# Air-water flows in partially-filled conduits

# Entrainement d'air dans les écoulements en conduites avec surface libre

H. CHANSON, MIAHR, *Senior Lecturer, Fluid Mechanics, Hydraulics and Environmental Engineering, Department of Civil Engineering, The University of Queensland, Brisbane QLD 4072, Australia.*

**ABSTRACT**

Supercritical open channel flows are characterised by a large amount of "white waters". Although free-surface aeration was investigated for steep chutes, flow aeration down partially-filled pipes received little attention. A re-analysis of partially-filled pipe data (VOLKART 1982,1985) is presented. The results indicate that the air concentration and velocity distributions can be estimated by simple expressions. The study provides new information for the design of storm waterways, sewers and tunnel spillways where flow aeration may occur.

**RÉSUMÉ**

Les éoulements à surface libre supercritiques sont sujets à un entraînement d'air à la surface libre ("eau blanche"). Bien que l'entraînement d'air dans les écoulements à surface libre soit bien connu dans les canaux rectangulaires à ciel ouvert, il n'en est pas de même pour les écoulements à surface libre dans les conduites. On présente une nouvelle analyse des résultats expérimentaux obtenus par VOLKART (1982 et 1985). On montre que l'on peut estimer par des fonctions simples les profils de concentrations en air et de vitesses. Ces résultats fournissent de nouvelles informations pour la conception de systèmes d'évacuation des eaux d'orage ou d'évacuateurs de crue eu galeric dans lesquels un entraînement d'air peut se produire.

## Introduction

The process of free-surface aeration in spillways, chutes and storm waterways was first studied because of the effects of entrained air on the thickness of the flow. Free-surface-aeration in steep-slope chutes was investigated thoroughly for wide rectangular channels (e.g. WOOD 1991). Air entrainment is observed also in partially-filled pipes (e.g. sewers, tunnel spillways) (fig. 1). There is little information however on the hydraulic characteristics of air-water flows in partially-filled conduits. The basic reference is the experimental work of VOLKART (1982,1985) who showed a significant amount of air entrainment. His work indicated that the flow properties differ from open chute flows. Additional bibliography includes FALVEY (1980), CHEPAIKIN and DONCHENKO (1984) and CHANSON (1992).

In this paper, it will be shown that the air concentration and velocity distributions in partially-filled conduits can be estimated by simple expressions. The results can be integrated to provide some information on the air flow rate entrained within the air-water flow.

Revision received June 12, 1997. Open for discussion till April 30, 1998.

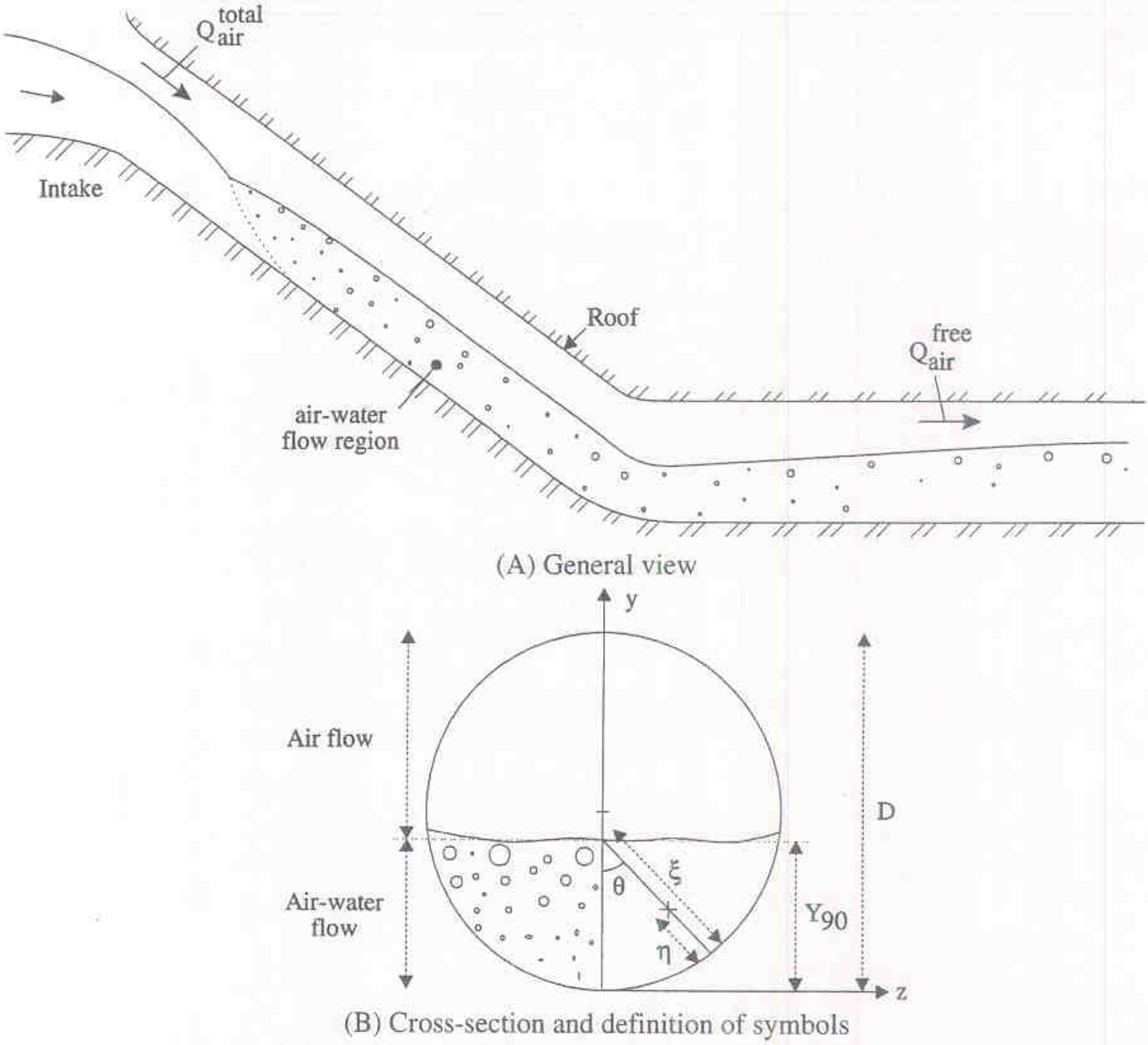

Fig. 1. Sketch of air-water flow in partially-filed pipe

*Definitions*

The local air concentration $C$ (i.e. the void fraction) is defined as the volume of air per unit volume. The interface between the air-water mixture and the atmosphere is defined as the iso-air concentration line $C = 90\%$.
In partially-filled pipe, the mean air concentration on the centreline is defined as:

$$(C_{mean})_{cl} = \left( \frac{1}{Y_{90}} \cdot \int_0^{Y_{90}} C \cdot dy \right)_{cl} \tag{1}$$

where $y$ is the distance normal to the bottom (fig. 1(B)) and $Y_{90}$ is the distance where $C = 0.9$.
The mean air content of the air-water flow is:

$$C_{mean} = \frac{1}{A_{90}} \cdot \int_0^{Y_{90}} \int_{-\sqrt{y \cdot (D-y)}}^{+\sqrt{y \cdot (D-y)}} C \cdot dz \cdot dy \tag{2}$$

where $z$ is the transverse direction (fig. 1(B)), $D$ is the internal pipe diameter and $A_{90}$ is the cross-section area of the air-water flow (i.e. $C < 90\%$). The clear water cross-section area is:

$$A_W = \int_0^{Y_{90}} \int_{-\sqrt{y \cdot (D-y)}}^{+\sqrt{y \cdot (D-y)}} (1-C) \cdot dz \cdot dy \tag{3}$$

and the mean flow velocity equals:

$$U_w = \frac{Q_w}{A_w} \tag{4}$$

where the total water discharge $Q_W$ satisfies the continuity equation for water:

$$Q_W = \int_0^{Y_{90}} \int_{-\sqrt{y \cdot (D-y)}}^{+\sqrt{y \cdot (D-y)}} (1-C) \cdot V \cdot dz \cdot dy \tag{5}$$

Table 1 Experimental flow conditions for VOLKART's (1982,1985) experiments.

| Run | $Q_W$ | Pipe diameter | Slope | x | $(Y_{90})_{cl}/D$ | $(C_{mean})_{cl}$ | $C_{mean}$ | Comments |
|---|---|---|---|---|---|---|---|---|
| | $m^3/s$ | D in m | degrees | m | ([a]) | ([a]), ([b]) | ([a]), ([c]) | |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | | (8) |
| Section 1 | 0.050 | 0.24 | 45.0 | 9.36 | 0.276 | 0.255 | 0.32 | VOLKART (1985), fig. 5 |
| Section 2 | 0.120 | 0.24 | 45.0 | 9.36 | 0.457 | 0.227 | 0.26 | VOLKART (1985), fig. 5 |
| Section 3 | 0.035 | 0.11 | 43.8 | 4.81 | 0.74 | 0.150 | 0.17 | VOLKART (1982), fig. 4 |
| Section 4 | 0.020 | 0.11 | 43.8 | 4.81 | 0.467 | 0.192 | 0.23 | VOLKART (1982), fig. 5 |
| Section 5 | 0.025 | 0.24 | 45.0 | 9.36 | 0.2105 | 0.251 | 0.30 | VOLKART (1985), fig. 5 |
| Section 6 | 0.075 | 0.24 | 45.0 | 9.36 | 0.351 | 0.219 | 0.24 | VOLKART (1985), fig. 5 |
| Section 7 | 0.280 | 0.24 | 11.6 | 36.3 | 0.82 | 0.180 | | VOLKART (1982), fig. 6 |
| Section 8 | 0.280 | 0.24 | 11.6 | 48.0 | 0.80 | 0.240 | 0.24 | VOLKART (1982), fig. 7 |

Notes:
D: internal conduit diameter; Slope: pipeline slope with the horizontal; x: distance from the pipe intake; Wall roughness: 0.1 mm; (a): calculated by the author from VOLKART's data; (b): defined as equation (1); (c): defined as equation (2).

## Air concentration and velocity distributions

VOLKART (1982,1985) measured air-water flow characteristics in circular pipes. Eight sets of data are well documented (table 1) and they are re-analysed in this paper. Flow conditions near transition to pipe flow are not considered and the present study is focused on flow conditions such as $Y_{90}/D < 0.7$.

For all re-analysed data (table 1), the free-surface (defined in term of 90% air content) is nearly horizontal in the transverse direction $z$ (fig. 2). To a first approximation, it is reasonable to assume:

$$Y_{90}(z) = (Y_{90})_{cl} = Y_{90} \tag{6}$$

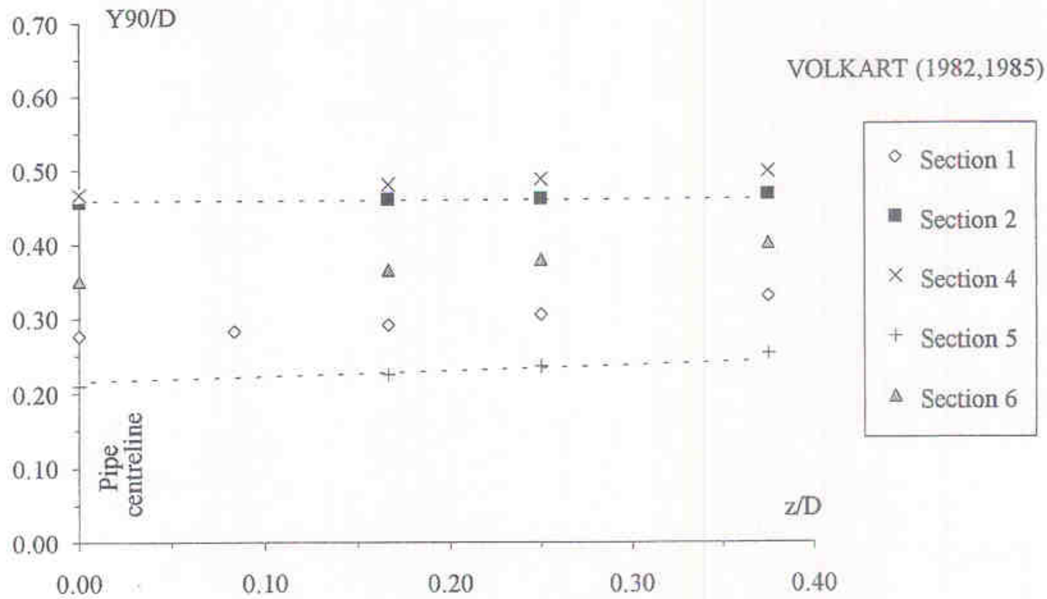


Fig. 2. Transverse free-surface profiles in self-aerated partially-filled pipe (data: table 1)

In the following the notation $Y_{90}$ will denote the centreline characteristic depth where $C = 0.9$.
At any position along the pipeline, the air-water cross-section (i.e. defined as $y < Y_{90}$) and its parameters (e.g. free-surface width, wetted perimeter) can be defined uniquely in term of the filling ratio $Y_{90}/D$ (see appendix A).
A re-analysis of all air concentration data was performed using four or five cross-sectional profiles for each of the eight experiments. Altogether thirty-seven air content profiles were re-analysed. The results (fig. 3) show that the air concentration distributions follow a similar shape as in chute flows and that they are independent of the lateral position. Figure 3 presents nine air concentration profiles characteristics of medium to large mean air contents.

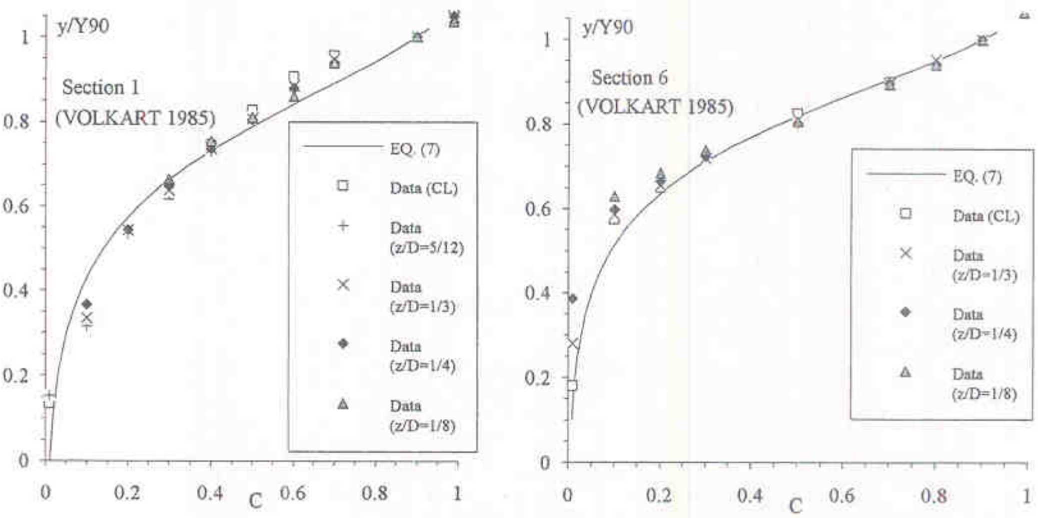


Fig. 3. Dimensionless air concentration distributions in self-aerated partially-filled pipes - Comparison between equation (7) and experimental data (flow conditions in table 1)

Altogether the air concentration profiles can be fitted with analytical solutions of the air bubble diffusion process as in open channel flows (e.g. WOOD 1991, CHANSON 1995a,b). Figure 3 presents a comparison between data and CHANSON's (1995a,b) model:

Table 2 Dimensionless coefficients of the air bubble diffusion model (after CHANSON 1995b).

| $(C_{mean})_{cl}$ | D' | K' |
|---|---|---|
| (1) | (2) | (3) |
| 0.01 | 0.007312 | 68.70445 |
| 0.05 | 0.036562 | 14.0029 |
| 0.10 | 0.073124 | 7.16516 |
| 0.15 | 0.109704 | 4.88517 |
| 0.20 | 0.146489 | 3.74068 |
| 0.30 | 0.223191 | 2.567688 |
| 0.40 | 0.3111 | 1.93465 |
| 0.50 | 0.423441 | 1.508251 |
| 0.60 | 0.587217 | 1.178924 |
| 0.70 | 0.878462 | 0.896627 |

$$C = 1 - \tanh^2\left(K' - \frac{y'}{2 \cdot D'}\right) \tag{7}$$

where $y' = y/Y_{90}$, and $K'$ and $D'$ are functions of the centreline mean air concentration $(C_{mean})_{cl}$ (table 2). Note that equation (7) was validated with local air concentrations ranging from 0 to 0.95 and mean air contents between 0.17 and 0.30 only.

Figure 3 implies that the air concentration distribution is a function only of $Y_{90}/D$ and $(C_{mean})_{cl}$. It is independent of the transverse distance $z$ from the centreline.

For flow conditions such as $Y_{90}/D < 0.7$, the velocity distributions exhibit a maximum near the free-surface on the centreline: i.e., at $y = Y_{90}$ and $z = 0$. The maximum velocity is denoted $V_{90}$. When the air flow region become small (i.e. $Y_{90}/D > 0.7$), the interactions between the air-water mixture and the air flow are no longer negligible. They affect the velocity distributions and the maximum velocity is observed below the free-surface on the centreline.

Velocity measurements from six experiments and six profiles each were re-analysed for $Y_{90}/D < 0.7$. For the thirty six velocity profiles, the three-dimensional velocity distribution was best approximated by a power law distribution:

$$\frac{V}{V_{90}} = \left(\frac{\eta}{\xi}\right)^{1/N} \tag{8}$$

where $\eta$ and $\xi$, defined on figure 1(B), are geometrical functions of $Y_{90}/D$ and of the Cartesian co-ordinates $(y,z)$. For the data of VOLKART (table 1, $Y_{90}/D < 0.7$) the exponent $N$ varies between 5.2 and 7.2 with a mean values of about 6.2. In air-water chute flows, the velocity distributions follow a 1/6-th power law (WOOD 1991, CHANSON 1993) and it is reasonable to assume also $N = 6$ for partially-filled pipe flows. Note that equation (8) was checked with local velocities between 0 and 10 m/s, maximum velocities ranging from 6 to 10 m/s and pipe diameters between 0.11 and 0.24 m. On figure 4 experimental data are compared with equation (8) for $N = 6$. The data are plotted for several angles $\theta$ as defined on figure 1(B) and indicated in the legend. Note that the velocity distribution is independent of the mean air content, of the ratio $Y_{90}/D$ and of the angle $\theta$ for $0 \le \theta \le 60$ degrees. Deviations from the power law are observed at the free-surface (i.e. $\theta = 90$ deg.). They might be caused by interactions with the air flow in the upper part of the pipe and possibly by measurement errors near the free-surface.

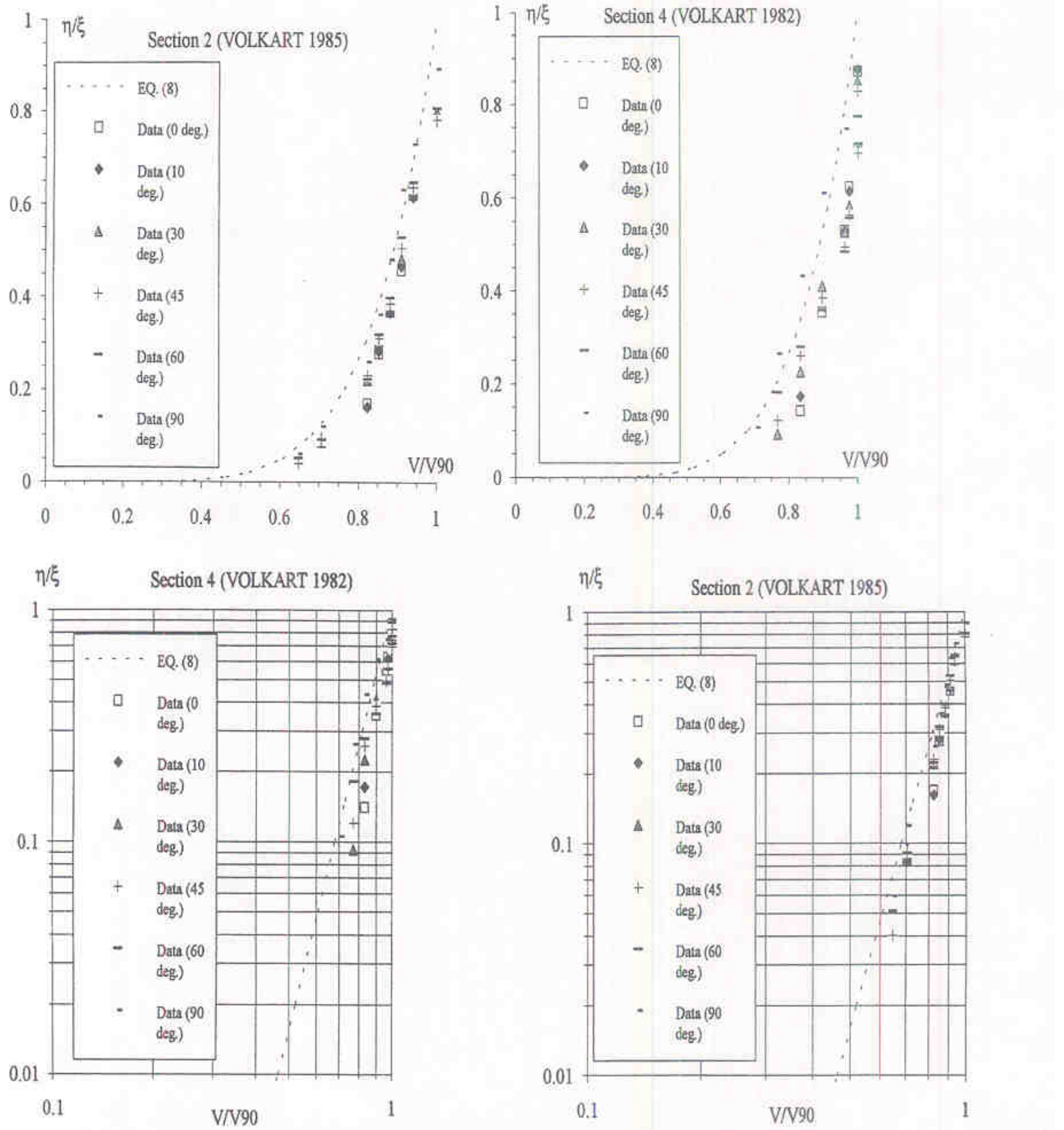


Fig. 4. Dimensionless velocity distributions in self-aerated partially-filled pipes – Comparison between equation (8) and experimental data (flow conditions in table 1) for several values of q (defined on fig. 1(B))

## Air-water flow parameters

At any position along the pipe line, the complete air-water flow properties (i.e. $C$ and $V$) can be deduced from the centreline mean air content $(C_{mean})_{cl}$ and the ratio $Y_{90}/D$ using equations (7) and (8). From the continuity equation, the maximum velocity $V_{90}$ can be computed as:

$$V_{90} = \frac{Q_W}{D^2} \cdot \left( \int_0^{Y_{90}/D} \int_{-\sqrt{y'' \cdot (1-y'')}}^{+\sqrt{y'' \cdot (1-y'')}} (1-C) \cdot \left(\frac{\eta}{\xi}\right)^{1/6} \cdot dz'' \cdot dy'' \right)^{-1} \tag{9}$$

where $y'' = y/D$ and $z'' = z/D$. As the air concentration and velocity distributions follow equations (7) and (8) respectively, it is possible to estimate the mean air content over the cross-section $C_{mean}$ (eq. (2)) and the air flow rate within the air-water mixture $Q_{air}{}^{aw}$:

Table 3 Air-water flow properties in self-aerated partially-filled pipes.

| $(C_{mean})_{cl}$ ↓ | $C_{mean}$ | | | | $V_{90}/U_w$ | | | | $\frac{Q_{air}^{aw}}{Q_w}$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $Y_{90}/D$ = | 0.14 | 0.35 | 0.49 | 0.7 | 0.14 | 0.35 | 0.49 | 0.7 | 0.14 | 0.35 | 0.49 | 0.7 |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) | (13) |
| 0 | 00 | 0 | 0 | 0 | 1.263 | 1.264 | 1.264 | 1.264 | 0 | 0 | 0 | 0 |
| 0.035 | 0.052 | 0.049 | 0.046 | 0.040 | 1.269 | 1.269 | 1.269 | 1.268 | 0.059 | 0.056 | 0.053 | 0.046 |
| 0.070 | 0.103 | 0.097 | 0.092 | 0.082 | 1.275 | 1.275 | 1.275 | 1.274 | 0.124 | 0.117 | 0.111 | 0.097 |
| 0.105 | 0.151 | 0.144 | 0.137 | 0.124 | 1.280 | 1.280 | 1.280 | 1.279 | 0.194 | 0.183 | 0.174 | 0.155 |
| 0.140 | 0.198 | 0.190 | 0.182 | 0.166 | 1.285 | 1.285 | 1.285 | 1.284 | 0.268 | 0.255 | 0.243 | 0.218 |
| 0.175 | 0.243 | 0.233 | 0.225 | 0.208 | 1.290 | 1.290 | 1.290 | 1.289 | 0.349 | 0.332 | 0.317 | 0.287 |
| 0.210 | 0.286 | 0.275 | 0.267 | 0.248 | 1.295 | 1.295 | 1.294 | 1.293 | 0.433 | 0.414 | 0.397 | 0.362 |
| 0.245 | 0.325 | 0.315 | 0.306 | 0.288 | 1.298 | 1.299 | 1.298 | 1.297 | 0.523 | 0.500 | 0.481 | 0.441 |
| 0.280 | 0.363 | 0.352 | 0.343 | 0.325 | 1.302 | 1.302 | 1.302 | 1.301 | 0.616 | 0.590 | 0.569 | 0.524 |
| 0.315 | 0.397 | 0.387 | 0.378 | 0.360 | 1.304 | 1.305 | 1.305 | 1.304 | 0.712 | 0.683 | 0.660 | 0.611 |
| 0.350 | 0.429 | 0.419 | 0.410 | 0.392 | 1.307 | 1.307 | 1.307 | 1.307 | 0.810 | 0.779 | 0.753 | 0.701 |
| 0.385 | 0.458 | 0.448 | 0.440 | 0.422 | 1.309 | 1.309 | 1.309 | 1.309 | 0.910 | 0.877 | 0.849 | 0.793 |
| 0.420 | 0.499 | 0.490 | 0.482 | 0.466 | 1.311 | 1.311 | 1.311 | 1.311 | 1.072 | 1.035 | 1.004 | 0.942 |
| 0.455 | 0.533 | 0.524 | 0.517 | 0.501 | 1.312 | 1.313 | 1.313 | 1.313 | 1.224 | 1.184 | 1.151 | 1.083 |
| 0.490 | 0.563 | 0.555 | 0.547 | 0.533 | 1.313 | 1.314 | 1.314 | 1.314 | 1.376 | 1.333 | 1.298 | 1.226 |
| 0.525 | 0.597 | 0.589 | 0.582 | 0.569 | 1.314 | 1.314 | 1.315 | 1.315 | 1.576 | 1.530 | 1.492 | 1.414 |
| 0.560 | 0.628 | 0.621 | 0.615 | 0.603 | 1.314 | 1.315 | 1.315 | 1.315 | 1.792 | 1.743 | 1.702 | 1.619 |
| 0.595 | 0.654 | 0.648 | 0.642 | 0.631 | 1.313 | 1.314 | 1.315 | 1.315 | 2.003 | 1.951 | 1.907 | 1.819 |
| 0.630 | 0.689 | 0.683 | 0.679 | 0.669 | 1.312 | 1.313 | 1.314 | 1.314 | 2.338 | 2.282 | 2.235 | 2.141 |
| 0.665 | 0.718 | 0.713 | 0.709 | 0.700 | 1.310 | 1.312 | 1.312 | 1.312 | 2.673 | 2.614 | 2.565 | 2.465 |

$$Q_{air}^{aw} = \int_{0}^{Y_{90}} \int_{-\sqrt{y \cdot (D-y)}}^{+\sqrt{y \cdot (D-y)}} C \cdot V \cdot dz \cdot dy \qquad (10)$$

The full set of calculations was integrated numerically using the above analysis for the free-surface profile, air concentration distribution and velocity distribution (i.e. eq. (6), (7) and (8)). The calculations are presented in table 3 and on figures 5 and 6. The results indicate that maximum velocity V90, the mean air concentration $C_{mean}$ and the air flow rate within the air-water flow region $Q_{air}^{aw}$ are primarily functions of the centreline mean air content $(C_{mean})_{cl}$. And the filling ratio $Y_{90}/D$ has little effect.

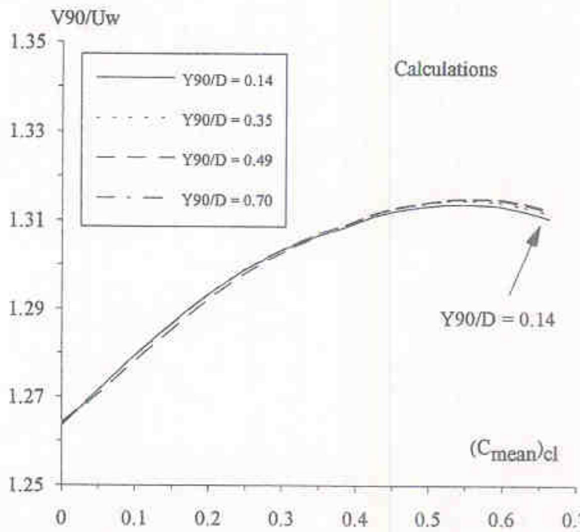


Fig. 5. Dimensionless maximum velocity $V_{90}/U_w$ in partially-filled pipe as a function of the centreline mean air content $(C_{mean})_{cl}$ and the filling ratio $Y_{90}/D$

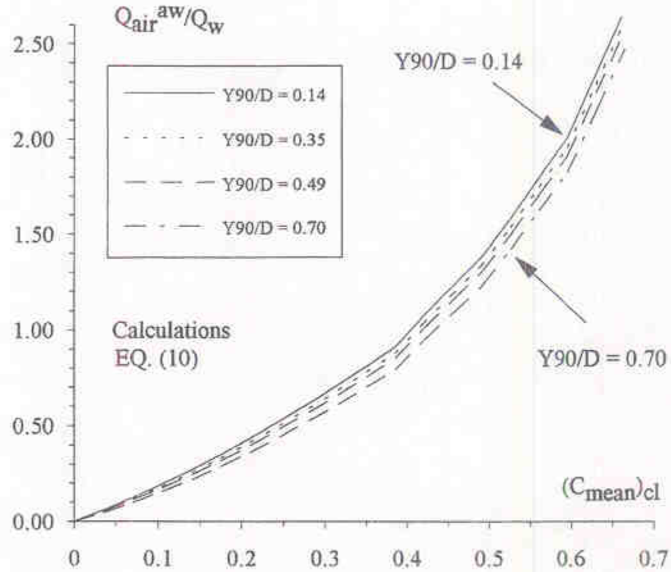


Fig. 6. Dimensionless air flow rate within the air-water flow region $Q_{air}^{aw}$ in partially-filled pipe as a function of the centreline mean air content $(C_{mean})_{cl}$ and the filling ratio $Y_{90}/D$

## Design considerations

The basic differences between air entrainment in open chutes and in partially-filled pipes are: 1- the risks of transition to pipe flow when the free-surface leaps the roof, 2- the limited amount of available air supply in the confined space, 3- the air flow above the air-water flow which may interact with the air-water flow and 4- the dependence of the free-surface width as a function of the filling ratio with a maximum exchange area for half-full pipe.

For designers a major design parameter is the total air discharge flowing in the pipe. This air flow rate must be brought into the pipe by manholes, air vents and at the pipe intake. In a partially-filled pipe the total air flow rate $Q_{air}^{total}$ equals:

$$Q_{air}^{total} = Q_{air}^{aw} + Q_{air}^{free} \tag{11}$$

where $Q_{air}^{aw}$ is the air flow within the air-water mixture flow and $Q_{air}^{free}$ is the air discharge in the "free" space above the air-water flow (fig. 1). The air flow (above the free-surface) is caused by the exchange of momentum with the flowing fluid and the associated drag of the free-surface, and eventually by the downstream air entraining capacity of the air-water flow (fig. 1).

For a pipe of constant diameter, slope and roughness, and for a given water discharge, the maximum air flow rate can be estimated for the equilibrium flow conditions. At uniform equilibrium flow conditions, the air flow $Q_{air}^{free}$ is caused only by the shear stress at the free-surface. The air flow boundary conditions are: $V$ = air-water velocity at $y = Y_{90}$ and $V = 0$ at the pipe walls. The air flow is a type of Couette flow and a rough estimate of the air discharge is:

$$Q_{air}^{free} \sim \frac{1}{2} \cdot V_{90} \cdot \left(\frac{\pi}{4} \cdot D^2 - A_{90}\right) \tag{12}$$

where $V_{90}$ is estimated by equation (9) and in table 3.

Equations (10) to (12) provide means to predict the total air flow rate at uniform equilibrium flow conditions as a function of the water discharge, pipe diameter, equilibrium mean air concentration and flow resistance. A practical example is developed in appendix B.

In absence of intermediary air vents along the pipe, designers must carefully consider possible compressibility effects along the pipe. Compressibility effects must be analysed for both the air flow above the free-surface and the air flow within the flowing air-water mixture. In air the choking effects and unbearable noise associated with compressibility effects are well known. Usually the compressibility effects can be neglected if the Mach number remains less than 0.3[1]. In air-water flows, CAIN (1978) indicated that the sound celerity (in the two-phase flow mixture) is related to the void fraction and falls to a minimum value of about 20 m/s for around 10% air content. The effects of fluid compressibility in air-water flows are not well documented and still somewhat subject to discussions.

## Conclusions

The re-analysis of free-surface aeration in partially-filled pipes indicates that the air-water flow properties can be estimated from simple expression (eq. (6), (7) and (8)). The air-water flow properties are functions only of the centreline mean air content and of the filling ratio $Y_{90}/D$. Such results provide means to estimate the air flow rate entrained within the air-water flow (table 3). The knowledge of the entrained-air flow rate will assist engineers in designing air intake and possible air vents.

Compressibility effects in the air flow is a serious concern. Although the air flow rate within the air-water mixture can be estimated, there is still little experimental information on the Couette-type flow in the air flowing above the free-surface.

## Acknowledgements

The writer thanks his students MM. S. GILCHRIST and J. LOCKE for their assistance in the project. The financial support of Australian Research Council (Ref. No. A89331591) is acknowledged.

[1] The assumption of incompressible fluid can be justified if the ratio of the kinetic energy over the internal energy is small. For air flowing at 100m/s and at 20°C, the ratio is about 0,024.

## Appendix A. Geometrical properties for circular pipe flows

Defining y as the direction normal to the centreline bottom and z the transverse direction, the experimental data of VOLKART (1982,1985) suggest that the free-surface (defined in term of 90% air content) is nearly flat and that $Y_{90}(z) = (Y_{90})_{CL} = Y_{90}$ (fig. 2).

At a position $x$ along the pipe, the free-surface depth $Y_{90}$, the free-surface width $B_{90}$, wetted perimeter $P_{90}$ and cross-section area $A_{90}$ of the air-water flow (i.e. defined as $y < Y_{90}$) are related by:

$$\frac{Y_{90}}{D} = \frac{1}{2} \cdot \left(1 + \cos\left(\frac{\Delta}{2}\right)\right) \quad \text{(A1)}$$

$$\frac{B_{90}}{D} = \sin\left(\frac{\Delta}{2}\right) \quad \text{(A2)}$$

$$\frac{P_{90}}{D} = \pi \cdot \left(1 - \frac{\Delta}{2 \cdot \pi}\right) \quad \text{(A3)}$$

$$\frac{A_{90}}{D^2} = \frac{\pi}{4} \cdot \left(1 - \frac{\Delta - \sin\Delta}{2 \cdot \pi}\right) \quad \text{(A4)}$$

where $\Delta$ is a dimensionless parametric variable.

### Appendix B. Uniform equilibrium flow conditions in partially filled pipes

Considering a long pipe (slope: 22.5 degrees, internal diameter: 1 m, friction factor: $f = 0.03$), let us compute the air-water flow properties at equilibrium for $Q_w = 2.59$ m³/s.
In uniform equilibrium flow, the momentum equation implies:

$$\frac{f}{8} \cdot U_W^2 \cdot P_w = g \cdot A_w \cdot \sin\alpha \qquad \text{(B1)}$$

For the proposed example, geometrical considerations lead: $A_W = 0.198$ m² and $U_W = 13.1$ m/s.
VOLKART's data (table 1) suggest that the mean equilibrium air content on the pipe centreline is about the same as for open channels. E.g., in table 1, sections 7 and 8 are located far downstream of the pipe intake and uniform equilibrium flow is most likely. At these locations, $(C_{mean})_{cl}$ is about 0.18 to 0.24 for $\alpha = 11.6$ degrees. In comparison, $C_{mean}$ is about 0.24 for $\alpha = 15$ degrees in uniform equilibrium flows down open chutes (WOOD 1991, CHANSON 1993).
In equilibrium flows down chutes, the mean air content for a 22.5 degree slope is 0.31 (WOOD 1991, CHANSON 1993). Assuming $(C_{mean})_{cl} \sim 0.31$, the filling ratio is about: $Y_{90}/D \sim 0.435$.
Table 3 gives then the air-water flow parameters as a function of $(C_{mean})_{cl}$ and $Y_{90}/D$:

$$C_{mean} = 0.38,\ V_{90}/U_w = 1.305,\ V_{90} = 17.1 \text{ m/s}$$
$$Q_{air}^{aw} = 0.67,\ Q_{air}^{free}/Q_w = 1,51,\ Q_{air}^{total} = 5.64 \text{ m}^3/\text{s}.$$

### References/Bibliographie


CAIN, P. (1978). "Measurements within Self-Aerated Flow on a Large Spillway." *Ph.D. Thesis*, Ref. 78–18, Dept. of Civil Engrg., Univ. of Canterbury, Christchurch, New Zealand.

CHANSON, H. (1988). "A Study of Air Entrainment and Aeration Devices on a Spillway Model." *Ph.D. thesis*, Ref. 88-8, Dept. of Civil Engrg., University of Canterbury, New Zealand.

CHANSON, H. (1992). "Air Entrainment on Chute and Tunnel Spillways." *Proc. 11th Australasian Fluid Mechanics Conference AFMC*, Vol. 1, Paper 1D-3, Hobart, Australia, pp. 83–86.

CHANSON, H. (1993). "Self-Aerated Flows on Chutes and Spillways." *Jl of Hyd. Engrg.*, ASCE, Vol. 119, No. 2, pp. 220-243. Discussion: Vol. 120, No. 6, pp. 778–782.

CHANSON, H. (1995a). "Air Bubble Entrainment in Free-surface Turbulent Flows. Experimental Investigations." *Report CH46/95*, Dept. of Civil Engineering, University of Queensland, Australia, June, 368 pages.

CHANSON, H. (1995b). "Air Bubble Diffusion in Supercritical Open Channel Flow." *Proc. 12th Australasian Fluid Mechanics Conference AFMC*, Sydney, Australia, R.W. BILGER Ed., Vol. 2, pp. 707–710.

CHEPAIKIN, G.A., and DONCHENKO, E.G. (1986). "Certain Effects of Flow Aeration in Spillways." *Gidrotekhnicheskoe Stroitel'stvo*, No. 11, pp. 19-22 (Hydrotechnical Construction, 1987, Plenum Publ., pp. 636–640).

FALVEY, H.T. (1980). "Air-Water Flow in Hydraulic Structures." *USBR Engrg. Monograph*, No. 41, Denver, Colorado, USA.

VOLKART, P.U. (1982). "Self-Aerated Flow in Steep, Partially Filled Pipes." *Jl of Hyd. Div.*, ASCE, Vol. 108, No. HY9, pp. 1029–1046.

VOLKART, P.U. (1985). "Transition from Aerated Supercritical to Subcritical Flow and Associated Bubble Deaeration." *Proc. 21st IAHR Congress*, Melbourne, Australia, pp. 2–7.

WOOD, I.R. (1991). "Air Entrainment in Free-Surface Flows." *IAHR Hydraulic Structures Design Manual No. 4*, Hydraulic Design Considerations, Balkema Publ., Rotterdam, The Netherlands, 149 pages.

**Notations**

$A$ cross-section area ($m^2$);
$A_W$ clear-water cross-section area ($m^2$);
$A_{90}$ cross-section area ($m^2$) of the air-water flow (i.e. $y < Y_{90}$);
$B_{90}$ free-surface width (m) at $y = Y_{90}$;
$C$ air concentration defined as the volume of air per unit volume of air and water; also called the void fraction;
$C_{mean}$ mean air concentration defined in term of 90% air content (eq. (2));
$D$ pipe diameter (m);
$D'$ dimensionless diffusivity;
$f$ Darcy friction factor;
$g$ gravity constant ($m/s^2$);
$K'$ dimensionless integration constant;
$N$ exponent of the power-law velocity distribution;
$P_W$ wetted perimeter (m);
$P_{90}$ air-water flow "wetted" perimeter (m);
$Q$ discharge ($m^3/s$);
$Q_{air}^{aw}$ air-water mixture flow;
$Q_{air}^{free}$ air discharge ($m^3/s$) in the "free" space between the free-surface and the roof;
$Q_{air}^{total}$ total air flow rate ($m^3/s$) in a pipe;
$U_W$ mean flow velocity (m/s);
$V$ velocity (m/s);
$V_{90}$ maximum velocity (m/s) on the centreline at $y = Y_{90}$;
$x$ longitudinal flow distance (m) from intake;
$Y_{90}$ distance (m) measured normal to the bottom where $C = 0.9$;
$y$ direction (m) normal to the flow direction and to the pipe bottom;
$y'$ dimensionless distance: $y' = y/Y_{90}$;
$y''$ dimensionless distance: $y'' = y/D$;
$z$ transverse distance (m) from the channel centreline;
$z''$ dimensionless distance: $z'' = z/D$;
$\alpha$ channel slope;
$\Delta$ parametric variable;
$\theta$ angle defined in figure 1(B);
$\eta$ distance defined in figure 1(B);
$\xi$ distance defined in figure 1(B);

Subscript

*air* air flow;
*cl* centreline flow;
*w* water flow;
90 flow conditions for $C = 0.9$.