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Technical note

Effect of Froude number on bubble clustering in a hydraulic jump

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ABSTRACT
The study of bubble clustering processes may provide a significant insight into turbulent air–water flows. Previous studies investigated these processes in plunging jets, dropshafts and hydraulic jumps. This research investigates the bubble clustering process in hydraulic jumps using experimental data collected in a rectangular horizontal flume with partially developed inflow conditions for inflow Froude numbers in the range 6.5–14.3. Two criteria for cluster identification were applied: one criterion was based upon a comparison of the local instantaneous water chord time with the median water chord time, whereas the second identified a cluster if the water chord time was smaller than the air chord time of the preceding bubble, i.e. a bubble was in the near-wake of the leading bubble. The results highlight significant patterns in clusters production both over the flow depth and the distance from the jump toe. The effect of the inflow flow Froude number on the clustering process is also discussed.

Keywords: Air entrainment, bubble clustering process, Froude number, hydraulic jump, laboratory experiments

1 Introduction

A hydraulic jump is a sudden rapid transition from super- to sub-critical flow (Long *et al.* 1991, Mossa 1999, Chanson 2007). It is characterized by a significant amount of energy dissipation and air entrainment. The jump roller is formed by two distinct air–water regions, namely an air–water shear region and a recirculation region above (Figs 1 and 2). Within the air–water shear layer, momentum transfer from the high-velocity jet flow to the recirculation region above may be observed, as well as significant interactions between the entrained air and turbulence. These lead to complicated processes including bubble break-up, coalescence and clustering. The clustering process is related to the inhomogeneous bubble distribution, which has preferential concentration forming coherent structures termed clusters. In a bubbly flow, a cluster may be defined as a group of two or more bubbles with a clear separation from other bubbles up- and downstream of the cluster. In hydraulic engineering, previous investigations studied the clustering process in plunging jets (Chanson *et al.* 2006), stepped chutes (Chanson and Toombes 2002), a dropshaft (Gualtieri and Chanson 2004, 2007b), and the hydraulic jump (Chanson 2007, Gualtieri and Chanson 2007b).

Two criteria were applied to assess the occurrence of bubble clusters in hydraulic jumps. The comparative results highlight significant patterns in cluster production both over depth and distance from the jump toe. The influence of F_1 on clustering process is also discussed.

2 Experimental set-up: channel and instrumentation

The laboratory experiments were performed at the University of Queensland in a horizontal channel, 3.2 m long and 0.25 m wide (Fig. 1). Both bottom and sidewalls were made of glass panels. This channel was fed by a constant head tank. The discharge was measured with a 90° V-notch weir which was calibrated on-site with a volume-per-time technique. The water depths were measured to ±0.2 mm using rail-mounted pointer gages. The experiments were carried out for an inflow (subscript 1) Froude number $\mathsf{F}_1 = V_1/(g \times d_1)^{0.5}$ in the range 6.5–14.3, with the inflow depth d_1 and the inflow velocity V_1 ranging

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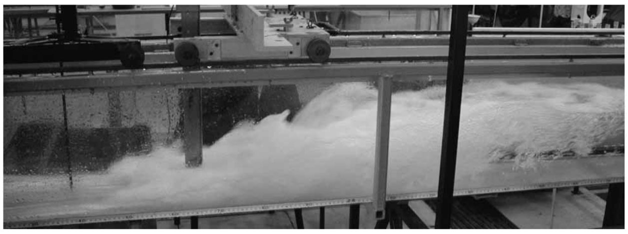

Figure 1 Hydraulic jump for $\mathsf{F}_1 = 14.3$

from 0.0119 to 0.0128 m and from 2.23 to 4.87 m/s, respectively. The air–water flow properties were measured with a single-tip conductivity probe, consisting of a sharpened rod (platinum wire Ø = 0.35 mm) which was insulated except for its tip and set into a metal supporting tube. It was excited by an electronic system designed with a response time <10 μs and calibrated with a square wave generator. The vertical probe position was adjusted by 0.1 mm increments.

The experiments yielded the void fraction C and the bubble count rate F over the depth at various distances from the jump toe on the channel centreline (Gualtieri and Chanson 2007a). The air concentration or void fraction C is the proportion of time during which the probe tip is in air, whereas the bubble count rate F = number of bubbles impacting the probe tip per second. Herein, the probe tip was horizontal and aligned with the main flow direction. The probe was scanned (subscript *scan*) at $F_{scan} = 20$ kHz during $T_{scan} = 45$ s at each sampling location. Preliminary clear-water velocity measurements using a Prandtl-Pitot tube of diameter Ø = 3.3 mm demonstrated that the supercritical inflow was partially developed for all tests with a relative boundary layer thickness δ/d_1 in the range 0.5–0.6 (Fig. 2).

3 Clustering analysis: criteria, results and discussion

Various approaches were proposed to identify a cluster structure within the air–water flow. One approach is based upon the analysis of water chord between two subsequent air particles. If two bubbles are closer than a characteristic time/length scale, they can be considered as a cluster (Chanson and Toombes 2002, Gualtieri and Chanson 2004, 2007b). This time/length scale may be related to the water chord statistics or to the bubble size itself, since bubbles within that distance are in the near-wake and may be influenced by the leading particle (Chanson and Toombes 2002, Chanson *et al.* 2006, Gualtieri and Chanson 2007b). In the hydraulic jump, it is difficult to ascertain the direction of motion of each individual bubbles, and the analysis must be conducted in terms of chord times.

Two criteria were herein applied to detect the occurrence of clusters in the air–water flow:

- Water chord between two subsequent air particles was compared with the median water chord recorded in the point of measurement. According to Criterion 1, a cluster was detected if:

$$t_{ch\text{-}w} < \left(\frac{1}{10}\right) t_{ch\text{-}w\text{-}median} \quad (1)$$

  where $t_{ch\text{-}w\text{-}median}$ is the median water chord time,

- Water chord time between two subsequent air particles was compared with the air chord of the preceding bubble recorded in the point of measurement. According to Criterion 2, a cluster was detected if:

$$t_{ch\text{-}w} < \eta\, t_{ch\text{-}ab} \quad (2)$$

  where $t_{ch\text{-}ab}$ is the air chord time of the leading bubble with η the parameter characterizing the wake timescale of the leading bubble, which for pseudo-spherical particles is in the range 0.5–2.0. It was assumed herein that $\eta = 1$.

The results of the clustering analysis were expressed in terms of the dimensionless number of clusters per second $(N_c/s) \times (d_1/V_1)$, if N_c is equal to the number of clusters detected in the measurement point over a sampling time s, percentage of clustered bubbles relative to the total number of detected

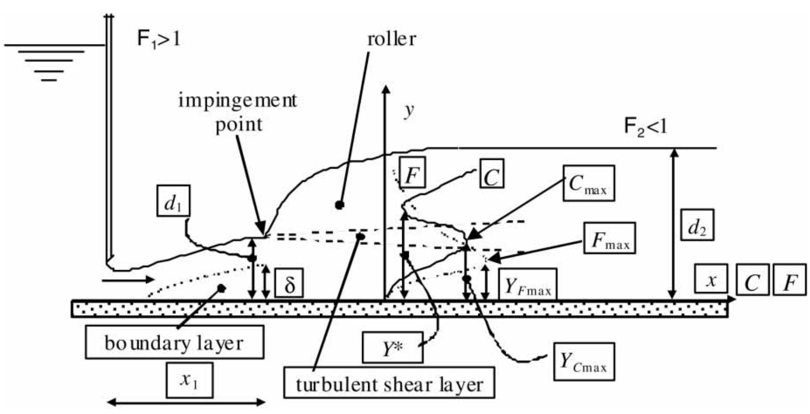

Figure 2 Sketch of hydraulic jump flow with partially developed inflow conditions

bubbles and number of bubbles per cluster. The locations with the maximum clustering in terms of these properties were compared with the locations where the local void fraction and bubble count rate maxima, $C_{\max}$ and $F_{\max}$, respectively, were recorded.

The existence of clusters is related to jet break-up, coalescence and bubble wake interference. As the bubble response time is significantly smaller than the characteristic time of the flow, bubble clustering tends to be caused primarily by bubble trapping in vortical structures. In plunging jet and hydraulic jumps, such large-scale vortices are generated in the developing shear layers. As the vortical, coherent structures are advected downstream, they grow up in size by vortex pairing and contribute to further clustering (Gualtieri and Chanson 2007a).

Figure 3 shows vertical distributions of the number of clusters per second N_c for $\mathsf{F}_1 = 6.51$ for the two cluster criteria (Eqs 1 and 2), where y is the vertical elevation above the invert and d_1 the inflow depth. In Fig. 3, the horizontal axis is the dimensionless number of clusters per second $(N_c/s) \times (d_1/V_1)$. Figures 4 and 5 show results of the clustering analysis for $\mathsf{F}_1 = 10.8$ and 14.3. Overall, the clustering analysis regrouped 269 records from 18 vertical profiles (Table 1).

Earlier studies demonstrated that an air diffusion region exists in which the void fraction distributions follow an analytical solution of the classical advection–diffusion equation (Chanson 1995, Murzyn *et al.* 2007). Above this air diffusion layer, i.e. for $y > Y^*$, where Y^* is the upper vertical boundary of the air diffusion layer, there is the upper free-surface region where the void fraction increases rapidly to the unity (Fig. 2).

The dimensionless number of clusters per second varied between the two considered criteria. For $\mathsf{F}_1 = 6.5$, it ranged from 0.0026 to 0.0102 and from 0.0025 to 0.0153 for the Criteria 1 and 2, respectively. For $\mathsf{F}_1 = 10.8$, it ranged from 0.0031 to 0.0116 and from 0.0023 to 0.0355 for the Criteria 1 and 2, respectively. Finally, for $\mathsf{F}_1 = 14.3$, it was ranging from 0.0035 to 0.0103 and from 0.0037 to 0.0383 for the Criteria 1 and 2, respectively.

Generally, the lowest values were observed at the largest distance from the jump toe for all F_1 and for both cluster criteria. The lowest values were quite similar for both criteria. In the average, the dimensionless number of clusters per second was for Criterion 1 about 0.0065, 0.0088 and 0.0086 for $\mathsf{F}_1 = 6.5$, 10.8 and 14.3, respectively, whereas it was 0.0079, 0.0207 and 0.0223 for $\mathsf{F}_1 = 6.5$, 10.8 and 14.3, respectively, for Criterion 2, indicating that the clustering process tends to increase with F_1.

For Criterion 2, the location of the maximum dimensionless number of clusters per second $Y_{Nc\text{-}\max}$ was mostly close to the location of maximum bubble count rate in the shear region, i.e. $Y_{F\max}/d_1$ (Table 1). Usually, this location is higher than that of the maximum void fraction, i.e. $Y_{C\max}/d_1$ (Gualtieri and Chanson 2007a, Murzyn *et al.* 2007). Figure 6 shows the dimensionless longitudinal profiles of maximum dimensionless number of clusters per second in hydraulic jump flows. The values from Criterion 2 were always larger than those of Criterion 1. Independently of the clustering criterion, the maximum number of clusters per second decreased with increasing distance from the jump toe and decreased with decreasing inflow Froude number F_1 at a given dimensionless distance $(x - x_1)/d_1$.

The results include further the percentage of clustered bubbles. The averaged percentage of clustered bubbles for Criterion 1 was about 32, 20 and 14% if $\mathsf{F}_1 = 6.5$, 10.8 and 14.3, respectively. For Criterion 2, it was 35, 41 and 41% if $\mathsf{F}_1 =$ 6.5, 10.8 and 14.3, respectively. Overall the results indicate that the percentage of clustered bubbles was in average of about 20 and 39% for Criteria 1 and 2, respectively. Furthermore, the average number of bubbles per cluster was about 2.3 and 2.5 for Criteria 1 and 2, respectively, demonstrating that cluster structures are mostly formed by two bubbles. The percentage of clusters made of two bubbles was from 79 to 94% with an overall average value of 88% for Criterion 1 and from 70 to 91% with an overall average value of 81% for Criterion 2. These results are consistent with those obtained for dropshafts and stepped chutes. In a dropshaft, the percentage of clusters formed by two bubbles ranged from 76 to 100% with an average of 92% for Criterion 1, whereas for Criterion 2 it was in the range from 64 to 96%, with an average of 80%. For skimming and transition flows, the clusters made of two bubbles accounted for nearly 68 and 78% of all clusters, respectively (Chanson and Toombes 2002), and between 79 and 84% of all clusters in circular plunging jet flows (Chanson *et al.* 2006).

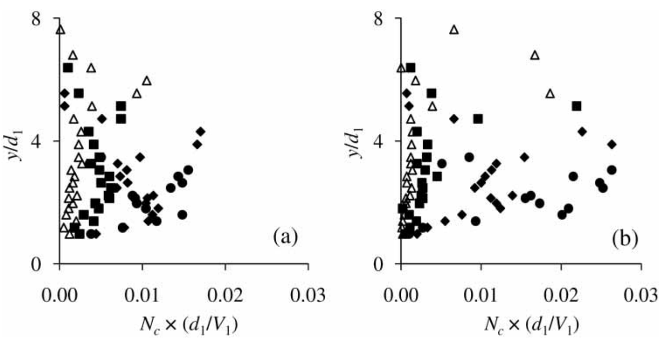

Figure 3 Number of clusters for $\mathsf{F}_1 = 6.51$ and $(x - x_1)/d_1 = 4.17$ (●), 8.33 (◆), 12.5 (■) and 16.7 (△), based on Criterion (a) 1 and (b) 2

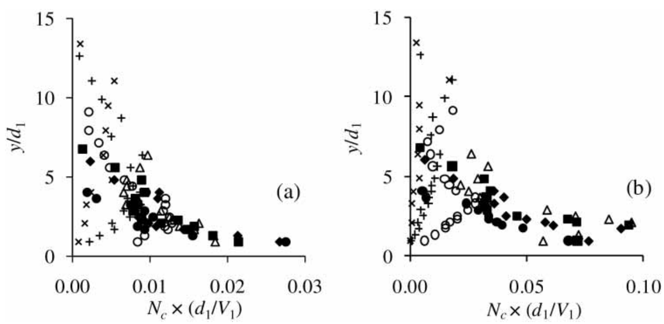

Figure 4 Number of clusters for $\mathsf{F}_1 = 10.8$ and $(x - x_1)/d_1 = 3.91$ (●), 7.81 (◆), 11.7 (■), 15.6 (△), 27.3 (○), 39.1 (+) and 50.8 (×), based on Criterion (a) 1 and (b) 2

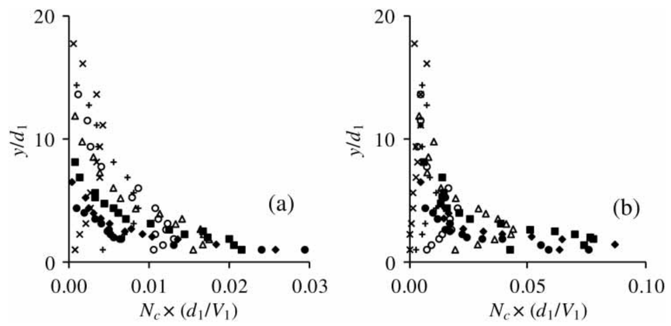

Figure 5 Number of clusters for $F_1 = 14.3$ and $(x - x_1)/d_1 = 4.20$ (●), 8.40 (◆), 16.8 (■), 29.4 (△), 42.0 (○), 54.6 (+) and 67.2 (×), based on Criterion (a) 1 and (b) 2

Overall the comparison between the two cluster criteria indicated that the formation of cluster structures is a frequent feature of the air–water flow in the hydraulic jump flows and a significant number of bubbles travelled inside a cluster structure. Criterion 2, based upon the near-wake concept, may be considered as more relevant because it relies on a comparison between the *local* characteristic flow scales, namely the water chord and the air chord of the preceding bubble. Criterion 1 provides a comparison between a *local* characteristic time, such as the water chord time, and a *time-averaged* characteristic time of the flow, such as the median value of the water chord time recorded at a certain point. The locations for $N_{c\text{-max}}$ provided by Criterion 2 are inside the turbulent shear layer implying that the clustering process is most intense where maximum turbulence exists. Finally, both criteria confirmed that most cluster structures were formed by only two bubbles.

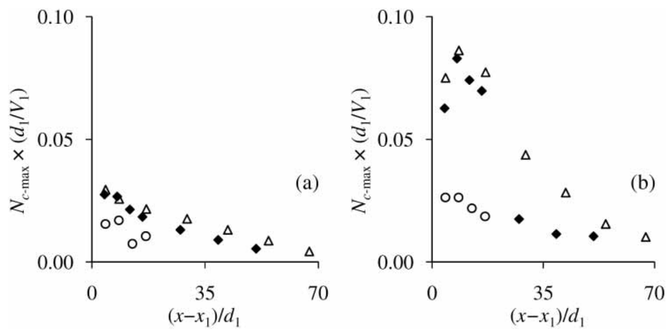

Figure 6 $N_{c\text{-max}}$ for $F_1 =$ (○) 6.51, (◆) 10.8 and (△) 14.3 and Criterion (a) 1, (b) 2

4 Conclusions

Clusters are a characteristic feature of interactions between turbulence, particles and bubbles in a hydraulic jump. They influence the surrounding flow field introducing enhanced velocity fluctuations and hydrodynamic interactions. Thus, the study of the clustering process provides significant insights into air–water flows of hydraulic engineering.

The results of a comprehensive clustering analysis are presented in which two criteria were applied to identify the presence of bubble cluster structures within temporal series of air bubbles and water particles. Criterion 1 is based on a comparison of the *local*, instantaneous water chord time with a *time-averaged* characteristic water timescale, whereas Criterion 2 identifies a cluster if a bubble is in the *near-wake* of the preceding bubble. The results highlight that the formation of cluster structures is a common characteristic in hydraulic jumps and that a large

Table 1 Comparison between $Y_{Nc\text{-max}}/d_1$, $Y_{C\max}/d_1$, $Y_{F\max}/d_1$ and Y^*/d_1

					$Y_{Nc\text{-max}}/d_1$	
F_1	$(x - x_1)/d_1$	$Y_{C\max}/d_1$	$Y_{F\max}/d_1$	Y^*/d_1	Criterion 1	Criterion 2
6.51	4.17	–	1.60	–	3.05	3.05
	8.33	2.85	1.97	3.47	4.30	4.30
	12.5	2.85	2.85	4.30	4.72	5.14
	16.7	3.26	3.26	4.72	5.97	5.55
10.8	3.91	–	0.91	–	0.91	1.30
	7.81	2.08	1.30	2.86	0.91	1.30
	11.7	1.69	1.30	3.25	0.91	1.30
	15.6	2.86	1.69	4.43	0.91	1.69
	27.3	3.65	3.25	6.38	2.28	3.25
	39.1	4.82	4.82	8.72	4.82	11.1
	50.8	–	11.1	9.50	11.1	11.1
14.3	4.20	1.40	0.98	2.24	0.98	0.98
	8.40	1.40	1.40	3.08	0.98	1.40
	16.8	2.24	1.82	4.76	0.98	1.82
	29.4	3.92	2.24	7.28	1.82	2.66
	42.0	6.02	3.50	9.38	1.82	4.34
	54.6	5.60	5.60	9.38	4.34	5.60
	67.2	8.12	8.12	9.38	11.1	16.11

proportion of the bubbles travel within cluster structures. Moreover, independent of the clustering criterion, the maximum number of clusters per second decreases with increasing distance from the jump toe and decreases with decreasing inflow Froude number F_1. This demonstrates the effect of F_1 on the clustering process in a hydraulic jump. The second cluster criterion appeared to be most effective because it relies on a comparison between the local characteristic flow times. The maximum clustering rate was observed within the turbulent shear layer, suggesting that the clustering process is most intense in the regions of large turbulent shear stresses. Both criteria indicate a majority of cluster structures consisting of only two bubbles, although the criterion solely relates to a longitudinal bubbly flow structure. The present results are consistent with earlier obtained in different air–water flows.

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Notation

C = void fraction (–)
d_1 = inflow depth (m)
F = bubble count rate (Hz)
F_1 = Froude number (–)
F_{scan} = sampling rate (Hz)
g = gravitational acceleration (m/s^2)
N_c = number of clusters (–)
$t_{ch\text{-}ab}$ = air chord time of the leading bubble (s)
$t_{ch\text{-}w}$ = water chord time (s)
$t_{ch\text{-}w\text{-}median}$ = median water chord time (s)
T_{scan} = sampling duration (s)
V_1 = inflow velocity (m/s)
x = streamwise distance from gate (m)
x_1 = streamwise distance of impingement point from gate (m)
y = vertical elevation (m)
$y_{C\max}$ = vertical location with maximum C (m)
$y_{F\max}$ = vertical location with maximum F (m)
$y_{Nc\text{-}\max}$ = vertical location with maximum N_c (m)
Y^* = upper vertical boundary of air diffusion layer (m)

Greek symbols

δ = boundary layer thickness (m)
η = timescale parameter of the wake of the leading bubble

Subscripts

c = cluster
max = maximum
scan = scanning
1 = refers to inflow condition

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