Unsteady Open Channel Flows. 2- Applications

A 0.2 m high small wave propagates downstream in a horizontal channel with initial flow conditions \( V = +0.1 \text{ m/s} \) and \( d = 2.2 \text{ m} \). Calculate the propagation speed of the small wave.

Solution
\[ U = +5.1 \text{ m/s}, C = 4.64 \text{ m/s}. \]
Textbook pp. 319-321.

Uniform equilibrium flow conditions are achieved in a long rectangular channel (\( W = 12.8 \text{ m} \), concrete lined, \( S_o = 0.0005 \)). The observed water depth is 1.75 m. Calculate the celerity of a small monoclinal wave propagating downstream. *Perform your calculations using the Darcy friction factor.*

Solution
\[ Q = 44.7 \text{ m}^3/\text{s}, U = +3.0 \text{ m/s}. \]

The flow rate in a rectangular canal (\( W = 3.4 \text{ m} \), concrete lined, \( S_o = 0.0007 \)) is 3.1 \( \text{ m}^3/\text{s} \) and uniform equilibrium flow conditions are achieved. The discharge suddenly increases to 5.9 \( \text{ m}^3/\text{s} \). Calculate the celerity of the monoclinal wave. How long will it take for the monoclinal wave to travel 20 km? *Perform your calculations using the Darcy friction factor.*

Solution
\[ d_2 = 1.6 \text{ m}, U = +1.88 \text{ m/s}, t = 10,660 \text{ s} \text{ (3 h.)}. \]

Considering a long, horizontal rectangular channel (\( W = 4.2 \text{ m} \)), a gate operation, at one end of the canal, induces a sudden withdrawal of water resulting in a negative velocity. At the gate, the boundary conditions for \( t > 0 \) are: \( V(x = 0, t) = -0.2 \text{ m/s} \). Calculate the extent of the gate operation influence in the canal at \( t = 1 \text{ hour} \). The initial conditions in the canal are: \( V = 0 \) and \( d = 1.4 \text{ m} \).

Water flows in an irrigation canal at steady state (\( V = 0.9 \text{ m/s} \), \( d = 1.65 \text{ m} \)). The flume is assumed smooth and horizontal. The flow is controlled by a downstream gate. At \( t = 0 \), the gate is very-slowly raised and the water depth upstream of the gate decreases at a rate of 5 cm per minute until the water depth becomes 0.85 m. (1) Plot the free-surface profile at \( t = 10 \text{ minutes} \). (2) Calculate the discharge per unit width at the gate at \( t = 10 \text{ minutes} \).

Solution (Textbook pp. 322-327)
The simple wave problem corresponds to a negative surge. In absence of further information, the flume is assumed wide rectangular. Let select a coordinate system with \( x = 0 \) at the gate and \( x \) positive in the upstream direction. The initial flow conditions are: \( V_0 = -0.9 \) and \( C_0 = 4.0 \text{ m/s} \). In the \((x, t)\) plane, the equation of the initial forward characteristics (issuing from \( x = 0 \) and \( t = 0 \)) is given:
\[
\frac{d t}{d x} = \frac{1}{V_0 + C_0} = 0.32 \text{ s/m}
\]
At \( t = 10 \text{ minutes} \), the maximum extent of the disturbance is \( x = 1,870 \text{ m} \). That is, the zone of quiet is defined as \( x > 1.87 \text{ km} \). At the gate (\( x = 0 \)), the boundary condition is: \( d(x=0, t_0 \leq t) = 1.65 \text{ m}, d(x = 0, t_0) = 1.65 - 8.33E-4t_0 \), for \( 0 < t_0 \leq 960 \text{ s} \), and \( d(x=0, t_0 \geq 960 \text{ s}) = 0.85 \text{ m} \). The second flow property is calculated using the backward characteristics issuing from the initial forward characteristics and intersecting the boundary at \( t = t_0 \).
\[ V(x=0, t_0) = V_0 + 2 \times (C(x=0, t_0) - C_0) \quad \text{Backward characteristics} \]

where \( C(x, t_0) = \sqrt{g \times d(x=0, t_0)} \).

At \( t = 10 \) minutes, the flow property between \( x = 0 \) and \( x = 1.87 \) km are calculated from:

\[ V(x, t = 600) + 2 \times C(x, t = 600) = V(x=0, t_0) + 2 \times C(x=0, t_0) \quad \text{forward characteristics} \]

\[ V(x, t = 600) - 2 \times C(x, t = 600) = V_0 - 2 \times C_0 \quad \text{backward characteristics} \]

where the equation of the forward characteristics is:

\[ t = t_0 + \frac{x}{V(x=0, t_0) + C(x=0, t_0)} \quad \text{forward characteristics} \]

These three equations are three unknowns: \( V(x, t = 600) \), \( C(x, t = 600) \) and \( t_0 = t(x=0) \) for the C1 characteristics. The results of the calculation at \( t = 12 \) minutes are presented in the below.

The flow rate at the gate is \(-2.56 \) m\(^2\)/s at \( t = 600 \) s. The negative sign shows that the flow direction is in the negative x-direction.

<table>
<thead>
<tr>
<th>( t_0 ) (x=0)</th>
<th>( d(x=0) )</th>
<th>( C(x=0) )</th>
<th>( V(x=0) )</th>
<th>Fr (x=0)</th>
<th>( x )</th>
<th>( V(x) )</th>
<th>( C(x) )</th>
<th>( d(x) )</th>
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</thead>
<tbody>
<tr>
<td>C1 (1)</td>
<td>C2 (2)</td>
<td>Fr (4)</td>
<td>t=10min (6)</td>
<td></td>
<td></td>
<td>t=10min (7)</td>
<td>t=10min (8)</td>
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<tr>
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<td>4.02</td>
<td>-0.90</td>
<td>-0.22</td>
<td>1873</td>
<td>-0.90</td>
<td>4.02</td>
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<tr>
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<td>3.96</td>
<td>-1.02</td>
<td>-0.26</td>
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<td>-0.29</td>
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<td>-2.23</td>
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<td>0</td>
<td>-2.23</td>
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<td>1.15</td>
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</table>

A 200 km long rectangular channel (W = 3.2 m) has a reservoir at the upstream end and a gate at the downstream end. Initially the flow conditions in the canal are uniform: \( V = 0.35 \) m/s, \( d = 1.05 \) m. The water surface level in the reservoir begins to rise at a rate of \( 0.2 \) m per hour for 6 hours. Calculate the flow conditions in the canal at \( t = 2 \) hours. Assume \( S_o = S_f = 0 \).

Solution

<table>
<thead>
<tr>
<th>( x )</th>
<th>( V(x) )</th>
<th>( C(x) )</th>
<th>( d(x) )</th>
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</thead>
<tbody>
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<td>23841</td>
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<td>21630</td>
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<tr>
<td>0</td>
<td>1.47</td>
<td>3.77</td>
<td>1.45</td>
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</tbody>
</table>

The flow situation corresponds to a positive surge formation. However the wave does not have time develop and steepen enough in 2 hours to form a discontinuity (i.e. surge front) ahead. In other terms, the forward characteristics issuing from the reservoir do not intersect for \( t \leq 2 \) hours.

2004 Examination paper

Considering the propagation of a tidal bore in an estuary, the river flow conditions prior to the bore arrival are: \( Q = 52 \) m\(^3\)/s, \( d = 1.15 \) m, \( B = 95 \) m. The river channel is assumed to be horizontal and rectangular. The tidal bore arrives and
propagate upstream. Its celerity is measured by an observer standing on the right bank and recorded as 3.35 m/s (positive upstream).

(a) Calculate the new flow depth and flow velocity immediately shortly the passage of the bore. Indicate clearly the direction of the flow after the passage of the bore.

(b) What type of bore would the observer see? Justify your answer.

Waters flow in a horizontal, smooth rectangular channel. The initial flow conditions are \( d = 2.1 \) m and \( V = +0.3 \) m/s. The flow rate is stopped by some gate closure at the downstream end of the canal. The downstream gate is closed slowly at a rate corresponding to a linear decrease in flow rate from 0.63 m\(^2\)/s down to zero in 15 minutes. (1) Predict the surge front development. (2) Calculate the free-surface profile at \( t = 1 \) hour after the start of gate closure.

A 5 m wide forebay canal supplies a penstock feeding a Pelton turbine. The initial conditions in the channel are \( V = 0 \) and \( d = 2.5 \) m. (1) The turbine starts suddenly operating with 6 m\(^3\)/s. Predict the water depth at the downstream end of the forebay canal. (2) What is the maximum discharge that the forebay channel can supply? Use a simple wave theory.

Solution (textbook pp. 335-339)

(1) \( d(x=0) = 2.24 \) m. (2) \( Q = -18.3 \) m\(^3\)/s.