Journal of Hydraulic Research Vol. 50, No. 2 (2012), pp. 244–246
http://dx.doi.org/10.1080/00221686.2012.654671

Discussion

Scale effects in physical hydraulic engineering models

By VALENTIN HELLER, Journal of Hydraulic Research, Vol. 49, No. 3 (2011), pp. 293–306

Discussers:
MICHAEL PFISTER (IAHR Member), *Laboratory of Hydraulic Constructions (LCH), Ecole Polytechnique Fédérale de Lausanne (EPFL), CH-1015 Lausanne, Switzerland.*
Email: michael.pfister@epfl.ch (author for correspondence)

HUBERT CHANSON (IAHR Member), *School of Civil Engineering, The University of Queensland, Brisbane, Queensland 4072, Australia.*
Email: h.chanson@uq.edu.au

Beside analytical approaches, physical modelling represents probably the oldest design tool in hydraulic engineering. It is thus a pleasure to see this Forum Paper in JHR. The Discussers focus on one aspect of the publication, thereby specifying the information of the Forum Paper.

Free surface flows are typically scaled with the Froude similitude keeping identical $\mathsf{F} = V/(gh)^{0.5}$ both in the model and in the prototype. The air transport in models is affected by scale effects because the internal flow turbulence, represented by the Reynolds number $\mathsf{R} = Vh/\nu$, is underestimated, while surface tension, represented by the Weber number $\mathsf{W} = (\rho V^2 h)/\sigma$, is overestimated (Chanson 2009), with V = flow velocity, g = gravity constant, h = flow depth, ρ = water density, σ = water surface tension, and ν = water kinematic viscosity. Because a strict dynamic similitude exists only at a full-scale, the underestimation of the air transport is minimized if limitations in terms of W or R are respected.

The Forum Paper overlooks a number of aspects and probably recommends too optimistic limitations. As stated in Table D1, the literature mentions limitations around $\mathsf{W}^{0.5} = 110$–170 and $\mathsf{R} = 1.0$–2.5×10^5. These values focus on air entrainment at hydraulic jumps, general chute air entrainment and aerated stepped spillway flows, as well as the air entrainment coefficient β and the streamwise bottom air concentration C_b generated by chute aerators. Pfister and Hager (2010a, b) identified an underestimation up to one magnitude in terms of C_b if $\mathsf{W}^{0.5} < 140$ (Fig. D1). There, the abscissa corresponds to the streamwise normalization given by these authors, and the trend lines correspond to the best fit of all C_b curves from tests with $\mathsf{W}^{0.5} \geq 140$, i.e. without significant scale effects.

As can be noted from Table D1, *two* criteria are often applied relating to the herein discussed scale effects, i.e. limiting values for $\mathsf{W}^{0.5}$ and R for a range of air–water flow parameters. This results in an over-determined system, as the two numbers depend on each other, besides F and the Morton number M. The latter characterizes the shape of bubbles or drops moving in a surrounding medium, solely as a function of the fluid properties and the gravity constant (Wood 1991, Chanson 1997). With a negligible inner bubble density, as is typical for air–water flows, the Morton number is with μ = dynamic water viscosity

$$\mathsf{M} = \frac{g\mu^4}{\sigma^3\rho} = \frac{\mathsf{W}^3}{\mathsf{F}^2\mathsf{R}^4} \tag{D1}$$

For air–water two-phase flows $\mathsf{M} = 3.89 \times 10^{-11}$. If using the Froude similitude: (1) M = constant, and (2) F is similar in the model and the prototype. Isolating these two numbers results in

$$\mathsf{MF}^2 = \frac{\mathsf{W}^3}{\mathsf{R}^4} \tag{D2}$$

For a given F, the right-hand side of Eq. (D2) thus has to be identical both in the model and in the prototype flows. The theoretical function MF^2 versus F is shown in Fig. D2(a). The theoretical MF^2 values (curve) are identical with the experimentally derived $\mathsf{W}^3/\mathsf{R}^4$ values (symbols; Pfister and Hager 2010a, 2010b), as expected from Eq. (D2). To visualize the limitations, Fig. D2(b) shows the measured $\mathsf{W}^{0.5}$ versus R, yet omitting the effect of

ISSN 0022-1686 print/ISSN 1814-2079 online
http://www.tandfonline.com

Table D1 Limitations to avoid significant scale effects in two-phase air–water flows under Froude similitude

Reference	Limitation	Air–water flow parameter	Application range
Kobus (1984)	$\mathsf{R} \geq 1.0 \times 10^5$	Air transport rate	Chute air entrainment
Koschitzky (1987)	$\mathsf{R} \geq 1.0 \times 10^5$	Air demand flow rate	Aerators, particularly β
Rutschmann (1988)	$\mathsf{W}^{0.5} \geq 110$	Air demand flow rate	Aerators, particularly β
Skripalle (1994)	$\mathsf{W}^{0.5} \geq 170$	Air demand flow rate	Aerators, particularly β
Boes (2000)	$\mathsf{R} \geq 1.0 \times 10^5$	Void fraction and interfacial velocity	Two-phase stepped spillway flow
Murzyn and Chanson (2008)	$\mathsf{R} > 1.0 \times 10^{5a}$	Void fraction, interfacial velocity, bubble count rate, turbulence intensity, bubble chord time	Hydraulic jumps
Felder and Chanson (2009)	$\mathsf{R} > 2.5 \times 10^{5a}$	Void fraction, interfacial velocity, bubble count rate, turbulence intensity, integral turbulent time scale, bubble chord size	Two-phase stepped spillway flow
Pfister and Hager (2010a)	$\mathsf{R} \geq 2.2 \times 10^5$, $\mathsf{W}^{0.5} \geq 140$	Void fraction	Aerators, C_b development

[a] Incomplete limitation since an asymptotic result was not achieved.

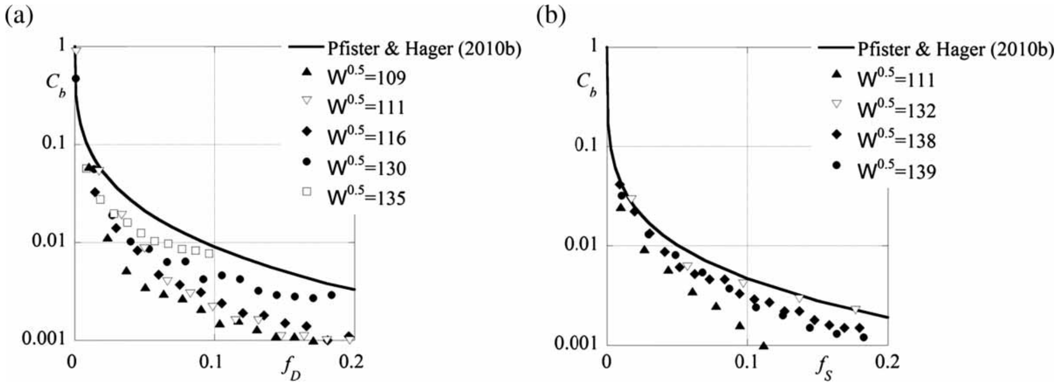

Figure D1 Bottom air concentration C_b curves versus normalization function, downstream of (a) deflector and (b) drop chute aerators, with trend line for unaffected tests and symbols for tests affected by scale effects

F, which is responsible for the data scatter. Note that all data affected by scale effects concentrate below the aforementioned limitations.

A transformation of Eq. (D1) gives the direct relation between W and R as

$$\mathsf{R} = \left(\frac{\mathsf{W}^3}{\mathsf{F}^2\mathsf{M}}\right)^{0.25} \tag{D3}$$

Inserting the limitations $\mathsf{W}^{0.5} = 110$, 140 and 170 from Table D1 in Eq. (D3) results in the related R-curves as a function of F, given in Fig. D3. Note that, for typical air–water chute flows with $5 \leq \mathsf{F} \leq 15$, scale effects are small if $\mathsf{W}^{0.5} > 140$ or $\mathsf{R} > 2$ to 3×10^5. The limits are not sensitive to F in this range, whereas more restrictive limitations of R have to be applied for smaller values of F.

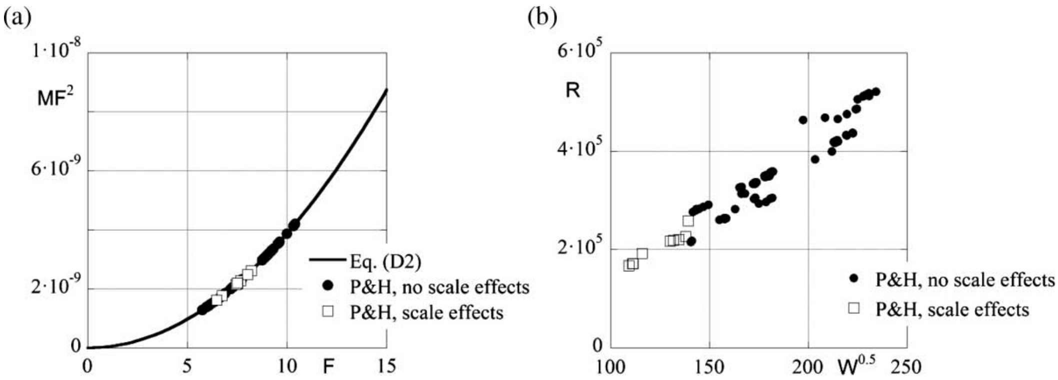

Figure D2 (a) MF^2 curve versus F (curve) and $\mathsf{W}^3/\mathsf{R}^4$ from measurements versus F (symbols), with P&H for Pfister and Hager (2010a, 2010b) and (b) R versus $\mathsf{W}^{0.5}$ ignoring effect of F

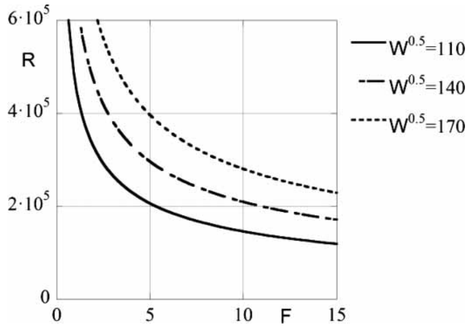

Figure D3 Visualization of Eq. (D3) for various $W^{0.5}$ giving R versus F

Further, the Forum Paper does not state the parameters required to assess scale effects. The limitations for scale effects in terms of turbulent properties and bubble sizes are more important than those in terms of void fraction and interfacial velocity (Chanson 2009). One may thus conclude that the limitations relevant for high-speed air–water two-phase flows using the Froude similitude are either $W^{0.5} > 140$ or $R > 2$ to 3×10^5. By considering only one limitation, the other is implicit.

References

Boes, R.M. (2000). Scale effects in modelling two-phase stepped spillway flow. *Hydraulics of stepped spillways*, 53–60, H.E. Minor, W.H. Hager, eds. Balkema, Rotterdam.

Chanson, H. (1997). *Air bubble entrainment in free-surface turbulent shear flows*. Academic Press, London UK.

Chanson, H. (2009). Turbulent air-water flows in hydraulic structures: dynamic similarity and scale effects. *Environ. Fluid Mechanics* 9(2), 125–142.

Felder, S., Chanson, H. (2009). Turbulence, dynamic similarity and scale effects in high-velocity free-surface flows above a stepped chute. *Experiments in Fluids* 47(1), 1–18.

Kobus, H. (1984). Local air entrainment and detrainment. Symp. *Scale Effects in Modelling Hydraulic Structures* 4.10, 1–10, H. Kobus, ed. Technische Akademie, Esslingen.

Koschitzky, H.-P. (1987). Dimensionierungskonzept für Sohlbelüfter in Schussrinnen zur Vermeidung von Kavitationsschäden (Design concept for chute aerators to avoid cavitation damage). *Mitteilung* 65. Institut für Wasserbau, TU: Stuttgart [in German].

Murzyn, F., Chanson, H. (2008). Experimental assessment of scale effects affecting two-phase flow properties in hydraulic jumps. *Experiments in Fluids* 45(3), 513–521.

Pfister, M., Hager, W.H. (2010a). Chute aerators I: Air transport characteristics. *J. Hydraulic Eng.* 136(6), 352–359.

Pfister, M., Hager, W.H. (2010b). Chute aerators II: Hydraulic design. *J. Hydraulic Eng.* 136(6), 360–367.

Rutschmann, P. (1988). Belüftungseinbauten in Schussrinnen (Chute aerators). *Mitteilung* 97. Laboratory of Hydraulics, Hydrology and Glaciology, D. Vischer, ed. ETH: Zurich [in German].

Skripalle, J. (1994). Zwangsbelüftung von Hochgeschwindigkeitsströmungen an zurückspringenden Stufen im Wasserbau (Forced aeration of high-speed flows at chute aerators). *Mitteilung* 124. Technische Universität, Berlin [in German].

Wood, I.R. (1991). Air entrainment in free-surface flows. IAHR *Hydraulic Structures Design Manual* 4. Balkema, Rotterdam.

Reply by the Author

The Discussion is appreciated. The Author thinks that it adds to the information presented in the Forum Paper and that it includes valuable additions to the limiting criteria to avoid significant scale effects in free surface air-water flows.

http://dx.doi.org/10.1080/00221686.2012.654672