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Technical Note

The compressibility of extra-high-velocity aerated flow

La compressibilité des écoulements aérés à très grande vitesse

H. CHANSON, Reader, *Fluid Mechanics, Hydraulics and Environmental Engineering, Department of Civil Engineering, The University of Queensland, Brisbane QLD 4072, Australia. E-mail: h.chanson@uq.edu.au*

ABSTRACT

Compressibility of high-velocity air–water flow is an important issue relevant to the design of high-velocity spillways. In this note, the re-analysis of existing model and prototype data suggests that transonic and supersonic flow conditions were achieved in a number of studies. The results imply that, in free-surface flows, compressibility effects have little impact neither on the air bubble diffusion process nor on the mixing layer characteristics.

RÉSUMÉ

La compressibilité des écoulements à grande vitesse de mélange air-eau est une question importante concernant la conception des déversoirs à grande vitesse. Dans cette note, une nouvelle analyse des données de modèles et de prototypes dont on dispose, suggère que, dans un certain nombre d'études, les états d'écoulement réalisés étaient transsoniques et supersoniques. Les résultats impliquent que, dans des écoulements à surface libre, les effets de compressibilité ont peu d'impact sur le processus de diffusion de bulles d'air et sur les caractéristiques de mélange des couches.

Keywords: Air entrainment, compressibility effects, high-velocity flows, air bubble diffusion, mixing layer, scale effects.

Compressibility of extra-high-velocity aerated flow is an important topic relevant to high-velocity spillway design. In confined air–water flows (e.g. bubbly pipe flows), the compressibility effects and the characteristics of supersonic gas–liquid flows are well known (e.g. Eddington, 1970). In free-surface air–water flow, the effects of fluid compressibility were less studied. Earlier contributions include Cain (1978) and Chanson (1997, pp. 24–26 and 278–280) in self-aerated open channel flows, while Ruggles *et al.* (1988) discussed the effects of bubble size on sound celerity. The present note is motivated by the earlier work of Zhao and Li (2000). The writer is critical of broad statements suggesting that compressibility effects might be substantial in air–water flow mixture when the Sarrau–Mach number exceeds 0.3. Such effects are not always true in self-aerated spillway flows.

A number of experiments in high-velocity air–water flows were conducted in New Zealand (Cain, 1974; Chanson, 1988) and in Switzerland (Volkart and Rutschmann, 1984). Detailed distributions of void fraction and air–water velocity were measured in each study, with air-water velocities ranging from 5 to 24 m/s. Dimensional results are presented in Fig. 1 for prototype and model data (Cain, 1974 and Chanson, 1988, respectively). The measurements of void fractions and air–water velocities are compared respectively with an analytical solution of the advective diffusion equation for air bubbles and a power law:

$$C = 1 - \tanh^2\left(k' - \frac{y}{2 * D' * y_{90}}\right) \quad (1)$$

$$\frac{u}{u_{90}} = \left(\frac{y}{y_{90}}\right)^{1/6} \quad (2)$$

where C is the void fraction, u is the air–water flow velocity, u_{90} is the characteristic velocity where $C = 0.90$, y is the distance normal to the invert, y_{90} is the characteristic distance where $C = 0.90$, D' and K' are functions of the mean void fraction only (Chanson, 1997) and the 1/6-th power law was found to be independent of the mean air content (Chanson, 1994).

Further experimental results are presented in dimensionless terms in Fig. 2 (prototype data: Cain, 1974; Volkart and Rutschmann, 1984). The dimensionless velocity is the ratio $Ma = u/a$ called the Sarrau–Mach number. The ratio of the fluid velocity to the sound celerity is commonly named after E. Mach who introduced it in 1887. It is also called the Sarrau number after Professor Sarrau who first used this ratio (Sarrau, 1884). The Sarrau–Mach number was originally called the Cauchy number as a tribute to A.L. de Cauchy. For $Ma > 1$ the flow is supersonic. Figure 2 shows that prototype flows may be locally supersonic. Model flows are most often subsonic and sometimes transonic, the Sarrau–Mach number *Ma* being sometimes greater than 0.3. For example, in Fig. 1b, the Sarrau–Mach number varies from 0 to 0.7.

In free-surface air–water flows, these experimental investigations suggest that compressibility effects are not significant. The air–water flow properties (void fraction and velocity distributions) have the same shape as in subsonic flows and they may be predicted the same analytical developments (e.g. Eqs (1) and (2), Chanson, 1997). Further the law of flow resistance is unchanged in transonic and supersonic self-aerated flows (Chanson, 1994). It is believed that the proximity of the 'free-surface' may facilitate the flow bulking and prevent the formation of sonic shock

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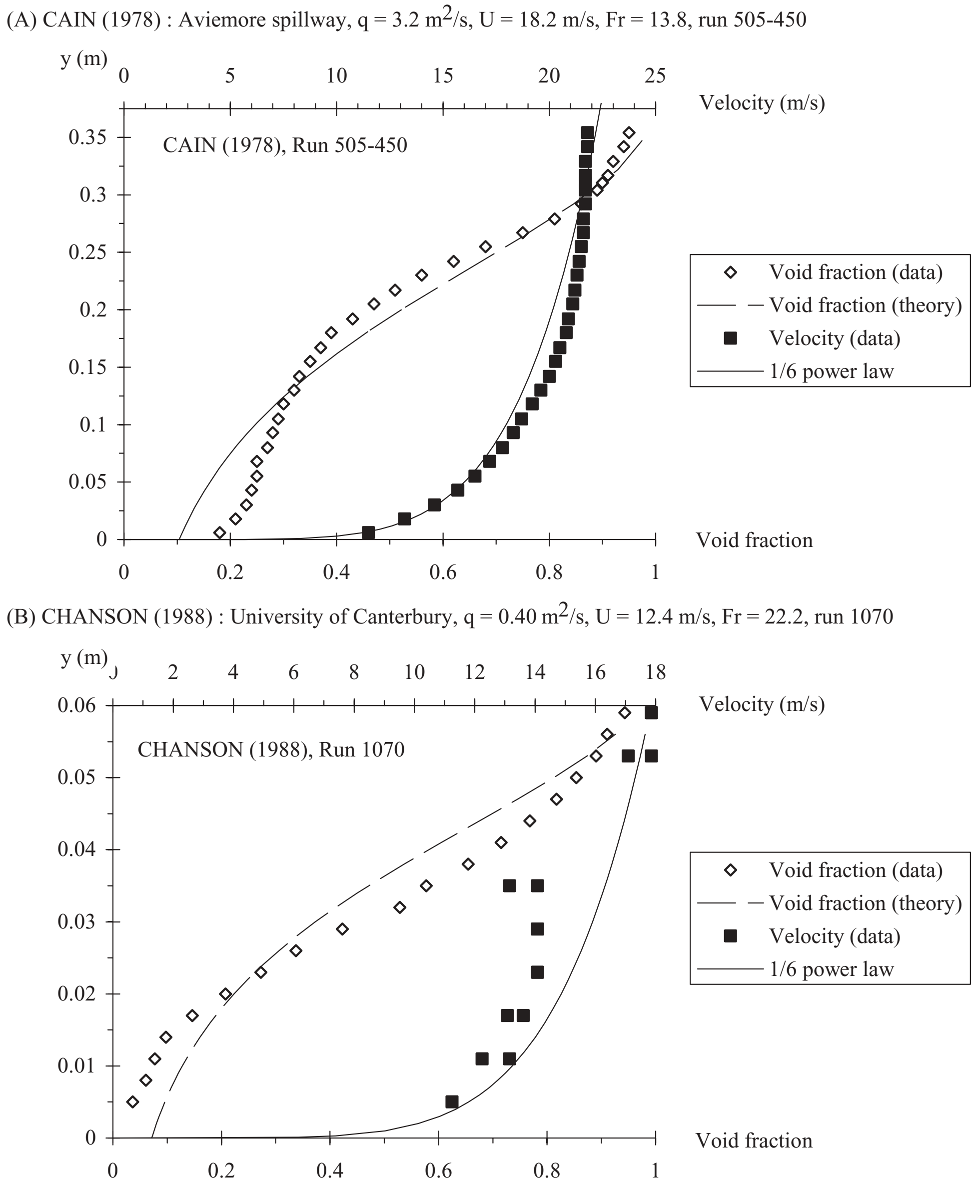

Figure 1 Air–water flow measurements.

waves. The ‘free-surface’ is not a fixed boundary and it fluctuates to accommodate the expansion of the flow bulk. Existing sets of experimental investigations (e.g. Figs 1 and 2) show no compressibility effects on the air bubble diffusion process nor on the mixing layer characteristics.

Discussion

In air–water flows, the minimum sound celerity is about 18–20 m/s for C $= 0.5$ at 10°C and $p = 1$ atm. Taking into account the variations of standard absolute pressures with altitudes, it may be demonstrated that the sound celerity in an air–water mixture decreases with increasing altitude (e.g. $a = 15$ m/s for $C = 0.5$ at 5000 m altitude). As a result compressibility effects might become more significant in high-altitude regions (e.g. in South-America, in Nepal).

The writer believes that the study of compressibility effects in high-velocity free-surface flows is important, but still at an embryonic stage. The topic should be brought to the attention to the research and engineering community for further basic studies.

Notation

a = Sound celerity in gas–liquid flow (m/s)
C = Void fraction, defined as the volume of air per unit volume of air and water
Fr = Froude number
p = Absolute pressure (Pa)
q = Water discharge per unit width (m^2/s)
u = Air–water flow velocity (m/s)
U = Depth-averaged flow velocity (m/s) (i.e. equivalent clear-water flow velocity)
y = Distance measured normal to the invert (m)

Greek symbols

θ = Bed (invert) slope

Subscript

90 = Characteristic parameter where $C = 90\%$

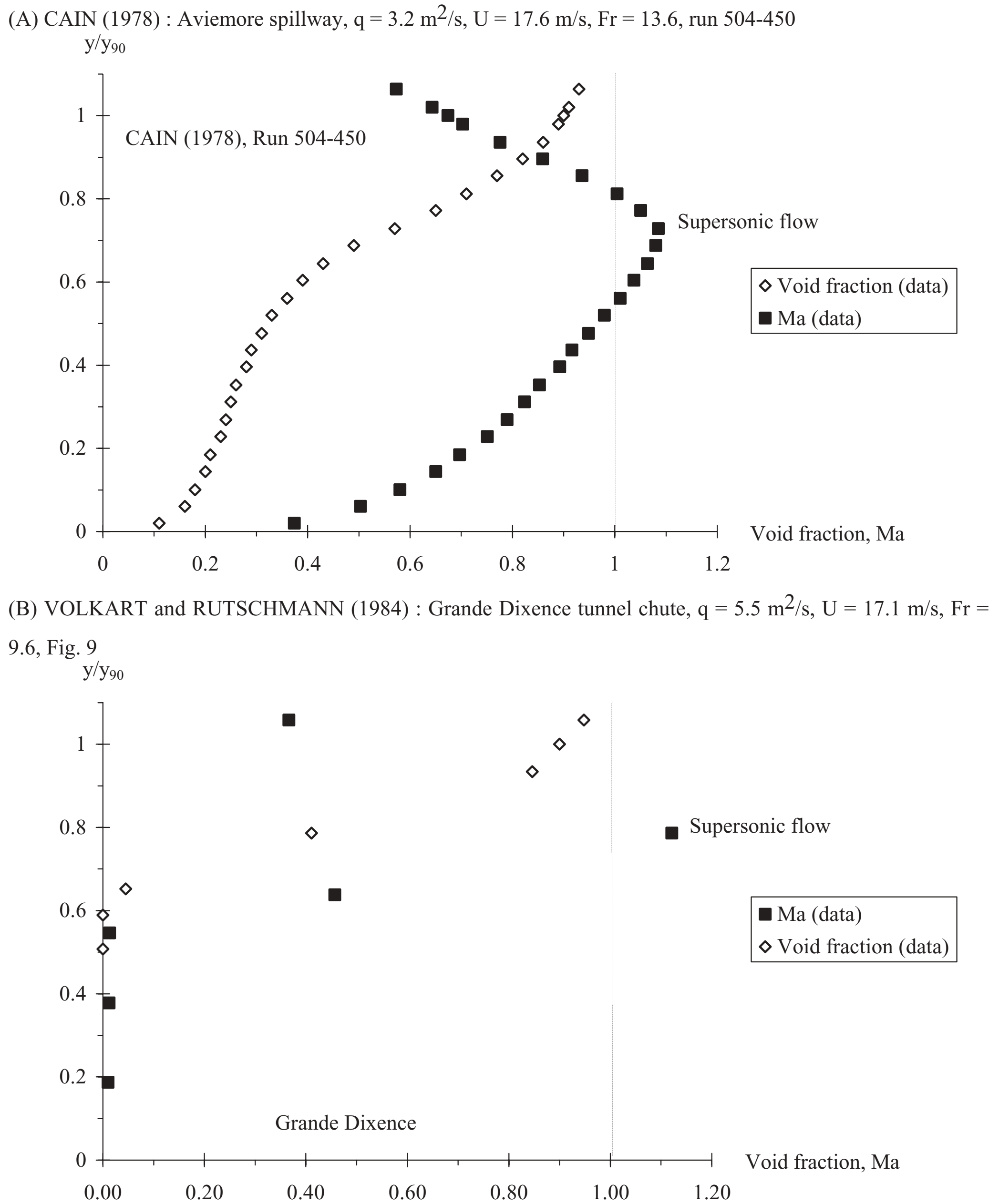

Figure 2 Dimensionless velocity distributions in terms of the Sarrau–Mach number.

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