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Discussion of “Energy Dissipation down a Stepped Spillway with Nonuniform Step Heights” by Stefan Felder and Hubert Chanson

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The authors investigated the residual energy down a stepped spillway for the nonuniform and uniform step heights. The discussers would like to comment on the analysis of the data for the residual energy and to present the residual energy of skimming flows in view of the aerated flow characteristics.

Analysis of Data for Residual Energy down a Stepped Spillway

On the basis of dimensional considerations, the functional relationship of the residual energy H_{res} for $R \geq 1.2 \times 10^5$ (Takahashi et al. 2005) can be expressed as

$$\frac{H_{\mathrm{res}}}{d_c} = \mathrm{f}\left(\frac{h}{d_c}, \frac{\Delta z_0}{d_c}, \theta\right) \tag{1}$$

Regarding the configurations A, B, and C for the skimming flow at the last step edge, H_{res}/d_c can be given as

$$\frac{H_{\mathrm{res}}}{d_c} = \mathrm{f}\left(\frac{h}{d_c}, \frac{\Delta z_0}{d_c}, \theta, \text{config. A, B, C}\right) \tag{2}$$

For $h/d_c \geq 0.5$ ($d_c/h \leq 2$) of the skimming flow with uniform step heights, the value of H_{res}/d_c is independent of h/d_c (Ohtsu et al. 2004), and Eq. (1) is expressed as

$$\frac{H_{\mathrm{res}}}{d_c} = \mathrm{f}\left(\frac{\Delta z_0}{d_c}, \theta\right) \tag{3}$$

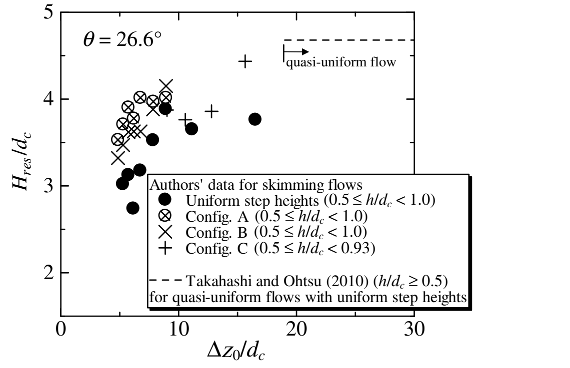

Fig. 1. Residual energy H_{res}/d_c of skimming flows for $h/d_c \geq 0.5$

Also, for $h/d_c \geq 0.5$ of the skimming flow with nonuniform step heights, Eq. (2) may be rewritten as

$$\frac{H_{\mathrm{res}}}{d_c} = \mathrm{f}\left(\frac{\Delta z_0}{d_c}, \theta, \text{config. A, B, C}\right) \tag{4}$$

Fig. 1 is obtained by arranging the authors’ data in accordance with Eqs. (3) and (4). Here, the broken line in Fig. 1 shows the residual energy for $h/d_c \geq 0.5$ in the quasi-uniform skimming flow region with uniform step heights (Takahashi and Ohtsu 2010). As shown in Fig. 1, H_{res}/d_c increases with $\Delta z_0/d_c$ in the nonuniform flow region, and H_{res}/d_c becomes constant for the quasi-uniform flow region. For a given $\Delta z_0/d_c$, the values of H_{res}/d_c for configurations A, B, and C are comparatively larger than those for uniform step heights. The scatter in the data of Fig. 4 in the original paper may mainly result from the effect of $\Delta z_0/d_c$ on H_{res}/d_c. However, further systematic experiments might be necessary to clarify the effect of $\Delta z_0/d_c$, h/d_c (or d_c/h), R, and configurations on H_{res}/d_c for a given $\theta (= 26.6°)$.

The rate of energy dissipation $\Delta H/H_{\max}[= 1 - (H_{\mathrm{res}}/d_c)/(\Delta z_0/d_c + 3/2)]$ for $h/d_c \geq 0.5$ of the skimming flow at the last step edge can be expressed by Eq. (5) for uniform step heights and by Eq. (6) for nonuniform step heights:

$$\frac{\Delta H}{H_{\max}} = \mathrm{f}\left(\frac{\Delta z_0}{d_c}, \theta\right) \tag{5}$$

$$\frac{\Delta H}{H_{\max}} = \mathrm{f}\left(\frac{\Delta z_0}{d_c}, \theta, \text{config. A, B, C}\right) \tag{6}$$

Fig. 2 is obtained by arranging the authors’ data in accordance with Eqs. (5) and (6), demonstrating that $\Delta H/H_{\max}$ increases

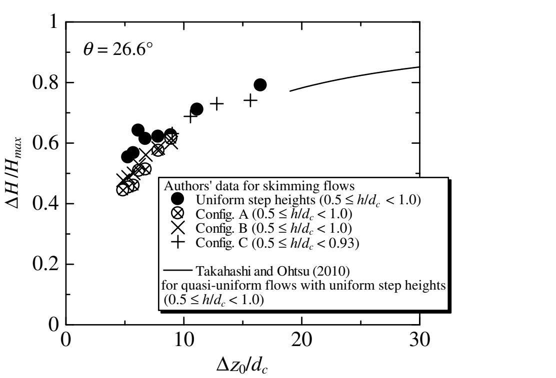

Fig. 2. Energy dissipation $\Delta H/H_{\max}$ of different step configurations for $0.5 \geq h/d_c$ of skimming flows

with $\Delta z_0/d_c$ for all configurations including uniform step heights. In addition, the values of $\Delta H/H_{max}$ for configurations A, B, and C are slightly smaller than those observed for uniform step heights.

Residual Energy of Aerated Skimming Flows with Uniform Step Heights

Considering the aerated flow characteristics, the residual energy (specific energy) E of aerated skimming flows above the pseudo-bottom is (Ohtsu et al. 2004)

$$E = \frac{\int_0^{Y_{90}} (\rho g y \cos\theta + p) V dy}{\int_0^{Y_{90}} \rho g V dy} + \frac{\int_0^{Y_{90}} (\frac{1}{2}\rho V^3) dy}{\int_0^{Y_{90}} \rho g V dy} \tag{7}$$

where $p = \int_y^{Y_{90}} \rho g \cos\theta dy$ = pressure; and $\rho = (1-C)\rho_w$ = density of the aerated flow. Using the clear-water depth d and the average clear-water velocity U_w, Eq. (7) can be expressed as (Ohtsu et al. 2005)

$$E = C_p d\cos\theta + C_v \frac{U_w^2}{2g} \tag{8}$$

where

$$C_p = \frac{\int_0^{Y_{90}} (\rho g y \cos\theta + p) V dy}{\int_0^{d} (\rho_w g y \cos\theta + p_w) U_w dy} = \frac{\int_0^1 [(1-C)Y + \int_Y^1 (1-C) dY] U dY}{(1 - \int_0^1 C dY) \int_0^1 (1-C) U dY} \tag{9}$$

$$C_v = \frac{\int_0^{Y_{90}} \frac{1}{2}\rho V^3 dy}{\rho_w q_w \frac{1}{2} U_w^2} = \frac{(1 - \int_0^1 C dY)^2 \int_0^1 (1-C) U^3 dY}{[\int_0^1 (1-C) U dY]^3} \tag{10}$$

with $p_w = \int_y^d \cdot \rho_w g \cos\theta dy$ as the clear-water pressure; $Y = y/Y_{90}$; and $U = V/V_{90}$. In Eq. (9), C_p is the ratio of the potential energy flux plus the work done by the pressure for the aerated flow to that for the clear-water flow. The ratio of the kinetic energy flux for the aerated flow to that for the clear-water flow is C_v. According to Eqs. (9) and (10), the values of the correction coefficients C_p and C_v depend on the profiles of $C(Y)$ and $U(Y)$. For nonaerated flow, $C_p = 1$ and C_v = energy coefficient for single-phase flow (the Coriolis coefficient).

The authors described that the air concentration profile $C(Y)$ for all configurations including uniform step heights is approximated by Eq. (4) in the authors' paper using the depth-averaged air concentration C_m. For the uniform step heights in the quasi-uniform and nonuniform skimming flows, the velocity profile of aerated flows $U(Y)$ may be approximated with the $1/N$th power law as

$$U = Y^{1/N} \quad \text{for } 0 \le Y \le 1 \tag{11}$$

For the quasi-uniform skimming flow with a uniform step height, the values of C_m and N can be obtained from the empirical equations for C_m and N as $C_m = 0.42$–0.49 and $N = 5.9$–9.8 for $\theta = 26.6°$ and $0.5 \le h/d_c \le 1.0$ (Takahashi and Ohtsu 2010). Thus, the profiles of $C(Y)$ and $U(Y)$ can be determined. Using Eqs. (9) and (10), the values of C_p and C_v are estimated as $C_p = 1.22$–1.35 and $C_v = 1.08$–1.04 for $\theta = 26.6°$ and $0.5 \le h/d_c \le 1.0$. In the nonuniform flow region for all configurations, if the magnitude and distribution of $C(Y)$ and $U(Y)$ are experimentally obtained, the values of C_p and C_v can be evaluated from Eqs. (9) and (10).

To determine the relationship between the residual energy (specific energy) of the aerated flow E and the conventional residual energy from the clear-water depth H_{res}, the ratio of E/H_{res} is obtained from Eq. (8) in this paper and from Eq. (5) in the original paper:

$$\frac{E}{H_{res}} = \frac{C_p d\cos\theta + C_v \frac{U_w^2}{2g}}{d\cos\theta + \frac{U_w^2}{2g}} = \frac{C_p (\frac{d}{d_c}) \cos\theta + \frac{C_v}{2} (\frac{d}{d_c})^{-2}}{(\frac{d}{d_c}) \cos\theta + \frac{1}{2} (\frac{d}{d_c})^{-2}} \tag{12}$$

For the quasi-uniform skimming flow with uniform step heights, the values of E/H_{res} can be evaluated from Eq. (12) with the values of C_p, C_v, and $d/d_c = [f/(8\sin\theta)]^{1/3}$ in which $f[= 8(d/d_c)^3 \sin\theta]$ is the friction factor and is given as $f = 0.14$ for $\theta = 26.6°$ and $0.5 \le h/d_c \le 1.0$ (Takahashi and Ohtsu 2010). The ratio of E to H_{res} results in $E/H_{res} = 1.09$–1.06 for $\theta = 26.6°$ and $0.5 \le h/d_c \le 1.0$, suggesting that the evaluated values of E may be more precise than those obtained from the conventional residual energy H_{res}. For the nonuniform flow region, the values of E/H_{res} for all configurations can be obtained from Eq. (12) with the values of C_p, C_v, and d/d_c.

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Closure to "Energy Dissipation down a Stepped Spillway with Nonuniform Step Heights" by Stefan Felder and Hubert Chanson

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The writers thank the discussers for their comments. Herein they develop on the dimensional considerations and residual head data.

In skimming flows above prototype stepped spillways, two key features are the strong free-surface aeration and air-water flow turbulence (Chanson 2001). In any dimensional analysis, the relevant parameters include the fluid properties and physical constants, the chute geometry and inflow conditions, the air-water flow properties, and the geometry of the steps (Chanson and Gonzalez 2005; Felder and Chanson 2009). A number of recent studies emphasized that the concept of dynamic similarity and scale effects are closely linked with the selection of relevant characteristic air-water flow properties (Chanson 2009). A critical aspect is the selection of the relevant length scales. Most physical studies of stepped spillways including the discussion assumed implicitly that the vertical step height is the characteristic length scale. This selection is inadequate for a stepped spillway with nonuniform step heights because there is more than one step height in the original paper. Traditional results obtained on stepped chutes with uniform step height might become unsuitable.

In the original paper, the residual head at the measurement section was calculated as

$$H_{\text{res}} = \cos\theta \int_0^{Y_{90}} (1-C)dy + \frac{\left[\int_0^{Y_{90}} (1-C)V dy\right]^2}{2g\left[\int_0^{Y_{90}} (1-C)dy\right]^2} \qquad (1)$$

where Y_{90} = characteristic depth where $C = 0.90$; C = void fraction; and V = interfacial velocity. In Eq. (1), right side, the first term is the depth-averaged pressure head and the second term is the kinetic energy head. The velocity correction term was assumed unity and the pressure distribution was assumed hydrostatic. The discussers pointed out nicely that the pressure gradient might differ locally from the hydrostatic pressure gradient because the streamlines might not be parallel to the average chute slope on a nonuniform stepped invert. There is, however, a lack of physical data in terms of pressure distributions to argue and to quantify the effect of the streamline curvature.

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