters are the rate of energy dissipation $\Delta H/H_{\max}$ and the residual energy H_{res}/d_c at the downstream end of the stepped spillway. $H_{\max}$ is the maximum upstream head above the downstream step edge: $H_{\max} = \Delta z_o + 3/2 \times d_c$, in which Δz_o = dam height above the spillway toe; ΔH = total head loss, $\Delta H = H_{\max} - H_{\text{res}}$; and H_{res} = residual head at the measurement section

$$H_{\text{res}} = d \times \cos\theta + \frac{U_w^2}{2 \times g} \quad (5)$$

In Eq. (5), the equivalent clear-water flow depth d and the flow velocity U_w is calculated from the air-water flow properties. The clear-water flow depth d is defined as

$$d = \int_{y=0}^{Y_{90}} (1 - C) \times dy \quad (6)$$

in which C = air concentration. By continuity, the flow velocity is

$$U_w = q_w / \int_0^{Y_{90}} (1 - C) \times dy \quad (7)$$

in which q_w is the water discharge per unit width. The energy dissipation and residual energy were calculated at the last step edge for all configurations.

For all flow configurations, a decreasing rate of energy dissipation with increasing discharge was observed, which is consistent with earlier studies on stepped spillways (Chanson 1995; Matos 2000). All the data sets were in close agreement, with some small differences suggesting the largest energy dissipation for a uniform spillway with 10-cm-high steps. The results implied that Stephenson's (1988) observation of a 10% increase in energy dissipation for nonuniform step heights was not fulfilled in the present study. Similar findings were seen in terms of the dimensionless residual head H_{res}/d_c measured at the last step edge (Fig. 4). Note that the residual head at the downstream end of the stepped chute might be slightly larger than the specific energy at the start of the apron. Hence, Fig. 4 is suitable for design purposes including nondesign flow conditions because it is conservative.

Fig. 4 shows some differences in the residual head for the different step configurations: the lowest residual head was achieved with uniform step height $h = 0.10$ m. For the smaller flow rates, the residual head decreased with increasing discharge for all experiments, while it was about constant for the largest flow rates. For the largest flow rates, the discharge was not fully developed at the downstream end of the spillway and the residual energy might be overestimated (Chanson 2001; Meireles and Matos 2009). In Fig. 4, the residual-head data are compared with some simple design criteria for moderate slope-stepped spillways: the upper dotted line expresses the median residual energy of a number of experimental data obtained for spillway slopes smaller than 15.9° and the lower dashed line the median values for stepped-spillway data with slopes $21.8 < \theta < 26.6°$ (Felder and Chanson 2009). The present findings agree with previous physical studies conducted with slopes between 3.4 and 26.6° and used for the median values shown in Fig. 4.

The energy dissipation is caused by momentum exchanges between the mainstream flow and cavity regions. The flow resistance

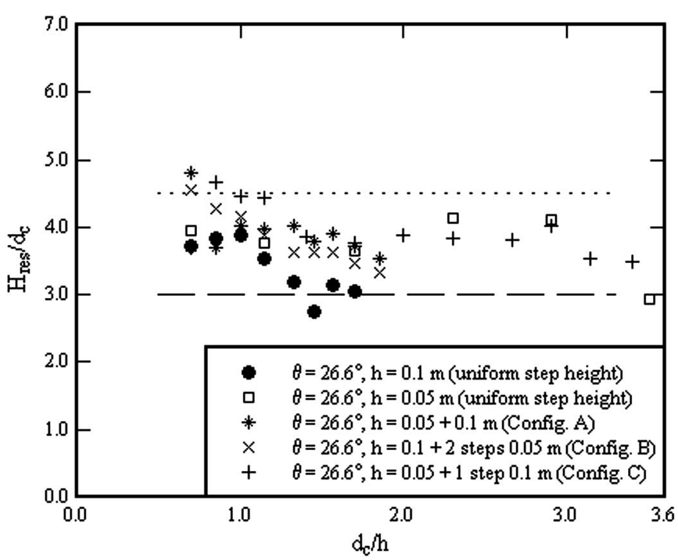

Fig. 4. Residual head for different step configurations in the present study; measurements at last step edge at downstream end $x = 2.01$ m; median values are shown by dotted and dashed lines

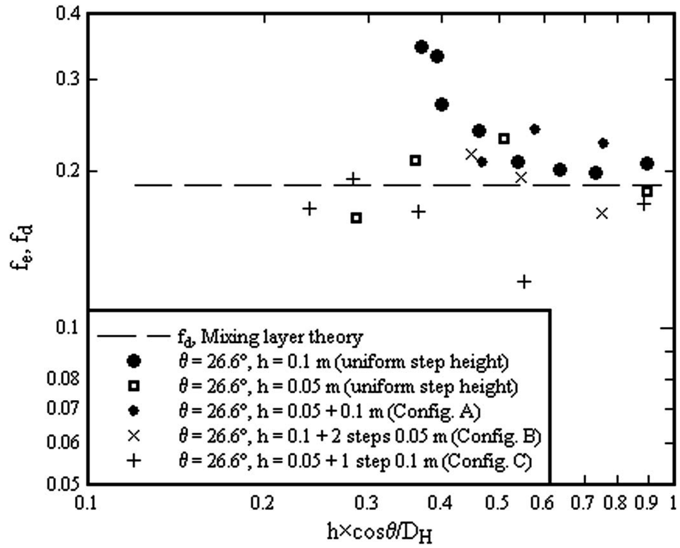

Fig. 5. Flow resistance for different step configurations (present study)

may be expressed in the form of an equivalent Darcy-Weisbach friction factor f_e, which is a dimensionless average-shear stress between the main stream and cavity recirculation:

$$f_e = \frac{8 \times \tau_0}{\rho_w \times U_w^2} = \frac{8 \times g \times S_f \times [\int_{y=0}^{Y_{90}}(1-C)dy]}{U_w^2} = \frac{8 \times g \times S_f \times d}{U_w^2} \tag{8}$$

in which S_f = friction slope. Eq. (8) is a simple rewriting of the backwater equation in which the friction slope S_f and flow velocity U_w were estimated taking into account the air entrainment over the air-water flow region. The experimental results are summarized in Fig. 5, in which the friction factor is plotted as a function of the dimensionless step-roughness height $h \times \cos\theta/D_H$ with D_H = equivalent pipe diameter. The data in Fig. 5 include skimming flow data for the configurations 10 cm and 5 cm (uniform step height), and skimming/transition flow data for the configurations A, B, and C as the notion of skimming/transition flow becomes more uncertain. In Fig. 5, the data are compared with the solution of a mixing-length model, $f_d = 1/(\pi^{1/2} \times K)$, in which $1/K$ = dimensionless rate of expansion of the shear layer (Chanson et al. 2002).

For all configurations, the Darcy-Weisbach friction factor varied between 0.12 and 0.37. The findings were consistent with the reanalyses of flow resistance data showing variations of Darcy friction factors between 0.1 and 0.35 (Felder and Chanson 2009).

Discussion

The results showed only small differences between all configurations in terms of flow pattern, energy dissipation, and flow resistance. The present findings indicate that the design of stepped spillways with nonuniform step heights does not enhance the energy dissipation at the chute's downstream end. However, the results present some useful information for alternative designs of stepped spillways. The introduction of larger steps, for example, might be triggered by nonhydraulic considerations, e.g., large step heights ($h > 2$ m) may be introduced to limit access on the stepped spillway by individuals or motorcycles (Chanson 2001). Another example is the Gold Creek Dam stepped spillway (1890). Designed with 19 steps of 0.76 m, the spillway was built with 12 steps of 1.5 m because of limited cement availability.

The current observations of the flow pattern indicated that the nonuniform stepped configurations might lead to some flow instabilities for smaller flow rates, larger flow depth, and stronger splashing. Fig. 2(d) illustrates such a situation in configuration C. Some further analyses might provide some additional information.

Conclusion

In the past 3 decades, a number of studies were performed to develop design guidelines for stepped spillways with flat horizontal steps. There is, however, a lack of understanding for stepped chutes with more complex design, in particular stepped spillways with nonuniform step heights, although some prototype stepped spillways have operated successfully for long periods. The present study investigated five configurations down a moderate slope-stepped chute (1V:2H), and the comparative performances were discussed for a range of dimensionless flow rates ($0.7 < d_c/h < 1.9$).

The basic results showed minor differences between all configurations in terms of flow pattern, energy dissipation, and flow resistance. The findings indicated that the rate of energy dissipation was about the same for uniform and nonuniform stepped configurations. The present work suggested that the design of stepped spillways with nonuniform step heights does not enhance the energy dissipation at the downstream end. The observations showed that the nonuniform stepped configurations might induce some flow instabilities for smaller flow rates. Altogether, the results provide some practical information for alternative designs of stepped spillways with nonuniform step heights.

Acknowledgments

The writers thank Dr. Jorge Matos (IST Lisbon) for his valuable comments. The financial support of the Australian Research Council (Grant DP0878922) is acknowledged.

Notation

The following symbols are used in this paper:

C = air concentration;
D_H = hydraulic diameter (m) or equivalent pipe diameter;
D_o = dimensionless constant function of the depth-averaged air concentration;
d = clear-water flow depth (m) measured normal to the pseudobottom formed by the step edges;
d_c = critical flow depth (m);
F^* = Froude number defined in terms of the step-cavity height;
f = Darcy-Weisbach friction factor;
f_d = dimensionless shear stress in a mixing layer;
f_e = equivalent Darcy-Weisbach friction factor in the air-water flow region;
g = gravity acceleration (m^2/s);
H = total head (m);
H_{max} = upstream total head (m);
H_{res} = residual head (m);
h = vertical step height (m);
K = dimensionless expansion rate of mixing layer;
K' = dimensionless constant function of the depth-averaged air concentration;
k_s = step cavity height (m), $k_s h \times \cos\theta$;

L_I = longitudinal distance (m) between the first step edge and the inception point of free-surface aeration;
q_w = water discharge per unit width (m^2/s);
R = Reynolds number defined as $\mathsf{R} = \rho_w \times U_w \times D_H/\mu_w$;
S_f = friction slope;
U_w = flow velocity $(m/s)'$;
x = longitudinal distance (m) from the first step edge;
Y_{90} = characteristic distance (m) in which $C = 0.90$;
y = distance (m) normal to the pseudobottom formed by the step edges;
ΔH = head loss (m);
Δx = longitudinal spacing (m) between probe sensors;
Δz_o = dam height (m);
θ = bed-slope angle with the horizontal, positive downwards;
μ_w = water dynamic viscosity (Pa s);
ρ_w = water density (kg/m^3); and
τ_o = boundary shear stress (Pa).

Subscript

c = critical flow conditions.

References

Carosi, G., and Chanson, H. (2008). "Turbulence characteristics in skimming flows on stepped spillways." *Can. J. Civ. Eng.*, 35(9), 865–880.

Chamani, M. R., and Rajaratnam, N. (1999). "Characteristics of skimming flow over stepped spillways." *J. Hydraul. Eng.*, 125(4), 361–368.

Chanson, H. (1995). *Hydraulic design of stepped cascades, channels, weirs and spillways*, Pergamon, Oxford, UK.

Chanson, H. (2001). "The hydraulics of stepped chutes and spillways." Balkema, Lisse, Netherlands.

Chanson, H., and Toombes, L. (2002). "Air-water flows down stepped chutes: Turbulence and flow structure observations." *Int. J. Multiphase Flow*, 28(11), 1737–1761.

Chanson, H., Yasuda, Y., and Ohtsu, I. (2002). "Flow resistance in skimming flows and its modelling." *Can. J. Civ. Eng.*, 29(6), 809–819.

Felder, S., and Chanson, H. (2009). "Energy dissipation, flow resistance and gas-liquid interfacial area in skimming flows on moderate-slope stepped spillways." *Environ. Fluid Mech.*, 9(4), 427–441.

Gonzalez, C. A., and Chanson, H. (2007). "Experimental measurements of velocity and pressure distribution on a large broad-crested weir." *Flow Meas. Instrum.*, 18(3–4), 107–113.

Hansen, K. D., and Reinhardt, W. G. (1991). *Roller-compacted concrete dams*, McGraw-Hill, New York.

Matos, J. (2000). "Hydraulic design of stepped spillways over RCC dams." *Int. Workshop on Hydraulics of Stepped Spillways*, H. E. Minor and W. H. Hager, eds., Balkema, Zürich, Switzerland, 187–194.

Meireles, I., and Matos, J. (2009). "Skimming flow in the nonaerated region of stepped spillways over embankment dams." *J. Hydraul. Eng.*, 135(8), 685–689.

Sorensen, R. M. (1985). "Stepped spillway hydraulic model investigation." *J. Hydraul. Eng.*, 111(12), 1461–1472.

Stephenson, D. (1988). "Stepped energy dissipators." *Proc., Int. Symp. on Hydraulics for High Dams*, Int. Association for Hydraulic Research, Madrid, Spain, 1228–1235.

Thorwarth, J., and Köngeter, J. (2006). "Physical model tests on a stepped chute with pooled steps—Investigations of flow resistance and flow instabilities." *Proc., Int. Symp. on Hydraulic Structures*, A. Marcano and A. Martinez, eds., Int. Association for Hydraulic Research, Madrid, Spain, 477–486.

Toombes, L., and Chanson, H. (2008). "Flow patterns in nappe flow regime down low gradient stepped chutes." *J. Hydraul. Res.*, 46(1), 4–14.